Corrigendum to

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Abstract

There is a small gap in the proof of Theorem 4.5 of the mentioned paper. Here the authors give the complete proof.

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In the mentioned paper, the so called limit rotation loop of the hyperbolic plane over a Euclidean field $K$, introduced in [1], is studied in its embedding in the 3-dimensional projective space over $K$ as a transversal of the coset space $G/D$, where $G = PSL_2(K)$ and $D$ is the subgroup of the hyperbolic rotations
fixing a given point \( o \). Moreover the automorphism group of such loop is determined in \textbf{Theorem 4.5}. This statement is correct but unfortunately the authors themselves have found a small gap in the proof. Therefore, for the sake of clarity and completeness, we have:

1. at page 5875, line 4, to replace \( PSL_2(K) \) with \( PGL_2(K) \);

2. in the proof of \textbf{Theorem 4.5},
   - to put \( \text{Aut}(PSL_2(K)) = \text{Aut}(PGL_2(K)) \) at page 5877, end of line 5, and
   - to complete lines 11-12 of the same page in the following way: "this result follows noticing that, for all \( G \in PGL_2(K) \), \( \psi_G(\Lambda^+) = \Lambda^+ \) implies that \( G \in PSL_2(K) \), and:"

For more clarity, we write here completely the corrected version of the mentioned theorem with its proof (pages 5876-5877):

\textbf{Theorem 4.5.} \textit{Let \( (H, \oplus) \) be the limit rotation loop of a general hyperbolic plane over a Euclidean field \( K \). Then}

\[ \text{Aut}(H, \oplus) \cong \Psi_D \rtimes \text{K} \]

\textit{where \( \Psi_D \) is the subgroup of \( \Psi \) made up of collineations of \( PG(3, K) \) derived from inner automorphisms corresponding to the elements of the group \( D \) and \( \text{K} \) is the group of pure semilinear collineations, namely made up of elements \( \alpha \in \text{K} \) such that}

\[ \overline{\alpha} : \left\{ \begin{array}{c} PG(3, K) \\ K^*(x_1, x_2, x_3, x_4) \end{array} \right\} \rightarrow PG(3, K) \]

\[ K^*(x_1^\alpha, x_2^\alpha, x_3^\alpha, x_4^\alpha) \]

\textit{where \( \alpha \in \text{Aut}(K, +, \cdot) \).}

\textit{Proof.} In the group \( PSL_2(K) \) consider the fibration made up of the centralizers of each element. It is well known that this fibration is characteristic, and, in our representation of the group \( PSL_2(K) \) as a subset of the pointset of the projective space \( PG(3, K) \), it corresponds precisely to the lines of \( PG(3, K) \) through the point \( 1 \). By [3, props 2.1 and 2.3] the group \( \text{Aut}(PSL_2(K)) = \text{Aut}(PGL_2(K)) \) is precisely the subgroup of collineations of the projective space fixing \( 1 \) and preserving the quadric \( Q \), moreover by [3, Thm 1] it holds

\[ \text{Aut}(PSL_2(K)) \cong \Psi \rtimes \text{K}. \]

Hence, according to the previous proposition, the subgroup \( T \leq \text{Aut}(PSL_2(K)) \) is isomorphic to a subgroup of collineations of \( PG(3, K) \), and to prove the statement it remains only to show that \( T \cong \Psi_D \rtimes \text{K} \). This result follows noticing that for all \( G \in PGL_2(K) \), \( \psi_G(\Lambda^+) = \Lambda^+ \) implies that \( G \in PSL_2(K) \), and:
1. for all $G \in PSL_2(K)$ it holds $\psi_G(D) = D$ if and only if $G \in D$, thus $\Psi_D = \{\psi_G \in \Psi \mid \psi_G(D) = D \text{ and } \psi_G(\Lambda^+) = \Lambda^+\};$

2. since the field $K$ is euclidean, each $\alpha \in K$ preserves the ordering in $K$, thus by 3.5 it preserves the half cone $\Lambda^+$; moreover it is straightforward to see that $\overline{\alpha}(D) = D$. 

\[ \square \]

References


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