Adaptive Detection and Time and Power Parameters Estimation of the Low-Frequency Random Pulse with Inexactly Known Duration against White and Correlated Hindrances

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Abstract

We carried out the synthesis and analysis of quasi-likelihood detection and estimation algorithms of band low-frequency Gaussian pulse with unknown time
delay, mathematical expectation and dispersion and inaccurately known duration, observed against white noise and band Gaussian disturbance with unknown intensity. While synthesizing receiving devices, we realized the adaptation to the intensities of active hindrances. Then we considered the influence of anomalous errors on the accuracy of time delay estimate. We also set the functionality of offered detector and measurer and the applicability boundaries of asymptotically exact formulas that characterize them through statistical modeling methods.

**Keywords:** Random pulse, signal detection and measurement, discontinuous parameter, maximum likelihood method, inconsistent estimate, disturbance with unknown intensity, detection and estimation characteristics, statistical computer modeling

1 Introduction

Search for the effective random processes analysis algorithms for the information-measuring systems is an actual in statistical radio engineering. In applied tasks of communication, technical diagnostics, pattern recognition, et al., the information signal allowing representation of a kind

$$s(t) = \xi(t) I \left( \frac{t - \lambda_0}{\tau_0} \right), \quad I(x) = \begin{cases} 1, & |x| \leq 1/2; \\ 0, & |x| > 1/2. \end{cases}$$  \hspace{1cm} (1)

can be often used as adequate model of the real random process [1-5]. In Eq. (1) it is designated: $\lambda_0$ – pulse appearance time, $\tau_0$ – pulse duration, $\xi(t)$ – realization of the stationary Gaussian random process with mathematical expectation (ME) $a_0$ and spectral density (SD)

$$G_\xi(\omega) = (2\pi D_0/\Omega_0) I(\omega/\Omega_0).$$

Here $\Omega_0$ is bandwidth, and $D_0$ is dispersion of the process $\xi(t)$.

We will consider process $\xi(t)$ fluctuations as “fast”, so the following condition is satisfied

$$\mu = \tau_0 \Omega_0 / 4\pi >> 1.$$  \hspace{1cm} (2)

Let us put that the signal (1) is distorted both self-noise of radio-electronic system approximated by white noise $n(t)$ with one-sided SD $N_0$ and additive external disturbance $v(t)$. As model of an external disturbance, we will choose the stationary centered Gaussian random process possessing the SD

$$G_v(\omega) = (\gamma_0/2) I(\omega/\Omega_1),$$  \hspace{1cm} (3)
where $\Omega_1 \geq \Omega_0$ is bandwidth, and $\gamma_0$ is SD value (intensity) of the process $v(t)$. An unintentional (interburst) interference, which has passed through input filter (preselector) of receiving device or barrage jamming, can afford examples of such disturbances [3-5].

In works [3, 4] reception efficiency of a signal (1) with unknown appearance time, ME, dispersion and inexact known duration was investigated, if SD $N_0$ and $\gamma_0$ of the active disturbances are a priori known, or its expected (predictable) values can be specified. However, intensity $\gamma_0$ of an external disturbance $v(t)$ can be unknown and not predicted commonly enough. Besides, the receiver tuned on rough values of parameters $N_0$ and $\gamma_0$ can not provide demanded quality of a random pulse (1) processing.

One of possible effective ways on overcoming of the specified difficulties is the use of adaptive analysis algorithms, which are invariant to SD values of active disturbances [5]. Below we demonstrate the technique to find the algorithms of detection and appearance time, ME and dispersion estimates of the signal (1) with inexact known duration, in case of the adaptation to the external disturbance of the unknown intensity. We also present the results of theoretical and experimental (by statistical computer modeling methods) investigations of working capacity of offered detector and measurer.

2 Output Signal of the Maximum Likelihood Receiver

We use a method of maximum likelihood for detection of the signal (1) and estimation of its parameters $\lambda_0$, $a_0$, $D_0$. If the pulse (1) duration is known, then according to [6] maximum likelihood receiver (MLR) should form solving statistics – a logarithm of the functional of likelihood ratio (FLR) $M(\lambda, \tau_0, a, D, \gamma)$ – for all $\lambda \in [A_1, A_2]$, $a \in (-\infty, \infty)$, $D \geq 0$, $\gamma \geq 0$. Let realization $x(t) = s(t) + n(t) + v(t)$ (hypothesis $H_1$) or $x(t) = n(t) + v(t)$ (hypothesis $H_0$) is passed to the MLR input. Then if condition (2) holds, the logarithm of FLR for hypothesis $H_1$ against alternative $H$: $x(t) = n(t)$ has the appearance [3-5]

$$M(\lambda, \tau_0, a, D, \gamma) = \frac{dL_1(\lambda, \tau_0)}{(N_0 + \gamma)(N_0 + \gamma + d)} + \frac{\gamma L_3}{N_0(N_0 + \gamma)} + \frac{2aL_2(\lambda, \tau_0)}{N_0 + \gamma + d} - \frac{a^2 \tau_0}{N_0 + \gamma + d} - \mu \left[ \ln \left(1 + \frac{\gamma + d}{N_0}\right) -(K - 1) \ln \left(1 + \frac{\gamma}{N_0}\right) \right],$$

(4)

Here $d = 4\pi D/\Omega_0$, $K = T\Omega_1/\tau_0 \Omega_0$. 

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\[
L_1(\lambda, \tau) = \int_{\tau-\tau/2}^{\lambda+\tau/2} y_0^2(t) dt, \quad L_2(\lambda, \tau) = \int_{\lambda-\tau/2}^{\lambda+\tau/2} x(t) dt, \quad L_3 = \int_0^T y_1^2(t) dt. \quad (5)
\]
and \( y_i(t) = \int_{-\infty}^{\infty} x(t') h_i(t-t') dt' \), \( i = 0, 1 \) is output signal (response) of the filter with transfer function \( H_i(\omega) \), which satisfies to a condition \( |H_i(\omega)|^2 = I(\omega/\Omega_i) \), on the observable data realization \( x(t) \).

As it is known [6], MLR make a decision on presence of a useful signal comparing absolute (greatest) maximum magnitude of the logarithm of FLR (4) with a threshold \( c \), which is calculated according to the chosen detection optimality criterion. Hence, the value

\[
M = \sup_{\lambda \in [\lambda_1, \lambda_2], a, D, \gamma} M(\lambda, \tau_0, a, D, \gamma) = \sup_{\lambda \in [\lambda_1, \lambda_2]} L(\lambda, \tau_0)
\]

should be compared with a threshold \( c \). Here \( L(\lambda, \tau_0) = \sup_{a, D, \gamma} M(\lambda, \tau_0, a, D, \gamma) \).

If random pulse (1) duration is inexacty known, then MLR output signal can be written down as follows [5]

\[
L^*(\lambda) = L(\lambda, \tau^*) = \frac{\tau^* E_N}{N_0} \left\{ K^* \ln \left[ \frac{(K^*-1)L_3}{K^*(L_3-L_1(\lambda, \tau^*))} \right] + \ln \left[ \frac{L_3-L_4(\lambda, \tau^*)}{(K^*-1)(L_4(\lambda, \tau^*)-L_2(\lambda, \tau^*)/\tau^*)} \right] \right\} \quad (6)
\]

after carrying out maximization procedure of the functional (4) on variables \( a, D, \gamma \). Here \( E_N = N_0 \Omega_0/4\pi \) is average power of noise \( n(t) \) within bandwidth of the process \( \xi(t) \), \( K^* = T\Omega_1/\tau^* \Omega_0 \), and \( \tau^* \) is expected (predictable) signal (1) duration, which is not equal to \( \tau_0 \) in general.

Following [6], in the presence of the useful signal (1) we express the functionals \( L_i(\lambda, \tau^*) \), \( i = 1, 2 \) (5) as sum of regular \( S_i(l) \) and noise \( N_i(l) \) functions: \( L_i(\lambda, \tau^*) = S_i(l) + N_i(l) \), and functional \( L_3 \) as sum of deterministic \( S_3 \) and random \( N_3 \) values: \( L_3 = S_3 + N_3 \). Here \( S_i(l) = \langle L_i(\lambda, \tau^*) \rangle \), \( N_i(l) = L_i(\lambda, \tau^*) - \langle L_i(\lambda, \tau^*) \rangle \), \( S_3 = \langle L_3 \rangle \), \( N_3 = L_3 - \langle L_3 \rangle \), \( l = \lambda/\tau_0 \) is dimensionless parameter, and averaging \( \langle \cdot \rangle \) is carried out on realizations \( x(t) \) under the fixed values \( \lambda_0, \tau_0, a_0, D_0 \) and \( \gamma_0 \). If condition (2) is satisfied, then we receive
\[ S_1(l) = \tau_0 E_N \left[ (1 + q_v)(1 + \delta_\tau) + \left( q_0 + \eta^2 / 2 \right) C(l - l_0, \delta_\tau) \right], \]
\[ S_2(l) = \tau_0 \eta N \sqrt{E_N/2} C(l - l_0, \delta_\tau), \quad S_3 = \tau_0 E_N \left[ q_0 + \eta^2 / 2 + K(1 + q_v) \right], \]
(7)
\[
\begin{align*}
\langle N_1(l_1)N_1(l_2) \rangle &= \left( \tau_0^2 E_N^2 / \mu \right) \left( 1 + q_v \right)^2 R_1(l_1, l_2) + q_0 \left( 2 + 2q_v + q_0 \right) + \eta^2 \left( 1 + q_v + q_0 \right) \right] R_2(l_0, l_1, l_2) , \\
\langle N_2(l_1)N_2(l_2) \rangle &= \left( \tau_0^2 E_N^2 / 2\mu \right) \left( 1 + q_v \right) R_1(l_1, l_2) + q_0 \left( 2 + 2q_v + q_0 \right) \right] , \\
\langle N_3^2 \rangle &= \tau_0^2 E_N^2 \left[ (1 + q_v + q_0)^2 + \eta^2 (1 + q_v + q_0) + (K-1)(1+q_v)^2 \right] / \mu .
\end{align*}
\]

Here \( q_0 = D_0 / E_N \), \( q_v = \gamma_0 / N_0 \), \( \eta^2 = 2a_0^2 / E_N \), \( l_0 = \lambda_0 / \tau_0 \), \( \delta_\tau = (\tau_1^* - \tau_0) / \tau_0 \) is relative detuning on random pulse (1) duration, \( K \) is defined from Eq. (4),
\[
C(x, y) = \begin{cases} 
1 + \min(0, y), & |x| \leq |y| / 2 , \\
1 + y / 2 - |x|, & |x| / 2 < |x| \leq 1 + y / 2 , \\
0, & |x| > 1 + y / 2 ,
\end{cases}
\]
\[
R_1(l_1, l_2) = \begin{cases} 
1 + \delta_\tau - |l_1 - l_2|, & |l_1 - l_2| \leq 1 + \delta_\tau , \\
0, & |l_1 - l_2| > 1 + \delta_\tau ,
\end{cases}
\]
\[
R_2(l_0, l_1, l_2) = \max \left\{ 0, \min \left[ l_0 + 1 / 2, l_1 + (1+\delta_\tau) / 2, l_2 + (1+\delta_\tau) / 2 \right] - \right. \\
-\max \left[ l_0 - 1 / 2, l_1 - (1+\delta_\tau) / 2, l_2 - (1+\delta_\tau) / 2 \right] \right\}.
\]

If a useful signal (1) is absent, then for functionals (5) we have
\[
S_1(l) = \tau_0 E_N \left( 1 + q_v \right) \left( 1 + \delta_\tau \right), \quad S_2(l) = 0, \quad S_3 = K \tau_0 E_N \left( 1 + q_v \right).
\]
(8)
\[
\begin{align*}
\langle N_1(l_1)N_1(l_2) \rangle &= \left( \tau_0^2 E_N^2 / \mu \right) R_1(l_1, l_2) , \\
\langle N_2(l_1)N_2(l_2) \rangle &= \left( \tau_0^2 E_N^2 / 2\mu \right) R_1(l_1, l_2) , \\
\langle N_3^2 \rangle &= K \tau_0^2 E_N^2 \left( 1 + q_v \right)^2 / \mu .
\end{align*}
\]

3 Random Pulse Parameters Estimation

In the beginning let us suppose that the signal (1) is present with probability 1 in the observable realization. Then, following [5], estimates \( \lambda_q, a_q \) and \( D_q \) of appearance time \( \lambda_0 \), ME \( a_0 \) and dispersion \( D_0 \) of a random pulse (1) with inexact duration adapted for intensity \( \gamma_0 \) of an external disturbance, can be written down as
\[\lambda_q = \arg \sup_{\lambda \in [\lambda_1, \lambda_2]} L^*(\lambda),\]  \hspace{1cm} (9)

\[a_q = L_2(\lambda_q, \tau^*)/\tau^*, \quad D_q = \max \left\{ 0, \left[ K^* L_1(\lambda_q, \tau^*) - L_3 \right]/\tau^* \left( K^* - 1 \right) - \left[ L_2(\lambda_q, \tau^*)/\tau^* \right]^2 \right\}.\]  \hspace{1cm} (10)

It should be noted that adaptation on parameter \( \gamma_0 \) allows to receive time and power parameters estimates of a signal (1), which are invariant to the SD \( N_0 \) of white noise also. We name Eqs. (9), (10) as quasi-likelihood estimates (QLEs).

Really, if \( \tau^* = \tau_0 \) (\( \delta_1 = 0 \)), then QLEs (9), (10) go over to corresponding maximum likelihood estimates (MLEs) [5].

Let us find the characteristics of QLEs \( \lambda_q, a_q \) and \( D_q \). For this purpose we introduce the value
\[\varepsilon = \mu^{-1/2},\]  \hspace{1cm} (11)

which is small parameter if condition (2) holds. Then, taking into account Eq. (7) the functional (6) can be presented in a kind of

\[L^*(l) = \mu(1 + \delta_1) \left\{ K^* \ln \left[ \frac{(K^* - 1)(\tilde{S}_3 + \varepsilon \tilde{N}_3)}{K^*(\tilde{S}_3 + \varepsilon \tilde{N}_3 - \tilde{S}_1(l) - \varepsilon \tilde{N}_1(l))} \right] + \ln \left[ \frac{\tilde{S}_3 + \varepsilon \tilde{N}_3 - \tilde{S}_1(l) - \varepsilon \tilde{N}_1(l)}{(K^* - 1)\left( \tilde{S}_1(l) - \varepsilon \tilde{N}_1(l) - \left( \tilde{S}_2(l) + \varepsilon \tilde{N}_2(l)/\sqrt{2} \right)^2 \right)} \right] \right\}.\]  \hspace{1cm} (12)

Here \( \tilde{S}_1(l) = S_1(l)/\tau_0 E_N(1 + q_\nu)(1 + \delta_1), \quad \tilde{S}_2(l) = S_2(l)/\tau_0 E_N(1 + q_\nu)(1 + \delta_1), \quad \tilde{N}_1(l) = N_1(l)\sqrt{\mu}/\tau_0 E_N(1 + q_\nu)(1 + \delta_1), \quad \tilde{N}_2(l) = N_2(l)\sqrt{2\mu}/\tau_0 E_N(1 + q_\nu)(1 + \delta_1) \) are normalized functions, and \( \tilde{S}_3 = S_3/\tau_0 E_N(1 + q_\nu)(1 + \delta_1) \) and \( \tilde{N}_3 = N_3\sqrt{\mu}/\tau_0 E_N(1 + q_\nu)(1 + \delta_1) \) are normalized values.

We develop Eq. (12) as Maclaurin series in \( \varepsilon \) and focus on the first term of expansion depending on realization of the observable data \( x(l) \). As a result, for \( \varepsilon \to 0 \) we have
\[L^*(l) \approx \mu(1 + \delta_1) \left[ \tilde{S}(l) + \varepsilon \tilde{N}(l) \right],\]  \hspace{1cm} (13)

where
\[\tilde{S}(l) = K^* \ln \left[ \frac{(K^* - 1)\tilde{S}_3}{K^*(\tilde{S}_3 - \tilde{S}_1(l))} \right] + \ln \left[ \frac{\tilde{S}_3 - \tilde{S}_1(l)}{(K^* - 1)(\tilde{S}_1(l) - \tilde{S}_2(l))} \right].\]  \hspace{1cm} (14)
is normalized regular function, and

\[
\tilde{N}(l) = \frac{K\tilde{S}_1(l) - (K^*-1)\tilde{S}_2(l) - \tilde{S}_3}{[\tilde{S}_1(l) - \tilde{S}_2(l)] [\tilde{S}_3 - \tilde{S}_1(l)]} \tilde{N}_1(l) + \frac{2\tilde{S}_2(l)\tilde{N}_2(l)}{\tilde{S}_1(l) - \tilde{S}_2(l)} + \frac{\tilde{S}_3 - K^*\tilde{S}_1(l)}{\tilde{S}_3 [\tilde{S}_3 - \tilde{S}_1(l)]} \tilde{N}_3
\]

(15)

is normalized noise function. Let us define the signal-to-noise ratio (SNR) for the sensed signal. To that end, we note that regular function \( \tilde{S}(l) \) (14) has the flat top by length \( |\delta| \), which is situated on the interval \( [l_0 - |\delta|/2, l_0 + |\delta|/2] \). In particular, \( \tilde{S}(l) \) is maximum under \( l = l_0 \). Then, according to [6] output SNR \( \hat{\xi} \) is determined as

\[
\hat{\xi}^2 = \frac{\mu \tilde{S}^2(l_0)}{\langle \tilde{N}^2(l_0) \rangle} = \frac{\mu}{\sigma^2} \left\{ \frac{K}{1 + \delta} \ln \left[ \frac{2K(1+q_v) + 2q_0 + \eta^2}{K^2(1+q_v) - (2q_0 + \eta^2)\min(0, \delta)/(K-1-\delta)} \right] + \ln \left[ \frac{(1+\delta)(2(1+q_v) - (2q_0 + \eta^2)\min(0, \delta)/(K-1-\delta))}{2(1+q_v)(1+\delta) + (2q_0 + \eta^2\max(0, \delta)/(1+\delta))(1+\min(0, \delta))} \right]^2 \right\} + (16)
\]

where

\[
\hat{\sigma}^2 = \frac{[2(K-1)q_0 - \eta^2]^2 + 4K^2\eta^2(1+q_v + q_0) + 4(K-1)^2(2q_0 + \eta^2)^2[(K-1)(1+q_v)^2 - \delta(1+q_v + q_0)(1+q_v + q_0 + \eta^2)]}{(1+\delta)(2(K+q_v) + q_0 + \eta^2)^2} + \frac{4(K-1)^2(2q_0 + \eta^2)^2[(K-1)(1+q_v)^2 - \delta(1+q_v + q_0)(1+q_v + q_0 + \eta^2)]}{2(K+q_v) + q_0 + \eta^2} \]

if \( \delta \leq 0 \), and
\[ \delta^2 = \frac{4\eta^2(1 + q_v)^2}{(1 + q_v)^2} \left[ (1 + q_v)(1 + \delta_v) + q_0 \right] + \left[ \frac{2q_0 + \eta^2\delta_v}{(1 + \delta_v)} \right] \left[ \frac{2q_0 + \eta^2\delta_v}{(1 + \delta_v)} \right] \times \]
\[ \times \left[ (1 + q_v + q_0) \left[ (1 + q_v + q_0 + \eta^2) + \delta_v (1 + q_v)^2 \right] + 4n^2 (1 + q_v)(1 + q_v + q_0) \right] + \]
\[ \times \left[ 2(1 + \delta_v) \left[ (1 + q_v)(1 + \delta_v) + q_0 \right] + \eta^2\delta_v \right]^2 \]
\[ + \left( 2q_0 + \eta^2 \right)^2 \left[ 2q_0(1 + q_v + q_0) + \eta^2 (1 + q_v + 2q_0) \right] \]
\[ \frac{2(1 + q_v)(1 + \delta_v)^2 \left[ 2(1 + q_v) + 2q_0 + \eta^2 \right]}{2(1 + q_v)^2 \left[ 2(1 + q_v) + 2q_0 + \eta^2 \right]} \]
\[ \left( 2q_0 + \eta^2 \right) \left[ 4q_0^2 - 2q_0 \left( 1 + q_v \right) (1 + 4\delta_v) - 2n^2 \right] \left( 1 + q_v \right) \left( 3n^2 - 4(1 + q_v)(1 + \delta_v) \right) \]
\[ \frac{2(1 + q_v)(1 + \delta_v)^2 \left[ 2(1 + q_v) + 2q_0 + \eta^2 \right]}{2(1 + q_v)^2 \left[ 2(1 + q_v) + 2q_0 + \eta^2 \right]} \]
\[ = \frac{4(2q_0 + \eta^2)}{(1 + q_v) \left[ 2(1 + q_v) + 2q_0 + \eta^2 \right]} \left[ (1 + q_v + q_0)^2 + 4\delta_v (1 + q_v)(1 + q_v + 2q_0) \right] \]
\[ \frac{2(1 + q_v)(1 + \delta_v)^2 \left[ 2(1 + q_v) + 2q_0 + \eta^2 \right]}{2(1 + q_v)^2 \left[ 2(1 + q_v) + 2q_0 + \eta^2 \right]} \],

if \( \delta_v > 0 \). From Eq. (16) follows that under \( q_0 > 0 \), \( q_v \geq 0 \), \( \delta_v > -1 \) and \( \mu \to \infty \) SNR \( \hat{z}^2 \to \infty \) for any finite values \( n \).

While analyzing measure \( \left( 9 \right) \), \( \left( 10 \right) \), we divide all possible estimates of the appearance time into two classes: reliable and anomalous \( \left[ 6 \right] \). The normalized estimate \( l_0 = \lambda_q / \tau_0 \) is reliable, if it lies within the interval \( \Gamma_S = \left[ l_0 - \delta_v / 2, l_0 + 1 + \delta_v / 2 \right] \), where regular function \( \left( 14 \right) \) depends on true value of the estimated parameter \( l_0 \). If the estimate \( l_q \) is out of the interval , i.e. \( l_q \notin \Gamma_N = \Gamma \setminus \Gamma_S \), \( \Gamma = \left[ \Lambda_1, \Lambda_2 \right] \), \( \Lambda_{1,2} = \Lambda_{1,2} / \tau_0 \), then the estimate and corresponding estimation error are named as anomalous \( \left[ 6 \right] \).

Under \( \hat{z} \to \infty \), a reliable estimate \( l_q \) \( \left( 9 \right) \) possesses values with probability 1 from an interval \( \Gamma_0 = \left[ l_0 - |\delta_v| / 2 - \delta, l_0 + |\delta_v| / 2 + \delta \right] \), where \( \delta \ll 1 \). Therefore, when \( \hat{z} \gg 1 \), it is enough to study behavior of functional \( \left( 13 \right) \) within the interval \( \left[ l_0 - |\delta_v| / 2, l_0 + |\delta_v| / 2 \right] \) and in small neighborhoods of points \( l_0 \pm |\delta_v| / 2 \) for characteristics calculation of a reliable estimate \( l_q \). We designate \( \Delta = 0 \), if \( |l_i - l_0| \leq |\delta_v| / 2 \), and \( \Delta = \max \left\{ \left\| \delta_v \right\| / 2 - \left\| l_i - l_0 \right\|, \left\| \delta_v \right\| / 2 - \left\| l_i - l_0 \right\|, \left\| l_i - l_2 \right\| \right\} \), if \( |l_i - l_0| > |\delta_v| / 2 \), \( i = 1, 2 \). Then, under \( \Delta \ll 1 \) the following asymptotic decompositions for Eqs. \( \left( 14 \right), \left( 15 \right) \) are valid.
\[
\tilde{S}(l) = \frac{K}{1 + \delta_t} \ln \left[ \frac{2K(1 + q_v) + 2q_0 + \eta^2}{K(2(1 + q_v) - (2q_0 + \eta^2) \min(0, \delta_t)/(K - 1 - \delta_t))} \right] + \\
+ \ln \left[ \frac{(1 + \delta_t)(2(1 + q_v) - (2q_0 + \eta^2) \min(0, \delta_t)/(K - 1 - \delta_t))}{2(1 + q_v)(1 + \delta_t) + (2q_0 + \eta^2) \max(0, \delta_t)/(1 + \delta_t)) \max(0, \delta_t)/(1 + \delta_t))} \right] + \\
+ A_S \min(0, |\delta_t|)[2 - l - l_0] + o(\Delta),
\]

where \( A_S \) is nonessential constant, \( \hat{\sigma} \) is defined from Eq. (16), and

\[
A_S = \frac{(K - 1)[2q_0^2 + \eta^2(2 + 2q_v + q_0)] - \eta^2 \delta_t \left(2 + 2q_v + q_0 + \eta^2/2\right)}{(1 + \delta_t)(1 + q_v + q_0) \left[2(K - 1)(1 + q_v) - \delta_t(2(1 + q_v + q_0) + \eta^2)\right]},
\]

\[
\hat{\sigma}_1^2 = \frac{q_0^2 + \eta^2(1 + q_v)}{(1 + \delta_t)^2(1 + q_v + q_0)^2} + \\
\frac{\delta_t(2q_0 + \eta^2)}{(1 + \delta_t)^2(1 + q_v + q_0)} \left[4q_0(1 - K)(1 + q_v) - \delta_t \left[2q_0(1 + q_v + q_0) - \eta^2(1 + q_v - q_0)\right]\right],
\]

\[
\hat{\sigma}_2^2 = \frac{q_0^2 \left(2 + 2q_v + q_0\right)}{(1 + \delta_t)^2(1 + q_v + q_0)^2} + \\
\frac{\delta_t \left(2q_0 + \eta^2\right)}{(1 + \delta_t)^2(1 + q_v + q_0)^2} \left[4q_0(1 - K)(1 + q_v) - \delta_t \left[2q_0(1 + q_v + q_0) - \eta^2(1 + q_v - q_0)\right]\right] + \\
\frac{2\delta_t \left[q_0^2 + \eta^2(1 + q_v + q_0)\right] + q_0 \left(q_0(4 + 4q_v + q_0)(q_0 + \eta^2) + 2n^2(1 + q_v)^2\right]}{(1 + \delta_t)^2(1 + q_v)^2(1 + q_v + q_0)^2} + \\
\frac{\delta_t^2 \left(2q_0 + \eta^2\right)}{(1 + \delta_t)^2(1 + q_v)^2(1 + q_v + q_0)} \left[4q_0(1 - K)(1 + q_v) - \delta_t \left[2q_0(1 + q_v + q_0) + \eta^2\right]\right],
\]

if \( \delta_t \leq 0 \), and

\[
A_S = \frac{4q_0^2(1 + \delta_t) + 2\eta^2 \left[q_0(1 + 2\delta_t) + 2(1 + q_v)(1 + \delta_t)\right]}{2(1 + \delta_t)(1 + q_v) \left[2(1 + \delta_t)(1 + q_v)(1 + \delta_t) + q_0\right] + \eta^4 \delta_t},
\]

\[
\hat{\sigma}_1^2 = \frac{\left[2q_0 + \eta^2(1 + \delta_t)\right]^2 + 4\eta^2(1 + q_v)}{2\left(1 + \delta_t\right)(1 + q_v)(1 + \delta_t) + q_0 + \eta^2 \delta_t},
\]

(18)
\[
\hat{\sigma}^2 = \left[ 2q_0 + \eta^2 \delta_\tau / (1 + \delta_\tau) \right] \left[ 2(2 + 2q_v + q_0) \right] q_0^2 + \eta^2 (1 + q_v + q_0) \bigg] + (1 + q_v)^2 \left[ 2(1 + \delta_\tau) \times \\
+ \left[ q_0(2 + 2q_v + q_0) + \eta^2 (1 + q_v + q_0) \right] \eta^2 \delta_\tau / (1 + \delta_\tau) \bigg] + 4q_0 \eta^2 (1 + q_v)^2 \right].
\]

if \( \delta_\tau > 0 \).

Let us suppose that the value \( \delta \) is such that expressions (17) can be approximated by dominant terms of asymptotic decompositions with required accuracy. Then, on the basis of results [7], it is possible to write down the approximate expressions for conditional (under fixed \( l_0 \)) bias \( b_0(l_q | l_0) = \langle l_q \rangle - l_0 \) and variance \( V_0(l_q | l_0) = \langle (l_q - l_0)^2 \rangle \) of a reliable estimate \( l_q \) (9) as

\[
b_0(l_q | l_0) \approx 0, \\
V_0(l_q | l_0) = \frac{\hat{\sigma}^2}{8} + \exp \left( \frac{2\kappa B_1 |\delta_1|}{B_2^2} \right) \left[ 1 - \Phi \left( \frac{2\kappa \sqrt{B_1 |\delta_1|}}{B_2} \right) \right] + \\
+ \left[ \frac{8\kappa^2 B_1^2}{B_2^2} |\delta_1| \right] \left[ 1 - \frac{4B_1}{3B_2} - \frac{4B_1}{B_2} \delta^2 B_1 - B_2 \right] + \left[ \frac{|\delta_1|}{\kappa^2} \left( \frac{3B_2}{2} - 8B_1 \right) + \frac{13B_2^2}{4\kappa^4} \right] + \\
+ \left[ \frac{|\delta_1| B_1}{2\pi} \right] \left[ 4\kappa \delta^2 B_1 B_2^2 \left( 3B_1 - B_2 \right) + \left| \delta_1 \right| \left( 3 - \frac{22B_1}{3B_2} \right) + \frac{13B_2^2}{2\kappa^3} \right].
\]

Here

\[
\kappa = \sqrt{\frac{A_S}{\sqrt{\delta_1^2 + \delta_2^2}}}, \quad B_1 = \begin{cases} 2, & \delta_\tau \leq 0, \\
2\delta_1^2 / (\delta_1^2 + \delta_2^2), & \delta_\tau > 0,
\end{cases} \quad B_2 = \frac{2\delta_1^2 + \delta_2^2}{\delta_1^2 + \delta_2^2},
\]

and \( \Phi(x) = \int_{-\infty}^{x} \exp(-t^2/2)dt/\sqrt{2\pi} \) is probability integral.

Formulas (19) are received in the assumption [7] that correlation time \( 1 + \delta_\tau \) of noise function \( \tilde{N}(l) \) (15) surpasses width \( |\delta_1| \) of the flat top of regular function \( \tilde{S}(l) \) (14), i.e.

\[
\delta_\tau > -1/2,
\]
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and their accuracy increases with $\mu$ and $\hat{\tau}$. Assuming in Eq. (19) $\delta_\tau = 0$, we receive variance of the normalized reliable MLE of appearance time of a signal (1) with a priori known duration [5].

It follows from [3] and Eq. (19), that characteristics of the normalized QLE (9) adapted to the unknown value $\gamma_0$ of the external disturbance $v(t)$ SD (3) coincide for a case $\delta_\tau > 0$ or coincide asymptotically (with increase of parameter $K$) for a case $\delta_\tau < 0$ with corresponding characteristics of the normalized QLE of appearance time of a signal (1) received under a priori known value $\gamma_0$ of the SD (3). Therefore, in the conditions of high posterior accuracy the loss in quality of the adaptive estimate of pulse (1) appearance time because of intensities ignorance of disturbance and white noise are absent asymptotically (under $K \gg 1$). Under output SNR $\hat{\tau} \to \infty$ variance $V_0(l_q|l_0)$ value of a reliable estimate $l_q$ goes to $\delta_\tau^2/8$, i.e. even if there are very small random distortions of a signal (1), adaptive QLE (9) is not consistent in a case of nonzero detunings on duration $\tau_0$.

Now let us consider the threshold (i.e. taking into account anomalous errors) characteristics of the estimate (9). The allowance of anomalous errors is necessary, if prior interval reduced length [6] $m = \bar{\lambda}_2 - \bar{\lambda}_1$ of possible values of the appearance time $l_0$ is much greater than the range of the reliable estimate interval $\Gamma_S$, i.e.

$$m \gg 1.$$  \hspace{1cm} (21)

As reliable and anomalous estimate decisions are incompatible events, then conditional bias $b(l_q|l_0)$ and variance $V(l_q|l_0)$ of the QLE $l_q$ with anomalous errors can be presented in the form of [6]

$$b(l_q|l_0) = P_0b_0(l_q|l_0) + (1 - P_0)b_a(l_q|l_0) = (1 - P_0)b_a(l_q|l_0),$$

$$V(l_q|l_0) = P_0V_0(l_q|l_0) + (1 - P_0)V_a(l_q|l_0),$$

where $P_0 = P[|l_q - l_0| \leq 1 + \delta_\tau/2]$, $b_0(l_q|l_0)$ and $V_0(l_q|l_0)$ are probability and conditional bias and variance (19) of the reliable estimate correspondingly, $b_a(l_q|l_0)$ and $V_a(l_q|l_0)$ are conditional bias and variance of the anomalous estimate. According to [6, 7], if condition (21) is satisfied, then

$$b_a(l_q|l_0) = (\bar{\lambda}_2 + \bar{\lambda}_1)/2 - l_0,$$

$$V_a(l_q|l_0) = (\bar{\lambda}_2^2 + \bar{\lambda}_1\bar{\lambda}_2 + \bar{\lambda}_1^2)/3 - (\bar{\lambda}_2 + \bar{\lambda}_1)l_0 + l_0^2,$$
and for probability $P_0$ we can write down

$$P_0 \approx \int_{-\infty}^{+\infty} F_N(u) dF_S(u).$$

(23)

Here $F_S(u) = P[H_S < u]$ and $F_N(u) = P[H_N < u]$ are distribution functions of absolute maxima $H_S$ and $H_N$ of the functional $L^*(l)$ (12) in the intervals of reliable $\Gamma_S$ and anomalous $\Gamma_N$ estimates correspondingly. We suppose that SNR (16) is great enough, therefore for calculation of probability (23) asymptotically exact approximations of subintegral functions $F_N(u)$ and $F_S(u)$ calculated under $u \rightarrow \infty$ can be used [6, 7].

For $\tau^* = \tau_0$ the probability of a reliable estimate was found in [5]. In this connection we consider a case $\tau^* \neq \tau_0$ ($\delta_\xi \neq 0$). In the beginning we define the probability $F_N(u)$. As $T > \Lambda_2 - \Lambda_1$ and $\Omega_4 \geq \Omega_0$, then the inequality $K > m$ is always satisfied. Therefore, under $m \gg 1$ (21) it is necessary to accept

$$K \gg 1,$$

(24)

and for characteristics (7) of the functional $L_3$ (5) approximations (8) are valid. Using Eqs. (8), for $l \in \Gamma_N$ we overwrite the functional $L^*(l)$ (12) in a kind of

$$L^*(l) = \mu(1 + \delta_\tau) \left\{ K^* \ln \left[ \frac{(K^* - 1) \left( K^* + \varepsilon \sqrt{K^*/(1 + \delta_\tau)} \right) N_{30}}{K^* + \varepsilon \sqrt{K^*/(1 + \delta_\tau)} N_{30} - 1 - \varepsilon N_{10} l/(1 + \delta_\tau)} \right] + \ln \left[ \frac{K^* + \varepsilon \sqrt{K^*/(1 + \delta_\tau)} N_{30} - 1 - \varepsilon N_{10} l/(1 + \delta_\tau)}{1 - \varepsilon N_{10} l/(1 + \delta_\tau)} \right] \right\} ,$$

(25)

Here $N_{10}(l/(1 + \delta_\tau)) = \tilde{N}_1(l) \sqrt{1 + \delta_\tau}$, $N_{20}(l/(1 + \delta_\tau)) = \tilde{N}_2(l) \sqrt{1 + \delta_\tau}$ are asymptotically Gaussian (under $\mu \rightarrow \infty$) and Gaussian random processes with characteristics $\langle N_{10}(l) \rangle = \langle N_{20}(l) \rangle = 0$, $\langle N_{10}^2(l) \rangle = \langle N_{20}^2(l) \rangle = 1$, $\langle N_{10}(l_1) N_{10}(l_2) \rangle = \langle N_{20}(l_1) N_{20}(l_2) \rangle = \max \left( 0, 1 - |l_1 - l_2| \right)$, and $N_{30}$ asymptotically (with increasing $\mu$) Gaussian random variable with zero ME and unit dispersion.

Taking into account Eq. (2), we develop Eq. (25) as Maclaurin series in small parameter $\varepsilon$ (11) and are limited to the two first expansion terms depending on
realization of the observable data $x(t)$. As a result, under $\varepsilon \to 0$ we have

$$L^*(l) = \frac{1}{2} \left[ \frac{K^*}{K^* - 1} \left[ \frac{N_{30}}{\sqrt{K^*}} - N_{10} \left( \frac{l}{1 + \delta_\tau} \right) \right]^2 + N_{20}^2 \left( \frac{l}{1 + \delta_\tau} \right) \right] + o(\varepsilon). \quad (26)$$

We neglect expansion terms of order $\varepsilon$ and less here and introduce the random process of a kind

$$N_{40}(l) = \left[ \frac{N_{30}}{\sqrt{K^*}} - N_{10}(l) \right] \sqrt{K^*/(K^* - 1)},$$

which are approximately Gaussian [1] with ME $\langle N_{40}(l) \rangle = 0$ and correlation function $\langle N_{40}(l_1)N_{40}(l_2) \rangle = [K^* \max(0,1-|l_1-l_2|) - 1]/(K^* - 1)$ if condition (2) holds, and in addition $\langle N_{40}(l_1)N_{20}(l_2) \rangle \approx 0$. Then, under $\mu >> 1$, $K >> 1$ the probability $F_N(u)$ (23) we can present as follows:

$$F_N(u) = P[\sup_{l \in \Gamma_N} L^*(l) < u] = P[\sup_{l \in \Gamma_N} \zeta(l) < \sqrt{u}],$$

where $\zeta(l) = \sqrt{X_1^2(l) + X_2^2(l)}$ is stationary Rayleigh random process, for which the correlation coefficient $R(\Delta)$ of the quadratures $X_1(l)$, $X_2(l)$ can be presented as $R(\Delta) = 1 - |\Delta|$ under $\Delta \to 0$.

When inequality (21) is satisfied,

$$F_N(u) \approx P[\sup_{l \in \Gamma} \zeta(l) < \sqrt{u}]. \quad (27)$$

Using in Eq. (27) asymptotic (under $m \to \infty$, $u \to \infty$) approximation of the distribution function $F(u) = P[\sup_{l \in \Gamma} \zeta(l) < u]$ found, for example, in [8], we have

$$F_N(u) \approx \begin{cases} \exp\left[ -m(2u - 1)\exp\left( -u \right)/\left( 1 + \delta_\tau \right) \right], & u \geq 3/2, \\ 0, & u < 3/2. \end{cases} \quad (28)$$

Now let us pass to definition of the probability $F_S(u)$ (23), having used a technique stated in [7]. Under $\tau^* \neq \tau_0$ and $\hat{z} \to \infty$ the functional $L^*(l)$ allows representation (13), and reliable QLE $l_q$ possesses the values within an interval $\left[ l_0 - |\delta_\tau|/2, l_0 + |\delta_\tau|/2 \right]$ with the probability tending to 1. Taking into account Eqs. (17), (24), over this interval we have for regular function $\tilde{S}(l)$ and correlation function of the noise function $\tilde{N}(l)$:
\[ s(t) = \frac{2q_0 + \eta^2}{2(1 + q_\nu)(1 + \max(0, \delta_\tau))} - \ln \left[ 1 + \frac{1}{2(1 + q_\nu)(1 + \max(0, \delta_\tau))} \left( 2q_0 + \frac{\eta^2 \max(0, \delta_\tau)}{1 + \delta_\tau} \right) \right], \]

(29)

and output SNR (16) for the algorithm (9) is written down in a kind of

\[ z^2 = \hat{z}^2 \bigg|_{K \gg 1} = \frac{\mu}{\sigma^2} \left( \frac{2q_0 + \eta^2}{2(1 + q_\nu)(1 + \max(0, \delta_\tau))} - \ln \left[ 1 + \frac{1}{2(1 + q_\nu)(1 + \max(0, \delta_\tau))} \left( 2q_0 + \frac{\eta^2 \max(0, \delta_\tau)}{1 + \delta_\tau} \right) \right] \right). \]

(30)

In Eqs. (29), (30) \( \hat{\sigma}_1^2, \hat{\sigma}_2^2 \) is defined from Eqs. (18),

\[ \sigma_1^2 = \frac{q_0^2 + \eta^2(1 + q_\nu)}{(1 + \delta_\tau)^2(1 + q_\nu + q_0)^2}, \]

\[ \sigma_2^2 = q_0 \left\{ \frac{(2 + 2q_\nu + q_0)(q_0^2 + \eta^2(1 + q_\nu + q_0)) + \eta^2(1 + q_\nu)^2}{(1 + \delta_\tau)^2(1 + q_\nu + q_0)^2} \right\}, \]

and \( \sigma^2 = \left[ q_0^2 + \eta^2(1 + q_\nu + q_0) \right] / (1 + q_\nu)^2(1 + \delta_\tau), \) if \( \delta_\tau \leq 0, \) and

\[ \sigma^2 = \left\{ \frac{2q_0 + \eta^2 \delta_\tau/(1 + \delta_\tau)}{2(1 + \delta_\tau)} \times \left( 1 + \left( \frac{q_0 + \eta^2}{1 + q_\nu} \right) \right)^2 + 4\eta^2(1 + q_0/(1 + q_\nu)) \right\} \times \left[ (1 + q_\nu)(1 + \delta_\tau) + q_0 + \eta^2 \delta_\tau \right]^{1/2} \]

if \( \delta_\tau > 0. \) As follows from [3] and Eq. (30), SNR \( z \) for the algorithm (9) adapted for unknown SD value of an external disturbance coincides with output SNR for the algorithm synthesized under a priori known SDs of an external disturbance and white noise.

If \( z \gg 1 \) and condition (2) holds, the probability \( F_S(u) \) can be presented as follows

\[ F_S(u) = P]\sup_{l \in \Gamma_S} L^*(l) < u \approx P]\sup_{l \in \Gamma_0} L^*(l) < u = P]\sup_{l \in [0, m_\eta]} r(l) < h(u). \]

(31)
Here \( m_S = |\delta_\tau|/(1 + \delta_\tau) \) if \( \delta_\tau < 0 \), and \( m_S = \delta_\tau^2/(1 + \delta_\tau) + \delta_\tau^2 \) if \( \delta_\tau > 0 \),

\[ h(u) = \left[u - \mu(1 + \delta_\tau)\tilde{S}(l_0)\right]/\sqrt{\mu_\sigma(1 + \delta_\tau)}, \quad \tilde{S}(l_0) \text{ and } \sigma \text{ are defined from Eqs. (29) and (30) correspondingly, and } r(l) \text{ is stationary centered Gaussian random process with correlation function } \langle r(l_1)r(l_2) \rangle = \max(0,1-|l_1-l_2|). \]

As it was shown in [7], the probability of threshold uncrossing by process \( r(l) \) realization over an interval by length \( \rho \leq 1 \) is defined as

\[
P\left[ \sup_{l \in [0,\rho]} r(l) < u \right] = \Psi_0(u, \rho) = \int_0^\infty \Phi\left( u - \frac{x(1-\rho)}{\sqrt{2(1-\rho)}} \right) \exp\left( -\frac{x^2}{2} \right) \frac{dx}{\sqrt{2\pi}} - \frac{\rho u}{\sqrt{2\pi}} \exp\left( -\frac{u^2}{2} \right) \Phi\left( u - \frac{\rho}{\sqrt{2-\rho}} \right) \exp\left( -\frac{u^2}{2-\rho} \right). \tag{32} \]

When inequality (20) is satisfied, in Eq. (31) we have \( m_S < 1 \) always. Then, using Eq. (32), for asymptotic approximation of function (31) the following expression, which accuracy increases with \( \mu \) and \( z \), is valid

\[
F_S(u) \approx \Psi_0[h(u), m_S]. \tag{33} \]

Substituting Eqs. (28), (33) in Eq. (23), under \( \delta_\tau \neq 0 \) we have finally

\[
P_0 = \frac{1}{\sigma(1 + \delta_\tau)\sqrt{2\pi\mu}} \int_{3/2}^\infty \exp\left[ -\frac{m(2u-1)}{(1 + \delta_\tau)} \right] \left[ 2 + m_S h^2(u) - 1 \right] \exp\left( -\frac{h^2(u)}{2} \right) \times \Phi\left[ h(u) - \frac{m_S}{\sqrt{2 - m_S}} \right] + \frac{m_S(2-m_S)}{2\pi} h(u) \exp\left( -\frac{h^2(u)}{2-m_S} \right) \] \tag{34} \]

Accuracy of the formula (34) increases with \( m, \mu \) and \( z \). It follows from [3] and Eqs. (19), (22), (34) that characteristics of the adaptive QLE (9) with anomalous errors coincide with corresponding characteristics of the signal (1) appearance time estimate synthesized under a priori known disturbance and white noise SDs. Therefore, if condition (24) is satisfied, then loss in accuracy of an adaptive appearance time estimate of a random pulse (1) is absent because of ignorance of intensities of an external disturbance and white noise.

Now let us find the characteristics of estimates \( a_q \) and \( D_q \) (10). We are limited to a condition of high posterior accuracy, when \( \hat{z} \gg 1 \) and the probability of anomalous error for appearance time estimation

\[
P_a = P\left[ l_q \notin \Gamma_S \right] = 1 - P_0 \tag{35} \]

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is negligible. In this case QLE \( l_q \) possesses the values within an interval \([l_0 - |\delta_\tau|/2, l_0 + |\delta_\tau|/2]\) with the probability tending to 1, so measurement error of the parameter \( l_0 \) does not exceed the value \(|\delta_\tau|/2\). Then for conditional biases \( b_0(a_q|a_0) \), \( b_0(D_q|D_0) \) and variances \( V_0(a_q|a_0) \), \( V_0(D_q|D_0) \) of the estimates \( a_q \), \( D_q \) (10) by direct averaging of functionals (5) on realizations \( x(t) \) we find

\[
b_0(a_q|a_0) = a_0 \max(0, \delta_\tau)/(1+\delta_\tau),
\]

\[
V_0(a_q|a_0) = b_0^2(a_q|a_0) + E_N \left[ 1 + q_0/(1+\max(0, \delta_\tau)) \right]/2\mu(1+\delta_\tau),
\]

\[
b_0(D_q|D_0) = -E_N \left[ \left( q_0 - \frac{\eta^2}{2(1+\delta_\tau)} \right) \frac{\max(0, \delta_\tau)}{1+\delta_\tau} \right] + \frac{1}{2\mu(1+\delta_\tau)} \frac{q_0}{1+\max(0, \delta_\tau)},
\]

\[
V_0(D_q|D_0) = E_N^2 \left[ \frac{1+q_0}{(1+\delta_\tau)} \right]^2 + \eta^2 \left( 1 + q_0 + \frac{q_0 \delta_\tau}{1+\delta_\tau} \right) \frac{\max(0, \delta_\tau)}{(1+\delta_\tau)^2} \frac{(1+q_0)^2}{K-1-\delta_\tau} + \frac{(1+q_0+q_0)^2}{(1+\delta_\tau)(1+\max(0, \delta_\tau))} - \left[ q_0 (2+2q_0 + q_0 + \eta^2 (1+q_0 + q_0)) \min(0, \delta_\tau) \right]/(K-1-\delta_\tau)^2 \right] + b_0^2(D_q|D_0).
\]

Accuracy of formulas (36) increases with \( \mu \) and \( \hat{\tau} \). If \( \delta_\tau = 0 \), then in Eq. (36) we go to expressions for characteristics of adaptive MLEs of ME and dispersion of a random pulse (1) with a priori known duration [5].

According to [3], Eq. (36) characteristics of the estimate \( a_q \) coincide and characteristics of the estimate \( D_q \) coincide asymptotically (with increasing \( K \)) with corresponding characteristics of the signal (1) ME and dispersion QLEs synthesized under a priori known SDs of a disturbance and white noise. Thus, loss in accuracy of adaptive estimates of power parameters of a random pulse (1) is absent asymptotically because of ignorance of intensities of an external disturbance and white noise. From the received results follows also that adaptive measurer (9), (10) can be efficiently applied at reception of a signal (1) against white noise with unknown SD (in the absence of external disturbance) if \( K \) can be made great enough.

### 4 Detection of the Random Pulse Signal

Let us define the type I (false alarm) \( \alpha \) and II (signal missing) \( \beta \) error pro-
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babilities [6]. Under $\tau^* = \tau_0$ the expressions found in [9] can be used for calculation of probabilities $\alpha$ and $\beta$. Therefore, we consider a case $\tau^* \neq \tau_0$ ($\delta_\tau \neq 0$) below. Firstly, we believe that the useful signal is absent. Then the false-alarm probability can be presented in a form of

$$\alpha = P[\sup_{l \in \Gamma} L^*(l) > c] = 1 - F_N(c),$$

where $F_N(c) = P[ L^*(l) < c ], \ l \in \Gamma$.

From Eqs. (6), (8) follows that functional $L^*(l)$ in the absence of a useful signal (1) coincides with functional (25). Therefore, according to Eqs. (26), (27) for function $F_N(c)$ approximation (28) can be used. Substituting Eq. (28) in Eq. (37), we find the approximate expression for false-alarm probability:

$$\alpha \approx \begin{cases} 1 - \exp[-m(2c-1)\exp(-c)/(1+\delta_\tau)], & c \geq 3/2, \\ 1, & c < 3/2. \end{cases}$$

Accuracy of the formula (38) increases with $m$ and $c$.

Now let us believe that the useful signal (1) is present at the detector input. Then the missing probability is determined as

$$\beta = P[\sup_{l \in \Gamma} L^*(l) < c].$$

We divide the whole interval of possible values of the parameter $l_0$ into two subintervals: $\Gamma_S$ and $\Gamma_N$, and under $m >> 1$ similarly to [6, 7] write down Eq. (39) as

$$\beta \approx F_N(c) F_S(c),$$

where $F_N(c), F_S(c)$ are defined from Eqs. (23). Using asymptotically exact approximations (28), (33) of functions $F_N(c)$ and $F_S(c)$, for missing probability we find

$$\beta \approx \exp[-m(2c-1)\exp(-c)/(1+\delta_\tau)] \Psi_0[h(c), m_S]$$

if $c \geq 3/2$, and $\beta \equiv 0$ if $c < 3/2$. Accuracy of this formula increases with $c$, $m$, $\mu$, $\varsigma$.

It follows from Eqs. (38), (40) that characteristics coincide of the detection algorithms adapted for unknown intensity of an external disturbance and synthesized for a priori known SDs of a disturbance and white noise. Besides, offered detector remains effective if the external disturbance is absent, and SD of white noise is a priori unknown.
5 Results of Statistical Modeling

For the determination of applicability borders of found asymptotically exact formulas for detection and estimation characteristics the statistical computer modeling of quasi-likelihood receivers action was carried out. During modeling, following a technique stated in [7], functional \( \tilde{L}_3 = L_3/\tau_0 N_0 \) value and samples of functionals \( \tilde{L}_1(l) = L_1(l)/\tau_0 N_0 \) , \( \tilde{L}_2(l) = L_2(l)/\sqrt{\tau_0 N_0} \) (5), \( L^*(l) \) (6) were formed over an interval \( [\tilde{\lambda}_1, \tilde{\lambda}_2] \) with step \( \Delta l = 0.01 \) in both presence and absence of a signal (1). Thus, relative mean square errors of the generated value \( \tilde{L}_3 \) and step approximations of continuous realizations of processes \( \tilde{L}_1(l) \), \( \tilde{L}_2(l) \) on the basis of discrete samples did not exceed 10 %. For each realization \( x(t) \) the greatest sample of the functional (6) was compared to a threshold \( c \) and false-alarm and missing probabilities are calculated. Besides, according to Eqs. (9), (10) estimates \( l_q \), \( a_q \), \( D_q \) were defined and variances of estimates were found.

In Figs. 1-5 some results of statistical modeling are presented where corresponding theoretical dependences are also shown. Each experimental value was received as a result of processing of not less than \( 10^4 \) realizations \( x(t) \) under \( \tilde{\lambda}_1 = 0 \), \( \tilde{\lambda}_2 = m \) , \( K = m + 1 \) , \( l_0 = (\tilde{\lambda}_1 + \tilde{\lambda}_2)/2 \) , \( z_0^2 = 2a_l^2\tau_0/N_0 = 10 \) , \( q_v = 1 \) and \( \delta_v = -0.1 \) (Figs. 1a-5a) or \( \delta_v = 0.1 \) (Figs. 1b-5b). By solid lines in Fig. 1 the theoretical dependences of false-alarm probability \( \alpha(c) \) (38) are traced, and in Fig. 2 theoretical dependences of missing probability \( \beta(q_0) \) (40) are plotted under \( m = 20 \) and \( \mu = 50 \) (curves 1), 100 (curves 2), 200 (curves 3). By squares, crosses and rhombuses the experimental values of detection characteristics for \( \mu = 50 \), 100 and 200 are designated here. The threshold \( c \) for missing probability calculation was chosen by Neumann-Pirson criterion, according to the specified level of false-alarm probability \( \alpha = 0.01 \) using the formula (38).

![Fig. 1. The theoretical and experimental dependences of false-alarm probability](image-url)
In Figs. 3 solid lines represent dependences (22) of the normalized conditional variance $\tilde{V}_l(q_0) = 12V(l_q, l_0)/m_a^2$ of the QLE $l_q$ taking into account anomalous errors, if $m = m_a = 20$. Here for $m = 1 + \delta_{\tau}$ analogous dependences (19) of the normalized conditional variance $\tilde{V}_{0l}(q_0) = 12V_0(l_q, l_0)/m_a^2$ of the reliable QLE $l_q$ are also drawn by dashed lines. Curves 1 are calculated with $\mu = 50$, 2 – 100, 3 – 200. The experimental values of variances $\tilde{V}_l$, $\tilde{V}_{0l}$ of the estimate $l_q$ with anomalous errors and reliable estimate $l_q$ are designated by squares, crosses, rhombuses and pluses, circles, triangles for $\mu = 50$, 100, 200 accordingly.

In Figs. 4 theoretical dependences (36) of normalized conditional variance $\tilde{V}_\eta(q_0) = 2V_0(a_q|a_0)/E_N$ of the QLE $a_q$ are drawn. The curve 1 corresponds $\mu = 50$, 2 – 100; 3 – 200. Experimental values of variances $\tilde{V}_\eta$ for $\mu = 50$, 100, 200 are shown by pluses, circles, triangles (when $m = 1 + \delta_{\tau}$ and the appearance time estimate is reliable), and by squares, crosses and rhombuses (when $m = 20$ and anomalous errors are possible measuring appearance time).
Finally, in Figs. 5 dependences of normalized conditional variance $\hat{V}_q(q_0) = V_0(D_q|D_0)/E_N^2$ (36) of the QLE $D_q$ are plotted by solid lines for $m = 20$, and by dashed lines – for $m = 1 + \delta_\tau$. Here there are drawn the experimental variance dependences $\tilde{V}_q$ received with both anomalous errors while measuring the appearance time of a random pulse ($m = 20$) and reliable appearance time estimate ($m = 1 + \delta_\tau$). Other designations in Figs. 5 are the same, as in Fig. 4.
Adaptive detection and time and power parameters estimation

As follows from Figs. 1-3, the theoretical dependences for probabilities $\alpha$ (38), $\beta$ (40) and variances $V_0(\ell_q \parallel \ell_0)$ (19), $V(\ell_q \parallel \ell_0)$ (22) well approximate experimental data, at least, under $\mu \geq 50$, $q_0 \geq 0.1$ and $z > 1…1.5$. With decreasing $q_0$, when SNR $z < 3…4$, the anomalous error probability $P_a$ (35) increases considerably and tends to 1. It leads to a jump-like (in comparison with reliable estimate) increase in QLE $I_q$ variance. With $q_0$ growth, when $z > 3…4$, variances $V_0(\ell_q \parallel \ell_0)$, $V(\ell_q \parallel \ell_0)$ coincide practically, and the appearance time estimate of a random pulse (1) becomes reliable with the probability close to 1.

According to Figs. 4, 5, formulas (36) for variances of QLEs $a_q$ and $D_q$ (10) well approximate corresponding experimental data, if $z \geq 2…3$.

5 Conclusion

The application of a maximum likelihood method, in a case when the values of the order of correlation time of a received fast-fluctuating random signal are neglected, allows to obtain simpler detectors and measurers of a band low-frequency Gaussian pulse with unknown time-power parameters and inexact known duration against white noise and band Gaussian disturbances. In particular, application of the adaptive approach for the reduction of prior uncertainly, and in view of external disturbance intensity, helps us to determine the detection and estimation algorithms of a band pulse signal, also invariant to white noise intensity. Characteristics of adaptive detection and appearance time

Fig. 5. The theoretical and experimental dependences of normalized variance of dispersion estimate
and mathematical expectation estimates of a random pulse coincide asymptotically with corresponding characteristics of detection and estimates for a case of a priori known spectral densities of a disturbance and noise. Then, if observation interval length is much greater compared to a useful signal duration or the external disturbance bandwidth surpasses essentially pulse random substructure bandwidth, then there are no sacrifices of accuracy under adaptive dispersion estimation due to ignorance of hindrance and white noise intensities.

The received results make it possible to perform a valid choice between offered and other detection and estimation algorithms of low-frequency random pulse signals with unknown time and power parameters and in the presence of pulse duration detuning observed against disturbances with unknown intensities depending on the available prior information and the required accuracy and simplicity of hardware detector or measurer implementation.

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References


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