Numerical and Mathematical Modeling of
Reactive Mass Transfer and Heat Storage
Installations of Palms Waste

Samir Men-La-Yakhaf
Team of modeling in fluid mechanics and environment, LPT, URAC 13
Faculty of sciences, Mohammed V University, Rabat-Agdal
B.P. 1014, Rabat, Morocco

Kamal Gueraoui
Team of modeling in fluid mechanics and environment, LPT, URAC 13
Faculty of sciences, Mohammed V University, Rabat-Agdal
B.P. 1014, Rabat, Morocco
Department of Mechanical Engineering, University of Ottawa, Ottawa, Canada
Corresponding author

Mohamed Driouich
Team of modeling in fluid mechanics and environment, LPT, URAC 13
Faculty of sciences, Mohammed V University, Rabat-Agdal
B.P. 1014, Rabat, Morocco

Mohamed Sammouda
Team of modeling in fluid mechanics and environment, LPT, URAC 13
Faculty of sciences, Mohammed V University, Rabat-Agdal
B.P. 1014, Rabat, Morocco

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Abstract
The treatment of palms waste is based on simple principle known as biodegradation. The main objective of this study is to improve the management of
bioreactors. We have studied the thermo-hydro-biological behavior of waste in the anaerobic phase and we have presented the equations of conservation of matter, energy and the thermodynamic functions. We have developed a numerical model to simulate the coupled behavior of bioreactors using the finite volume method.

**Keywords:** palms waste, anaerobic phase, numerical model, reactive mass transfer, heat transfer

**Nomenclatures**

- $K_{ip}$: intrinsic permeability (m²)
- $K_{rp}$: relative permeability (m²)
- $\mu_p$: dynamic viscosity of each phase (liquid, gas)
- $\rho_p$: density of each phase (liquid, gas) (kg/m³).
- $P_p$: pressure of each phase (liquid, gas) (Pa)
- $S_l$: liquid saturation.
- $S_r$: residual saturation
- $S_e$: effective saturation
- $M_g$: molecular mass (kg/mol)
- $R$: universal gas constant (J/mol.K)
- $n$: porosity of the medium
- $A_i$: fraction of the waste
- $C_g$: potential the production of biogas (m³/Kg)
- $\alpha_g$: term the production of biogas
- $V_p$: velocity of moving fluid (m/s)
- $Q$: volume flow (Wm⁻³)
- $(\rho C_p)r$: volumetric heat capacity (Jm⁻³.k⁻¹)
- $T$: Temperature, K
- $(\rho C_p)*$: volumetric heat capacity (Jm⁻³.k⁻¹)

**1 Introduction**

In this work, we have developed a numerical model based on the finite volume method (Benard and Eymard, 2005[8]; Eymard and al., 2000[9], Gueraoui and al. 1996[15]; Ghouli and al., 2009[16]) incorporating basic concepts from hydrodynamics and microbiology to simulate the hydraulic, thermal and biological behavior of anaerobic bioreactor landfills during the methanogenic phase.

The equations that govern our problem are the equations of conservation of matter, equation of energy, and the thermodynamic functions.
The development of such techniques requires knowledge of the parameters which influence the process and degradation reactions, conditioned by numerous environmental parameters. These parameters are now fairly well known, but their effects are still difficult quantifiable because of their mutual dependence (Aran, 2000)[1]. According to literature, the two main factors controlling the most waste degradation and biogas production are temperature, humidity of waste (Farquhar and Rovers (1973)[14], Rees (1980)[15], Jokela and al (1999), McDougall and Pyrah (1999)[10] and more particularly the distribution of leach-ate within the waste mass (Klink and Ham, 1982)[12].

Mathematical models which have been developed previously (El Fadel and Findikakis, 1996[9]; Vavilin et al., 2002[3]) are usually based on constant values of temperature in controlled laboratory conditions or controlled industrial digesters and so they can hardly be applied directly to landfills where temperature is the result of coupled processes. In these models the growth and decay rates of biomass are supposed to be constant at a given temperature.

2 Mathematical formulation

The main objective is the modeling of biphasic flows without a phase change. For this, we present the equations of conservation of matter, energy and the thermodynamic functions.

3 Generalized Darcy law

The velocity of the fluid phases flowing in a porous medium can be expressed by using the generalized Darcy law, Aran, C. (2000)[4]:

\[ V_p (m/s) = -\frac{K_{ip}K_{r,p}}{\mu_p} (\nabla P_p - \rho_p g) \] (1)

Mualem [13] proposes a simple analytical model to predict the relative permeability curves of water from the capillary pressure curves as a function of the effective saturation, associated with the model from Burdine [16], Brooks and Corey [15] gives:
The parameter $\lambda > 0$ describes the distribution of the pores. The low values of $\lambda$ indicate that the porous medium compound a single type of grain, while large values of $\lambda$ indicate the great non-uniformity of the porous medium and the effective saturation of Se is defined by:

$$S_e = \frac{S_l - S_r}{1 - S_r}$$

The density of gas can be defined as follows:

$$\rho_g = \frac{M_g P_g}{RT}$$

### 4 Conservation of mass

Consider the case where the porous medium is composed of the solid matrix, a liquid and a gas. The conservation of each constituent (liquid, gas) writes (Shabnam Gholamifard (2009))[1]:

$$\begin{cases}
\frac{\partial m_l}{\partial t} + \nabla \cdot (\rho_l V_l) = 0 \\
\frac{\partial m_g}{\partial t} + \nabla \cdot (\rho_g V_g) = \alpha_g
\end{cases}$$

(2)

with $m_l = n_S \rho_l$ and $m_g = n(1 - S_l) \rho_g$

Here $n$ is the porosity of the medium, and $\rho_l$, $\rho_g$ the density of liquid and gas phases. $V_p$ is the rate of filtration of P the phase defined by Darcy's law and $\alpha_g$ the term the production of biogas. This latter term is defined from the biological model of degradation and biogas production.

### 5 Production of biogas

The production of biogas depends on different parameters, such as the waste composition, the water content, temperature and density of the medium to be taken into account in the kinetic models biogas production. The production of biogas changes depending of degradable components waste and quantity. The production of biogas is defined by exponential law proposed by Halvodakis[9]. This law is malfunctioning for the first years degradation of a young waste, but applies very well to the anaerobic phase (Finidikakis and al) [12]:

$$\alpha_g = C_g \sum_{i=1}^{3} \frac{\Lambda_i(t + \Delta t) - \Lambda_i(t)}{\Delta t}$$

(4)
Here $A_i$ is the fraction of the waste, $C_g$ the potential the production of biogas $m^3 / Kg$ and $\alpha_g$ is the rate of biogas generation.

6 Conservation of energy

The macroscopic law of conservation of energy in the porous medium, taking into consideration the phenomenological Fourier law, is written as:

$$
\left(\rho C_p\right)^* \frac{\partial T}{\partial t} + V_p^* \left(\rho C_p\right)_f \nabla T = \nabla \left(\lambda^* \nabla T\right) + Q
$$

(5)

Here $T$ is the temperature (the solid phase and fluids phases), $V_p^*$ is the velocity of moving fluid given by Darcy’s law as follows:

$$
V_p^* = V_{p1}^* e_r + V_{p2}^* e_z
$$

and

$V_{p1}, V_{p2}$ are velocity components; $(\rho C_p)_f$ is the volumetric heat capacity ($Jm^{-3}.k^{-1}$) of the moving fluid; $Q$ ($Wm^{-3}$) denotes the volume flow of energy produced or consumed by the chemical reactions. Moreover, $(\rho C_p)^*$ ($Jm^{-3}.k^{-1}$) is equivalent volumetric heat capacity (at constant pressure), defined as the average, weighted by the volume fractions of the heat capacities of all three phases (solid matrix, water and gas).

The projection system of equations (5) in the cylindrical coordinates system is:

$$
\left(\rho C_p\right)^* \frac{\partial T}{\partial t} + (\rho C_p)_f \left( V_{p1} \frac{\partial T}{\partial r} + V_{p2} \frac{\partial T}{\partial z} \right) = \lambda^* \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \lambda^* \frac{\partial^2 T}{\partial z^2} + Q
$$

7 Method of resolution

The equations obtained previously do not admit analytical solutions or recourse to the numerical methods appears mandatory. We opted to the use of the finite volume method.[6][7][15]

All equations are discretized and the algebraic equations obtained are resolved using the dual scanning method. [16][17]
8 Resolution process primed

In order to reduce the computation time, we choose an initial profile that is fairly close to the actual profile.

9 Results

Fig 1: Temperature profile as a function of radius variable

Fig 2: temperature profile as a function of the height
Fig 3: Profile of the saturation according to the height

Fig 4: Profile of the production of biogas a function of time

Figure 1 shows the evolution of the temperature as a function of the radial variable. It is noted that the temperature is a decreasing function of this variable. This result can be interpreted by the fact that the maximum temperature on the driving axis decreases towards the wall to reach the value imposed thereon.

Figure 2 illustrates the change of the temperature as a function of the axial variable. It is found that the temperature increases in time. This can be explained by the fact that over time, the component is deposited, and therefore the interaction between them increases and therefore the temperature increases in time.

Figure 3 shows the evolution of the saturation according to axially variable. Note that this quantity decreases slowly with this variable, quite logical, since the saturation changes from its maximum value and decreases to the residual saturation of the medium.
Figure 4 shows the evolution of the production of biogas as a function of time. It is noticed that the production of biogas is an increasing function of this variable. This result can be explained by the fact that the biogas volume increases the course of time therefore the production of biogas is also increased.

10 Conclusion

We have developed a 2D mathematical model based on conservation equations and using a finite volume method to model bioreactor landfills. The coupled model contains a two-phase flow model based on Darcy’s law and a biological model based on the simplified Monod’s model [3],[4],[5] and considering the biogas production via degradation of the biodegradable solid wastes and VFA production. In contrast to general biological waste degradation models, this model considers the effect of temperature on growth rate of biomass.

References


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