Representation and Ranking of Fuzzy Numbers with Heptagonal Membership Function Using value and Ambiguity Index

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Abstract
In many real life problems, the parameters used to characterize the uncertainty or vagueness or impreciseness in information are either triangular or trapezoidal fuzzy numbers. But it is not always possible to restrict the membership function to either triangular or trapezoidal form. In this paper, a new form of non-normal fuzzy number named as Heptagonal fuzzy number is introduced and its arithmetic operations are defined. Further, an attempt has been made to give a new version of definition for value and ambiguity of non-normal fuzzy numbers and based on this new definition a parametric ranking method is proposed for ordering Heptagonal fuzzy numbers. Finally, an application of Heptagonal fuzzy number in fuzzy assignment problem is given and it is proved that the optimal value obtained using Heptagonal Fuzzy number is more optimal than the solution obtained by using Trapezoidal fuzzy numbers.
Keywords: Heptagonal fuzzy numbers, Value and Ambiguity, Non-normal fuzzy numbers, Ranking of fuzzy numbers, Fuzzy Assignment Problem.

1 Introduction

In most of the real life problems, information available are sometimes insufficient or sometimes available as vague linguistic descriptions such as “about 10 hours”, “high profit” etc. To deal with this, the problem has to be modeled with approximately available data. This can be done using Zadeh’s fuzzy set theory [11]. Lei [7] represented the processing time of operation in the fuzzy job shop scheduling problem with availability constraints using Triangular fuzzy numbers. Kumar et al. [6] used Triangular fuzzy numbers whereas Sagaya & Henry [10] used Trapezoidal fuzzy numbers to represent the fuzzy cost or fuzzy times in the Fuzzy Assignment Problem. Mahapatra [9] considered the cost of components as triangular fuzzy numbers in series system models with system reliability and cost. Kaur & Kumar [4] presented a model to solve the transportation problems in fuzzy environment using non-normal Generalized Trapezoidal fuzzy numbers. There are several papers in the literature in which the authors used triangular or trapezoidal fuzzy numbers in order to characterize the vague parameters that arise in real life problems. But in few cases where vagueness arise in seven different points, it is not possible to restrict the membership functions to take either triangular or trapezoidal form.

Hence in this paper, a new form of fuzzy number named as Heptagonal fuzzy number is introduced and the laborious tasks namely fuzzy arithmetic operations and ranking of the Heptagonal Fuzzy numbers are also proposed. This paper mainly focuses to deal with non-normal Heptagonal fuzzy number since it takes the degree of confidence of the decision maker’s opinions into account and also non-normal fuzzy numbers are more flexible and more intelligent than the normalized fuzzy numbers.

The paper is organized as follows: In section 2, the basic definitions and notations of fuzzy numbers are given. In section 3, the Heptagonal fuzzy numbers are defined and based on the function principle [1], the arithmetic operations such as addition, subtraction, scalar multiplication of heptagonal fuzzy numbers are defined. In section 4, a new version of definition for the pair of index called value and ambiguity of non-normal fuzzy numbers are given and based on the new definition, a parametric ranking method is proposed to order the Heptagonal fuzzy numbers. In section 5, an application of Heptagonal fuzzy numbers in fuzzy assignment problem is given with a numerical example.

2 Basic definitions

In this section, some basic definitions of fuzzy set theory and fuzzy numbers are reviewed [3].
Definition 2.1 Fuzzy Set
A fuzzy set is characterized by a membership function mapping the elements of a domain space or universe of discourse X to the unit interval [0,1]. (i.e) \( \mu_{\tilde{A}} : X \rightarrow [0,1] \)

Definition 2.2 Support of a fuzzy set
The support of a fuzzy set \( \tilde{A} \) in the universal set X is the set that contains all the elements of X that have a non-zero membership grade in \( \tilde{A} \).
That is, \( \text{Supp}(\tilde{A}) = \{ x \in X | \mu_{\tilde{A}}(x) > 0 \} \)

Definition 2.3 Core of a fuzzy set
The core of a fuzzy set \( \tilde{A} \) in the universal set X is the set that contains all the elements of X that exhibit a unit level of membership in \( \tilde{A} \).
That is, \( \text{Core}(\tilde{A}) = \{ x \in X | \mu_{\tilde{A}}(x) = 1 \} \)

Definition 2.4 \( \alpha \)–cut of fuzzy set
An \( \alpha \)–cut of a fuzzy set \( \tilde{A} \) is a crisp set \( A_{\alpha} \) defined as \( A_{\alpha} = \{ x \in X | \mu_{\tilde{A}}(x) \geq \alpha \} \)

Definition 2.5 Convex Fuzzy Set
A fuzzy set \( \tilde{A} \) is a convex fuzzy set if and only if each of its \( \alpha \)–cuts \( A_{\alpha} \) is a convex set.

Definition 2.6 Fuzzy Numbers
A fuzzy set \( \tilde{A} \) is a fuzzy number iff (i) For all \( \alpha \in (0,1] \) the \( \alpha \)–cuts \( A_{\alpha} \) is a convex set (ii) \( \mu_{\tilde{A}} \) is an upper semi continuous function (iii) \( \text{Supp}(\tilde{A}) \) is a bounded set in R (iv) The height of \( \tilde{A} = \max_{x \in X} \mu_{\tilde{A}}(x) = \omega > 0 \)

3 Heptagonal Fuzzy Numbers (HFN)
In this section, a new form of fuzzy number called as Heptagonal Fuzzy number (HFN) is introduced which can be effectively used in solving many decision making problems.

Definition 3.1
The parametric form of Heptagonal Fuzzy Number is defined as \( \tilde{H} = (f_1(r),g_1(t),g_2(t),f_2(r)) \), for \( r \in [0,k] \) and \( t \in [k,1] \) where \( f_1(r) \) and \( g_1(t) \) are bounded left continuous non decreasing functions over \([0,\omega_1]\) and \([k,\omega_2]\) respectively, \( f_2(r) \) and \( g_2(t) \) are bounded left continuous non increasing functions over\([0,\omega_1]\) and \([k,\omega_2]\) respectively and \( 0 \leq \omega_1 \leq k, k \leq \omega_2 \leq 1 \)

Definition 3.2
The membership function for the heptagonal fuzzy number \( \tilde{H} = (h_1,h_2,h_3,h_4,h_5,h_6,h_7;k,\omega) \) is defined as follows
Remark 3.1
(i) The above defined heptagonal fuzzy number becomes normal heptagonal fuzzy number if \( \omega = 1 \)
(ii) If \( k = 0 \), then the heptagonal fuzzy number reduces to triangular fuzzy number \((h_3, h_4, h_5)\) and if \( k = 1 \), it reduces to trapezoidal fuzzy number \((h_1, h_2, h_6, h_7)\)

3.1 Graphical representation of Heptagonal Fuzzy number

![Graphical representation of Heptagonal Fuzzy Numbers](image)

Definition 3.3
For \( \alpha \in (0,1] \), the \( \alpha \)-cut of HFN, \( \tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7; k, \omega) \) is defined as

\[
[\tilde{H}]_{\alpha} = \begin{cases} 
[h_1 + \frac{\alpha}{k} (h_2 - h_1), & h_7 - \frac{\alpha}{k} (h_7 - h_6)] \\
[h_3 + \frac{\alpha - k}{\omega - k}(h_4 - h_3), & h_5 - \frac{\alpha - k}{\omega - k}(h_5 - h_4)]
\end{cases} 
\text{ for } \alpha \in [0, k]
\]

\[
[\tilde{H}]_{\alpha} = \begin{cases} 
[h_1 + \frac{\alpha - k}{\omega - k}(h_2 - h_1), & h_7 - \frac{\alpha - k}{\omega - k}(h_7 - h_6)] \\
[h_3 + \frac{\alpha - k}{\omega - k}(h_4 - h_3), & h_5 - \frac{\alpha - k}{\omega - k}(h_5 - h_4)]
\end{cases} 
\text{ for } \alpha \in (k, \omega]
\]
Representation and ranking of fuzzy numbers

Remark 3.2
The $\alpha$–cuts of Heptagonal fuzzy number are convex sets and so the heptagonal fuzzy number is convex.

3.2 Need for Heptagonal Fuzzy numbers

If Heptagonal Fuzzy number is compared with most commonly used Trapezoidal or Triangular Fuzzy numbers, it looks complex both in its form as well as on its computational basis. But Heptagonal Fuzzy number gives additional possibility to represent imperfect knowledge what leads to model many real life problems in a more adequate way. HFN provides the flexibility to the decision maker to give his/her view using two different heights $k$ and $\omega$. Moreover the HFN represents the information in a detailed manner and also the vagueness can be represented in more realistic and natural way. The information is retained throughout the manipulation and the detailed information can be used by the decision maker for further analysis. HFN can find its applications in many optimization problems and in decision making problems which need seven parameters. In particular case of dynamics of tumor growth, the growth rate consists of seven points and is difficult to represent by using Triangular or Trapezoidal Fuzzy numbers. Therefore Heptagonal fuzzy number can find its vital applications in solving the problem.

3.3 Arithmetic Operations on Heptagonal Fuzzy numbers

In this section, Chen’s method based on the function principle [1] is used to develop arithmetic operations between heptagonal fuzzy numbers. The function principle is basically a point wise operation and is more useful than the extension principle for the fuzzy numbers with heptagonal membership function because the function principle preserves the membership form whereas it is not the case in extension principle.

3.3.1. Addition of two heptagonal fuzzy numbers

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; k_A, \omega_A)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7; k_B, \omega_B)$ be two heptagonal fuzzy numbers. First bring the HFN $\tilde{A}$ and $\tilde{B}$ to the common maximum by truncating the higher one, that is, take $k = \min\{k_A, k_B\}$ and $\omega = \min\{\omega_A, \omega_B\}$. Then the resulting number is clearly heptagonal fuzzy number which is illustrated in fig. 2 and the arithmetic operation addition is obtained as

$$\tilde{A} \oplus \tilde{B} = (c_1, c_2, c_3, c_4, c_5, c_6, c_7; k, \omega) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7; \min\{k_A, k_B\}, \min\{\omega_A, \omega_B\})$$
3.3.2. Scalar Multiplication of heptagonal fuzzy numbers

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; k_A, \omega_A)$ be a heptagonal fuzzy number and $\lambda$ be a scalar. The Scalar Multiplication is given by

$$\lambda \tilde{A} = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4, \lambda a_5, \lambda a_6, \lambda a_7; k_A, \omega_A) & \text{if } \lambda > 0 \\ (\lambda a_7, \lambda a_6, \lambda a_5, \lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; k_A, \omega_A) & \text{if } \lambda < 0 \end{cases}$$

3.3.3. Subtraction of two heptagonal fuzzy numbers

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; k_A, \omega_A)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7; k_B, \omega_B)$ be two heptagonal fuzzy numbers. The arithmetic operation subtraction is obtained as

$$\tilde{A} - \tilde{B} = \tilde{A} \oplus (\tilde{B})^{-1} = (a_1 - b_7, a_2 - b_6, a_3 - b_5, a_4 - b_4, a_5 - b_3, a_6 - b_2, a_7 - b_1; \min\{k_A, k_B\}, \min\{\omega_A, \omega_B\})$$

4 Ranking of Heptagonal Fuzzy numbers

Most of the ranking methods in the literature assume the membership functions to be normal and this is not adequate in many cases. To overcome the limitations, a new version of definition for a pair of index called value and ambiguity of non-normal fuzzy numbers is presented in this section. Also a parametric method based on the new definition is proposed to order the Heptagonal Fuzzy numbers which are non-normal in nature.

**Definition 4.1 [2]**

The value of the normal fuzzy number $\tilde{A}$ with respect to the parameters $\lambda$ and $r$ is defined as

$$V_\lambda(A; r) = (1 - \lambda)V_0(A; r) + \lambda V^*(A; r)$$

where lower value and upper
value of \( \tilde{A} \) are respectively \( V_\alpha(\tilde{A}; r) = \int_0^1 A^L_\alpha \ d(\alpha') \) and \( V^*(\tilde{A}; r) = \int_0^1 A^U_\alpha \ d(\alpha') \) and \( \alpha \)-cut of \( \tilde{A} = [A^L_\alpha, A^U_\alpha] \).

Using normalization procedure the definition for value can be extended from normal fuzzy numbers to non-normal fuzzy numbers as

**Definition 4.2**

Let \( \tilde{A} \) be a non normal fuzzy number with height \( \omega \) and its \( \alpha \)-cut \( A^L_\alpha, A^U_\alpha \). Define the value of \( \tilde{A} \) as \( V_\alpha(\tilde{A}; r) = (1 - \lambda) V_\alpha(\tilde{A}; r) + \lambda V^*(\tilde{A}; r) \) where

\[
V_\alpha(\tilde{A}; r) = \frac{1}{(\omega)^r} \int_0^\omega A^L_\alpha \ d(\alpha') \quad \text{and} \quad V^*(\tilde{A}; r) = \frac{1}{(\omega)^r} \int_0^\omega A^U_\alpha \ d(\alpha')
\]

**Definition 4.3**

If \( \tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7; k, \omega) \) is heptagonal fuzzy number, the lower and upper values of \( \tilde{H} \) are defined as

\[
V_\alpha(\tilde{H}; r) = \frac{1}{(\omega)^r} \int_0^\omega H^L_\alpha \ d(\alpha') \quad \text{and} \quad V^*(\tilde{H}; r) = \frac{1}{(\omega)^r} \int_0^\omega H^U_\alpha \ d(\alpha')
\]

The value of \( \tilde{H} \) is \( V_\alpha(\tilde{H}; r) = (1 - \lambda) V_\alpha(\tilde{A}; r) + \lambda V^*(\tilde{A}; r) \) where \( \lambda \) is a real number connected with the optimistic/pessimistic point of view of the decision maker and \( r > 0 \) attributes different weights to each level as per preference.

**Definition 4.4**

Ambiguity is a measure of vagueness, that is, the lack of precision in determining the exact value of a magnitude. The ambiguity of the heptagonal fuzzy number \( \tilde{H} \) is defined as \( \text{Amb}(A; r) = \frac{1}{(\omega)^r} \int_0^\omega \left( \frac{H^U_\alpha - H^L_\alpha}{2} \right) \ d(\alpha') \)

**Remark 4.1**

The value of the heptagonal fuzzy number \( \tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7; k, \omega) \) is given by \( \text{val}(\tilde{H}; r) = (1 - \lambda) V_\alpha(\tilde{H}; r) + \lambda V^*(\tilde{H}; r) \) where the lower value and upper value of \( \tilde{H} \) are given by \( V_\alpha(\tilde{H}; r) = \gamma \left( \frac{h_1 + rh_2}{r+1} \right) + h_3 (1 - \gamma) + \eta (h_4 - h_3) \) and

\[
V^*(\tilde{H}; r) = \gamma \left( \frac{h_2 + rh_6}{r+1} \right) + h_3 (1 - \gamma) - \eta (h_3 - h_4) \quad \text{where} \quad \gamma = \left( \frac{k}{\omega} \right)^r \quad \text{and} \quad \eta = \frac{r}{r+1} \left( 1 - \frac{k(\omega)^r - k^r}{r(\omega - k)} \right).
\]

**Remark 4.2**

The ambiguity of the heptagonal fuzzy number \( \tilde{H} \) is given by

\[
\text{Amb}(\tilde{H}) = \frac{1}{2} \left( V^*(\tilde{H}; r) - V_\alpha(\tilde{H}; r) \right)
\]
4.1 Summary of the proposed ranking method

The usual ranking method based on center of gravity, area compensation, centroid deal with the geometric shape of the fuzzy numbers, but do not take into account the personal opinion of the decision maker. But, the proposed ranking technique depends on two parameters \( \lambda \) and \( r > 0 \) which can be handled by the decision maker.

The parameter \( \lambda \) can be chosen by the decision maker according to his optimistic or pessimistic point of view. \( \lambda \in [0,1/2) \) if the decision maker is optimistic, \( \lambda \in (1/2,1] \) if the decision maker is pessimistic and \( \lambda = 1/2 \) if the decision maker is neutral in his view. The real number \( r > 0 \) allows the decision maker to attribute different weights to each level \( \alpha \) according to his preference.

For the two HFN \( \tilde{A} \) and \( \tilde{B} \), the HFN with higher value index is ranked above the HFN having lower value index. Suppose if the value index of both HFN are equal, then the HFN with lower ambiguity index is ranked above the HFN having higher ambiguity index. The index of ambiguity suggested will characterize the global spread of the membership function of HFN. The flowchart for the criterion adopted for ranking HFN using the pair of index is shown in Fig. 3

![Flow chart for the proposed parametric Ranking method](image)

5 Fuzzy Assignment Problem

Assignment problem is very often used in solving problems of engineering and management science. It is a special type of linear programming problem
which deals with assigning various jobs or tasks or sources to an equal number of persons or machines or destinations in such a way that the total assignment cost or processing time is minimized. Due to the high degeneracy nature of Assignment Problem, H.W.Kuhn [5] introduced a specially designed algorithm called Hungarian method for solving Assignment Problems in crisp environment. Pentico [8] provided a comprehensive survey of different variants of the assignment problems appearing in the literature over the past 50 years. If the parameter, cost of assigning the jobs to persons or tasks to machines are uncertain in nature then the fuzzy assignment problem will arise.

5.1 Mathematical formulation of Fuzzy Assignment Problem

The fuzzy assignment problem can be mathematically stated as follows

Minimize \( \tilde{Z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{C}_{ij} x_{ij} \)

Subject to \( \sum_{j=1}^{n} x_{ij} = 1, \ i = 1,2,...n \)
\( \sum_{i=1}^{n} x_{ij} = 1, \ j = 1,2,...n \)
\( x_{ij} \in \{0,1\} \)

where

- the assignment cost \( \tilde{C}_{ij} \) is the cost of assigning \( i^{th} \) person to the \( j^{th} \) job and the uncertainty in cost is here represented as heptagonal fuzzy number,
- the decision variable \( x_{ij} \) denotes the assignment of person \( i \) to the job \( j \), such that \( x_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ person is assigned to } j^{th} \text{ job} \\ 0 & \text{otherwise} \end{cases} \)
- The objective is to minimize the assignment cost such that only one person is assigned to one job.

5.2 Fuzzy Hungarian Algorithm to solve Fuzzy Assignment problem

In this section, the Fuzzy Hungarian algorithm given in [10] is extended to solve fuzzy assignment problem using Heptagonal Fuzzy numbers and using the proposed ranking technique given in section 4.

**Step 1:** Construct the fuzzy cost table from the given information and represent the uncertainty in cost by Heptagonal fuzzy numbers.

**Step 2:** If the number of rows is equal to the number of columns, then go to Step 3. Otherwise, add the required number of dummy rows or dummy columns with \( \tilde{0} \) heptagonal fuzzy cost so that the cost table becomes a square matrix.

**Step 3:** Calculate the ranking value of each cost in the fuzzy cost table. From each row, select the row minimum (that is, fuzzy cost with least ranking value in that row) and subtract each fuzzy cost in the row by the row minimum.
**Step 4:** From the resulting fuzzy cost table, select the column minimum (that is, fuzzy cost with least ranking value in that column) and subtract each fuzzy cost in the column by the column minimum. Now each row and column has at least one fuzzy zero.

**Step 5:** In the reduced fuzzy cost table obtained in Step 4, search for fuzzy optimal assignment as follows
i. Examine the rows successively until a row with a single fuzzy zero is found. Assign this fuzzy zero and cross off all other fuzzy zeros in its column. Continue this for all rows.
ii. Repeat the procedure for each column of reduced fuzzy cost table.
iii. If the row and/or column has two or more fuzzy zeros assign arbitrarily any one of these fuzzy zeros and cross off all other fuzzy zeros of that row or column.
iv. Repeat (i) through (iv) above successively until the chain of assigning or cross ends.

**Step 6:** If each row and each column has exactly only one fuzzy zero assignment, then the fuzzy optimal solution is reached. If not, go to step 7.

**Step 7:** Draw the minimum number of horizontal/vertical lines to cover all the fuzzy zeros of the reduced cost matrix and construct a new revised cost matrix as follows:
   a. Find the least fuzzy cost using ranking value among the entries that are not covered by any of the lines.
   b. Subtract this entry from all the uncovered entries and add the same to all the entries lying at the intersection of any two lines.

**Step 8:** Repeat the process from Step 5 to Step 7 until fuzzy optimal solution to the given fuzzy assignment problem is attained.

### 5.3 Numerical example

A manufacturing company manufactures a certain type of spare parts with three different machines. The company official has to execute three jobs with three machines. The information about the cost of assignment is imprecise and here Heptagonal Fuzzy numbers are used to represent the cost. The fuzzy assignment problem is given in Table 1.

<table>
<thead>
<tr>
<th>JOB</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
</tr>
</thead>
<tbody>
<tr>
<td>J1</td>
<td>(1,4,6,8,11,14,16,0,4,0,8)</td>
<td>(8,9,10,13,15,17,18,0,3,0,7)</td>
<td>(6,9,10,12,13,14,15,0,6,0,9)</td>
</tr>
<tr>
<td>J2</td>
<td>(1,1,4,6,18,22,24,26,0,6,0,9)</td>
<td>(9,13,15,18,20,22,23,0,5,0,8)</td>
<td>(1,5,18,20,22,24,27,28,0,6,1)</td>
</tr>
<tr>
<td>J3</td>
<td>(7,10,12,14,16,18,20,0,3,0,8)</td>
<td>(11,12,15,17,19,20,21,0,4,0,9)</td>
<td>(10,12,14,15,18,20,22,0,4,0,7)</td>
</tr>
</tbody>
</table>

*Table 1 Fuzzy Assignment Problem with Heptagonal Fuzzy Cost*
The ranking value of each fuzzy cost calculated by taking parameters as $\lambda = 0.5$ and $r = 1$ is given in Table 2.

<table>
<thead>
<tr>
<th>JOB</th>
<th>Machines</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M1</td>
<td>M2</td>
<td>M3</td>
<td></td>
</tr>
<tr>
<td>J1</td>
<td>$V(\tilde{c}_{11}) = 8.5$</td>
<td>$V(\tilde{c}_{12}) = 12.857$</td>
<td>$V(\tilde{c}_{13}) = 11.25$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{Amb}(\tilde{c}_{11}) = 3.75$</td>
<td>$\text{Amb}(\tilde{c}_{12}) = 2.643$</td>
<td>$\text{Amb}(\tilde{c}_{13}) = 2.583$</td>
<td></td>
</tr>
<tr>
<td>J2</td>
<td>$V(\tilde{c}_{21}) = 18.667$</td>
<td>$V(\tilde{c}_{22}) = 17.125$</td>
<td>$V(\tilde{c}_{23}) = 22$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{Amb}(\tilde{c}_{21}) = 4.667$</td>
<td>$\text{Amb}(\tilde{c}_{22}) = 4.0625$</td>
<td>$\text{Amb}(\tilde{c}_{23}) = 3.7$</td>
<td></td>
</tr>
<tr>
<td>J3</td>
<td>$V(\tilde{c}_{31}) = 13.906$</td>
<td>$V(\tilde{c}_{32}) = 16.555$</td>
<td>$V(\tilde{c}_{33}) = 15.714$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{Amb}(\tilde{c}_{31}) = 2.594$</td>
<td>$\text{Amb}(\tilde{c}_{32}) = 2.55$</td>
<td>$\text{Amb}(\tilde{c}_{33}) = 2.714$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Ranking value of Heptagonal Fuzzy cost for the parameters $\lambda = 0.5$ and $r = 1$

Row reduction gives the following reduced matrix

$$
\begin{pmatrix}
0 & (-8,-5,-1,5,9,13,17;0,3,0.7) & (-10,-5,-1,4,7,10,14;0,4,0.8) \\
(-12,-8,-4,0,7,11,17;0,5,0.8) & 0 & (-8,-4,0,4,9,14,19;0,5,0.8) \\
0 & (-9,-6,-1,3,7,10,14;0,3,0.8) & (-10,-6,-2,1,6,10,15;0,3,0.7)
\end{pmatrix}
$$

Column reduction gives the following reduced matrix

$$
\begin{pmatrix}
0 & (-8,-5,-1,5,9,13,17;0,3,0.7) & (-25,-15,-7,3,9,16,24;0,3,0.7) \\
(-12,-8,-4,0,7,11,17;0,5,0.8) & 0 & (-23,-14,-6,3,11,20,29;0,3,0.7) \\
0 & (-9,-6,-1,3,7,10,14;0,3,0.8) & 0
\end{pmatrix}
$$

Making the assignment and checking for optimality as given in algorithm in section 5.2 produce the optimal assignment as $J_1 \rightarrow M_1$, $J_2 \rightarrow M_2$, $J_3 \rightarrow M_3$ with the optimal assignment cost $\tilde{C} = (20,29,35,41,49,56,61;0,3,0.7)$ and its crisp value is 41.5.

5.4 Solving the Example using Trapezoidal Fuzzy cost

Represent the Heptagonal fuzzy cost in Table 1 by trapezoidal fuzzy cost and the converted problem is given in Table 3. It is solved using the technique adopted in [10]. The fuzzy optimal assignment is obtained as $J_1 \rightarrow M_1$, $J_2 \rightarrow M_2$, $J_3 \rightarrow M_1$ and the optimal assignment cost is $\tilde{C} = (20,29,56,61)$ with crisp value 42.17.
On comparison with the crisp value of fuzzy optimal cost obtained when using Heptagonal Fuzzy numbers, it is clear that the solution obtained for Fuzzy Assignment Problem using Heptagonal Fuzzy numbers is more optimal than the solution obtained using Trapezoidal fuzzy numbers.

### 6 Conclusion

In this paper, Heptagonal fuzzy number has been newly introduced and its arithmetic operations are defined. The triangular or trapezoidal forms of fuzzy numbers are not applicable in few cases where the uncertainties arise in seven different points and in such cases, HFN can be used to solve the problems. The proposed ranking method is suitable for the problems that contain both normal and non-normal Fuzzy numbers. Through a numerical example, it is proved that the solution obtained for Fuzzy Assignment Problem using Heptagonal Fuzzy numbers is more optimal than the solution obtained using Trapezoidal fuzzy numbers.

### References


Received: May 11, 2014