Corrigendum to

S. Pasotti, S. Pianta: *The limit rotation loop of a hyperbolic plane*,
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Abstract

There is a small gap in the proof of Theorem 4.5 of the mentioned paper. Here the authors give the complete proof.

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In the mentioned paper, the so called limit rotation loop of the hyperbolic plane over a Euclidean field $\mathbf{K}$, introduced in [1], is studied in its embedding in the 3-dimensional projective space over $\mathbf{K}$ as a transversal of the coset space $G/\mathcal{D}$, where $G = PSL_2(\mathbf{K})$ and $\mathcal{D}$ is the subgroup of the hyperbolic rotations

fixing a given point o . Moreover the automorphism group of such loop is determined in **Theorem 4.5**. This statement is correct but unfortunately the authors themselves have found a small gap in the proof.

Therefore, for the sake of clarity and completeness, we have:

1. at page 5875, line 4, to replace $PSL_2(\mathbf{K})$ with $PGL_2(\mathbf{K})$;
2. in the proof of **Theorem 4.5**,
 - to put $\text{Aut}(PSL_2(\mathbf{K})) = \text{Aut}(PGL_2(\mathbf{K}))$ at page 5877, end of line 5, and
 - to complete lines 11-12 of the same page in the following way: "this result follows noticing that, for all $G \in PGL_2(\mathbf{K})$, $\psi_G(\Lambda^+) = \Lambda^+$ implies that $G \in PSL_2(\mathbf{K})$, and:".

For more clarity, we write here completely the corrected version of the mentioned theorem with its proof (pages 5876-5877):

Theorem 4.5. *Let $(\mathbf{H}, \oplus)$ be the limit rotation loop of a general hyperbolic plane over a Euclidean field $\mathbf{K}$. Then*

$$\text{Aut}(\mathbf{H}, \oplus) \cong \Psi_{\mathcal{D}} \rtimes \overline{\mathbf{K}}$$

where $\Psi_{\mathcal{D}}$ is the subgroup of Ψ made up of collineations of $PG(3, \mathbf{K})$ derived from inner automorphisms corresponding to the elements of the group $\mathcal{D}$ and $\overline{\mathbf{K}}$ is the group of pure semilinear collineations, namely made up of elements $\overline{\alpha} \in \overline{\mathbf{K}}$ such that

$$\overline{\alpha} : \begin{cases} PG(3, \mathbf{K}) & \rightarrow PG(3, \mathbf{K}) \\ \mathbf{K}^*(x_1, x_2, x_3, x_4) & \mapsto \mathbf{K}^*(x_1^\alpha, x_2^\alpha, x_3^\alpha, x_4^\alpha) \end{cases}$$

where $\alpha \in \text{Aut}(\mathbf{K}, +, \cdot)$.

Proof. In the group $PSL_2(\mathbf{K})$ consider the fibration made up of the centralizers of each element. It is well known that this fibration is characteristic, and, in our representation of the group $PSL_2(\mathbf{K})$ as a subset of the pointset of the projective space $PG(3, \mathbf{K})$, it corresponds precisely to the lines of $PG(3, \mathbf{K})$ through the point $\mathbf{1}$. By [3, props 2.1 and 2.3] the group $\text{Aut}(PSL_2(\mathbf{K})) = \text{Aut}(PGL_2(\mathbf{K}))$ is precisely the subgroup of collineations of the projective space fixing $\mathbf{1}$ and preserving the quadric $\mathcal{Q}$, moreover by [3, Thm 1] it holds

$$\text{Aut}(PSL_2(\mathbf{K})) \cong \Psi \rtimes \overline{\mathbf{K}}.$$

Hence, according to the previous proposition, the subgroup $T \leq \text{Aut}(PSL_2(\mathbf{K}))$ is isomorphic to a subgroup of collineations of $PG(3, \mathbf{K})$, and to prove the statement it remains only to show that $T \cong \Psi_{\mathcal{D}} \rtimes \overline{\mathbf{K}}$. This result follows noticing that for all $G \in PGL_2(\mathbf{K})$, $\psi_G(\Lambda^+) = \Lambda^+$ implies that $G \in PSL_2(\mathbf{K})$, and:

1. for all $G \in PSL_2(\mathbf{K})$ it holds $\psi_G(\mathcal{D}) = \mathcal{D}$ if and only if $G \in \mathcal{D}$, thus $\Psi_{\mathcal{D}} = \{\psi_G \in \Psi \mid \psi_G(\mathcal{D}) = \mathcal{D} \text{ and } \psi_G(\Lambda^+) = \Lambda^+\}$;
2. since the field $\mathbf{K}$ is euclidean, each $\bar{\alpha} \in \overline{\mathbf{K}}$ preserves the ordering in $\mathbf{K}$, thus by 3.5 it preserves the half cone Λ^+ ; moreover it is straightforward to see that $\bar{\alpha}(\mathcal{D}) = \mathcal{D}$.

□

References

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- [3] S. Pianta and E. Zizioli, *Collineations of geometric structures derived from quaternion algebras*, J. Geom. **37** (1990), no. 1-2, 142–152.

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