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Comparison of Four Methods for Estimating the Weibull Distribution Parameters

Ivana Pobočíková and Zuzana Sedliačková

Department of Applied Mathematics
Faculty of Mechanical Engineering, University of Žilina
Univerzitná 1, 010 26 Žilina, Slovakia

Abstract

In this paper we study the performance of the least square method, the weighted least square method, the maximum likelihood method and the method of moments for estimating the Weibull distribution parameters. The comparison is based on the Monte Carlo simulation, the methods are compared in terms of the root mean square error and sample size n. The comparison shows that the maximum likelihood method and the method of moments provide similar estimates. We recommend the maximum likelihood method to estimate the Weibull distribution parameters due to its good properties. For very small sample sizes we recommend the weighted least square method.

Keywords: Weibull distribution, parameter estimation, least square method, weighted least square method, maximum likelihood method, method of moments, root mean square error

1 Introduction

The Weibull distribution is one of the widely used distributions in technical practice. It is often used in weather forecasting, in the theory of reliability and lifetime. This distribution was first introduced by the Swedish scientist Walodi Weibull (1887-1979), who used it in the theory of reliability.

We consider the two-parameter Weibull distribution. The probability density function of the Weibull distribution $W(c,\delta)$ with parameters $c>0$ and $\delta>0$, is given by

$$f(x)=\frac{c}{\delta^c}x^{c-1}\exp\left(-\left(\frac{x}{\delta}\right)^c\right), \tag{1}$$

where $x>0$, c is the shape parameter, and δ is the scale parameter.
The cumulative distribution function of the Weibull distribution is given by

$$F(x)=1-\exp\left(-\left(\frac{x}{\delta}\right)^c\right), \quad x>0. \tag{2}$$

The k^{th} moment $m_k, k=1,2,...,$ of the Weibull distribution is given by

$$m_k=\delta^k\Gamma\left(1+\frac{k}{c}\right), \tag{3}$$

where $\Gamma(a)$ is the gamma function defined by $\Gamma(a)=\int_0^\infty x^{a-1}e^{-x}\,dx, a>0$.

In this paper we study the performance of the methods for estimating the Weibull distribution parameters c and δ. Several methods are proposed to estimate the parameters. In this paper we consider the commonly used methods: the least square method (LSM), the weighted least square method (WLSM), the maximum likelihood method (MLM) and the method of moments (MOM). The LSM and the WLSM are commonly used due to their simplicity. The estimates of the parameters can be calculated easily by the closed-form formula. The MLM and the MOM are popular methods, but both methods are computationally demanding. In the case of the Weibull distribution neither method provides an explicit solution for the estimates of the parameters in the closed-form formula. The estimates of the parameters can be obtained only numerically. The performance of the methods is compared using the Monte Carlo simulation. The efficiency of the methods is compared based on the root mean square error (*RMSE*) criterion and the sample size n. Based on the simulation study we recommend the methods, which have better performance. The simulations and the calculation are performed in the Matlab.

There are some recent works on estimating the Weibull distribution parameters. Trustrum and Jayatilaka (1979) compared the LSM, the MLM and the MOM based on the Monte Carlo simulation. Bergman (1986), Faucher and Tyson (1988), Hung (2001), Lu, Chen and Wu (2004) recommend the WLSM, this method outperforms the LSM. Wu, Zhou and Li (2006) compared the LSM, the WLSM, the MLM and the MOM based on the Monte Carlo simulation. Chu and Ke (2012) compared the LSM and the MLM based on the numerical simulation study.

The rest of this paper is organized as follows. In Section 2 we introduce the estimation methods. In Section 3 the simulation study is provided and finally, in Section 4 we summarize our findings.

2 Estimation methods

2.1. Least square method

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the Weibull distribution $W(c,\delta)$ and let $x_1, x_2, \ldots, x_n$ be a realization of a random sample.

The cumulative distribution function (2) will be transformed to a linear function. From (2) by two logarithmic calculations we obtain

$$\ln\left[-\ln\left(1-F(x)\right)\right] = c\ln x - c\ln\delta. \qquad (4)$$

Let $Y = \ln\left[-\ln\left(1-F(x)\right)\right]$, $X = \ln x$, $\beta_1 = c$ and $\beta_0 = -c\ln\delta$. Then the equation (4) can be written as

$$Y = \beta_1 X + \beta_0. \qquad (5)$$

Now let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ be the order statistics of $X_1, X_2, \ldots, X_n$ and let $x_{(1)} < x_{(2)} < \ldots < x_{(n)}$ be observed ordered observations. To estimate the values of the cumulative distribution function $F(x)$ we use the mean rank

$$\hat{F}\left(x_{(i)}\right) = \frac{i}{n+1}, \qquad (8)$$

where i denotes the i^{th} smallest value of $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$, $i = 1, 2, \ldots, n$. Based on our previously study Pobočíková and Sedliačková (2012) this method can be improved the performance of the estimates.

The estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ of the regression parameters β_0 and β_1 minimize the function

$$Q(\beta_{0,}\, \beta_1) = \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 \ln x_{(i)})^2. \qquad (7)$$

Therefore, the estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ of the parameters β_0 and β_1 are given by

$$\hat{\beta}_1 = \frac{n\sum_{i=1}^{n} \ln x_{(i)} \ln\left[-\ln\left(1-\hat{F}\left(x_{(i)}\right)\right)\right] - \sum_{i=1}^{n} \ln x_{(i)} \sum_{i=1}^{n} \ln\left[-\ln\left(1-\hat{F}\left(x_{(i)}\right)\right)\right]}{n\sum_{i=1}^{n} \ln^2 x_{(i)} - \left(\sum_{i=1}^{n} \ln x_{(i)}\right)^2} \qquad (8)$$

$$\hat{\beta}_0 = \frac{1}{n}\sum_{i=1}^{n} \ln\left[-\ln\left(1-\hat{F}\left(x_{(i)}\right)\right)\right] - \hat{c}\frac{1}{n}\sum_{i=1}^{n} \ln x_{(i)} \;. \qquad (9)$$

The estimates $\hat{c}$ and $\hat{\delta}$ of the parameters c and δ are given by

$$\hat{c} = \hat{\beta}_1,$$
$$\hat{\delta} = \exp\left(-\frac{\sum_{i=1}^{n} \ln\left[-\ln\left(1-\hat{F}\left(x_{(i)}\right)\right)\right] - \hat{c}\sum_{i=1}^{n} \ln x_{(i)}}{\hat{c}\, n} \right). \tag{10}$$

2.2. Weighted least square method

The estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ of the regression parameters β_0 and β_1 minimize the function

$$Q(\beta_{0,}\, \beta_1) = \sum_{i=1}^{n} w_i (Y_i - \beta_0 - \beta_1 \ln x_{(i)})^2, \tag{11}$$

where w_i is the weight factor, $i = 1, 2, ..., n$. In this paper we use the weight factor proposed by Bergman (1986)

$$w_i = \left[\left(1-\hat{F}(x_{(i)})\right)\ln\left(1-\hat{F}(x_{(i)})\right)\right]^2, \quad i = 1, 2, ..., n. \tag{12}$$

Therefore, the estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ of the parameters β_0 and β_1 are given by

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i \ln x_{(i)} \ln\left[-\ln\left(1-\hat{F}\left(x_{(i)}\right)\right)\right] - \sum_{i=1}^{n} w_i \ln x_{(i)} \sum_{i=1}^{n} w_i \ln\left[-\ln\left(1-\hat{F}\left(x_{(i)}\right)\right)\right]}{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i \ln^2 x_{(i)} - \left(\sum_{i=1}^{n} w_i \ln x_{(i)}\right)^2}, \tag{13}$$

$$\hat{\beta}_0 = \frac{\sum_{i=1}^{n} w_i \ln\left[-\ln\left(1-\hat{F}\left(x_{(i)}\right)\right)\right] - \hat{c}\sum_{i=1}^{n} w_i \ln x_{(i)}}{\sum_{i=1}^{n} w_i}. \tag{14}$$

Then the estimates $\hat{c}$ and $\hat{\delta}$ of the parameters c and δ are given by

$$\hat{c} = \hat{\beta}_1,$$
$$\hat{\delta} = \exp\left(-\frac{\sum_{i=1}^{n} w_i \ln\left[-\ln\left(1-\hat{F}\left(x_i\right)\right)\right] - \hat{c}\sum_{i=1}^{n} w_i \ln x_i}{\hat{c}\sum_{i=1}^{n} w_i} \right). \tag{15}$$

2.3. Maximum likelihood method

The likelihood function of the Weibull distribution is given by

$$L(c,\delta)=\prod_{i=1}^{n} f(x_i,c,\delta)=\prod_{i=1}^{n}\frac{c}{\delta^c}x_i^{c-1}\exp\left(-\left(\frac{x_i}{\delta}\right)^c\right). \tag{16}$$

The maximum likelihood estimates $\hat{c}$ and $\hat{\delta}$ of the parameters c and δ maximize function (16) or, equivalently, the logarithm of the function (16)

$$\ln L(c,\delta)=n\ln c-n\,c\ln\delta-\frac{1}{\delta^c}\sum_{i=1}^{n}x_i^c+\left(c-1\right)\sum_{i=1}^{n}\ln x_i\,. \tag{17}$$

Differentiating (17) with respect to c and δ in turn and equating to zero, we obtain the equations

$$\begin{aligned}\frac{\partial\ln L(c,\delta)}{\partial\delta}&=-\frac{n\,c}{\delta}+\frac{c}{\delta^{c+1}}\sum_{i=1}^{n}x_i^c=0,\\ \frac{\partial\ln L(c,\delta)}{\partial c}&=\frac{n}{c}-n\ln\delta-\frac{\sum_{i=1}^{n}x_i^c\ln x_i-\ln\delta\sum_{i=1}^{n}x_i^c}{\delta^c}+\sum_{i=1}^{n}\ln x_i=0.\end{aligned} \tag{18}$$

On eliminating δ from equations (18) and simplifying we solve the equations

$$\delta=\left(\frac{1}{n}\sum_{i=1}^{n}x_i^c\right)^{1/c}, \tag{19}$$

$$\frac{1}{c}-\frac{\sum_{i=1}^{n}x_i^c\ln x_i}{\sum_{i=1}^{n}x_i^c}+\frac{1}{n}\sum_{i=1}^{n}\ln x_i=0\,. \tag{20}$$

The estimate $\hat{c}$ of the parameter c we obtain by solving (20) with respect to c. This equation has not analytical solution and must be solved numerically for c. We use the Newton method. As the starting point we use the result (10) obtained from the LSM. The estimate $\hat{\delta}$ of the parameter δ can be obtained using equation (19).

2.4. Method of moments

The estimates $\hat{c}$ and $\hat{\delta}$ of the parameters c and δ we obtain by equating the k^{th} moments m_k of the Weibull distribution defined by (3) with the corresponding sample k^{th} moments M_k. The sample k^{th} moment $M_k,\ k=1,2,...,$ is given by

$$M_k=\frac{1}{n}\sum_{i=1}^{n}x_i^k. \tag{21}$$

It is known, that the sample mean is $\overline{x}=M_1$. Then

$$\delta\Gamma\left(1+\frac{1}{c}\right)=\overline{x}\,, \tag{22}$$

$$\delta^2\Gamma\left(1+\frac{2}{c}\right)=\frac{1}{n}\sum_{i=1}^{n}x_i^2\,. \tag{23}$$

By dividing (23) by the square (22) we obtain

$$\frac{\Gamma\left(1+\frac{2}{c}\right)}{\Gamma^2\left(1+\frac{1}{c}\right)}=\frac{\frac{1}{n}\sum_{i=1}^{n}x_i^2}{\overline{x}^2}. \tag{24}$$

The estimate $\hat{c}$ of the parameter c we obtain by solving (24) with respect to c. This equation has not analytical solution and must be solved numerically for c. We use the Newton method. The starting point we use according to Ramírez and Carta (2005)

$$c=\left(\frac{\overline{x}}{\sqrt{s_x^2}}\right)^{1.086}, \tag{25}$$

where $\overline{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$ is the sample mean and $s_x^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i-\overline{x}\right)^2$ is the sample variance. The estimate $\hat{\delta}$ of the parameter δ can be estimated using equation (22).

3 Simulation study

In simulation study we generate the random samples from the Weibull distribution and compare the performance of the methods. We consider $c=0.5, 1.5, 2.5$, $\delta=1$ and sample sizes $n=5, 6, ..., 100$. For each combination c, δ and n we generate by the Monte Carlo simulation $N=5\,000$ random samples from the Weibull distribution. For each method under consideration we obtain $5\,000$ estimates $\hat{c}_1, \hat{c}_2, ..., \hat{c}_{5000}$ of the parameter c and $5\,000$ estimates $\hat{\delta}_1, \hat{\delta}_2, ..., \hat{\delta}_{5000}$ of the parameter δ. Then we compute the sample means $\overline{c}, \overline{\delta}$ and the sample variances s_c^2, s_δ^2

$$\overline{c}=\frac{1}{5000}\sum_{i=1}^{5000}\hat{c}_i, \qquad \overline{\delta}=\frac{1}{5000}\sum_{i=1}^{5000}\hat{\delta}_i, \tag{26}$$

$$s_c^2=\frac{1}{4\,999}\sum_{i=1}^{5000}\left(\hat{c}_i-\overline{c}\right)^2, \qquad s_\delta^2=\frac{1}{4\,999}\sum_{i=1}^{5000}\left(\hat{\delta}_i-\overline{\delta}\right)^2. \tag{27}$$

To compare the performance of the methods we compute the sample root mean square error (*RMSE*) given by

$$RMSE = \sqrt{\frac{1}{5000}\sum_{i=1}^{5000}\left[\left(\hat{c}_i - c\right)^2 + \left(\hat{\delta}_i - \delta\right)^2\right]}. \tag{28}$$

The estimates with smaller *RMSE* are preferred.

4 Results and discussion

For $c = 0.5$ the comparison shows that in general the *RMSE* of the MLM outperforms the other methods. The *RMSE* of the WLSM is slightly larger than the *RMSE* of the MLM. The *RMSE* of the MOM is the largest.

For the small sample size $5 \le n \le 15$ and for $c = 1.5,\ 2.5$ the comparison shows that in general the *RMSE* of the WLSM outperforms the other methods. The *RMSE* of the LSM, the MOM and the MLM are larger. The *RMSE* of the MLM is only slightly larger than the *RMSE* of the MOM, both methods are comparable.

For the sample size $15 < n \le 25$ and for $c = 1.5$ and also for the sample size $15 < n \le 23$ and for $c = 2.5$ the comparison shows that the *RMSE* of the WLSM outperforms the other methods. The MLM and the MOM are comparable methods in many cases in terms of the *RMSE*. The *RMSE* of the LSM is larger than the other methods.

For the sample size $n > 25$ and for $c = 1.5$ the comparison shows that the *RMSE* of the LSM is in many cases larger than the other methods. The MLM and the MOM are comparable methods in many cases in terms of the *RMSE*. The *RMSE* of the MOM is only slightly larger than the *RMSE* of the MLM. The *RMSE* of the WLSM is larger than these two methods.

For the sample size $n > 23$ and for $c = 2.5$ the comparison shows that the *RMSE* of the LSM is in many cases larger than the other methods. The MLM and the MOM are comparable methods in many cases in terms of the *RMSE*. The *RMSE* of the WLSM is larger than these two methods.

It is evident that as the sample size n increases the values of the *RMSE* of all methods decrease and hence the estimation precision of the parameters increases.

Figures 1, 2, 3 show illustrative plots of the *RMSE* for $n = 10, 11, ..., 100$ and considered values of the Weibull parameters c and δ. In Tables 1, 2, 3 are illustrated the results of the comparison for selected sample sizes and considered values of the Weibull parameters c and δ.

Table 1. Simulation results of the parameter estimation, $c = 0.5$, $\delta = 1$

Sample size	Method	$\overline{c}$	$\overline{\delta}$	s_c^2	s_δ^2	*RMSE*
n=5	LSM	0.4508	1.6745	0.0745	2.6689	1.7889
	WLSM	0.4319	1.6060	0.0651	2.4445	1.6974
	MLM	0.7235	1.2602	0.1798	1.5047	1.3423
	MOM	0.9528	1.7203	0.1483	2.6163	1.8676
n=10	LSM	0.4363	1.3946	0.0205	0.9475	1.0619
	WLSM	0.4345	1.3256	0.0182	0.8595	0.9939
	MLM	0.5879	1.1519	0.0308	0.6047	0.8162
	MOM	0.7543	1.5245	0.0385	0.9948	1.1717
n=20	LSM	0.4461	1.2092	0.0104	0.3563	0.6429
	WLSM	0.4572	1.1492	0.0093	0.3234	0.5973
	MLM	0.5367	1.0694	0.0106	0.2652	0.5310
	MOM	0.6491	1.3476	0.0184	0.4158	0.7598
n=30	LSM	0.4557	1.1618	0.0076	0.2252	0.5108
	WLSM	0.4713	1.1046	0.0066	0.1984	0.4655
	MLM	0.5256	1.0513	0.0062	0.1671	0.4203
	MOM	0.6143	1.2797	0.0121	0.2572	0.6004
n=40	LSM	0.4600	1.1211	0.0061	0.1545	0.4205
	WLSM	0.4774	1.0730	0.0051	0.1415	0.3904
	MLM	0.5181	1.0312	0.0046	0.1192	0.3537
	MOM	0.5924	1.2246	0.0104	0.1887	0.5080
n=50	LSM	0.4652	1.1087	0.0047	0.1198	0.3708
	WLSM	0.4823	1.0640	0.0040	0.1108	0.3452
	MLM	0.5147	1.0329	0.0034	0.0961	0.3174
	MOM	0.5783	1.2040	0.0084	0.1523	0.4566
n=60	LSM	0.4685	1.0921	0.0041	0.0994	0.3359
	WLSM	0.4837	1.0521	0.0033	0.0941	0.3169
	MLM	0.5116	1.0266	0.0027	0.0819	0.2923
	MOM	0.5691	1.1840	0.0073	0.1307	0.4203
n=70	LSM	0.4714	1.0777	0.0035	0.0793	0.2993
	WLSM	0.4871	1.0400	0.0029	0.0760	0.2840
	MLM	0.5103	1.0188	0.0024	0.0652	0.2607
	MOM	0.5607	1.1558	0.0069	0.1077	0.3776
n=80	LSM	0.4731	1.0699	0.0030	0.0724	0.2847
	WLSM	0.4890	1.0353	0.0026	0.0692	0.2705
	MLM	0.5093	1.0158	0.0020	0.0609	0.2514
	MOM	0.5567	1.1447	0.0062	0.1017	0.3634
n=90	LSM	0.4739	1.0649	0.0028	0.0591	0.2584
	WLSM	0.4896	1.0320	0.0023	0.0575	0.2469
	MLM	0.5076	1.0138	0.0018	0.0496	0.2272
	MOM	0.5513	1.1340	0.0058	0.0867	0.3362
n=100	LSM	0.4760	1.0563	0.0025	0.0533	0.2440
	WLSM	0.4898	1.0258	0.0022	0.0525	0.2355
	MLM	0.5067	1.0108	0.0016	0.0466	0.2198
	MOM	0.5482	1.1268	0.0053	0.0822	0.3253

Table 2. Simulation results of the parameter estimation, $c = 1.5$, $\delta = 1$

Sample size	Method	$\overline{c}$	$\overline{\delta}$	s_c^2	s_δ^2	*RMSE*
n=5	LSM	1.3407	1.0925	0.7156	0.1192	0.9320
	WLSM	1.2914	1.0764	0.6761	0.1170	0.9178
	MLM	2.1595	0.9934	1.8268	0.0993	1.5370
	MOM	2.1605	0.9966	1.7232	0.0985	1.5026
n=10	LSM	1.2949	1.0598	0.1758	0.0564	0.5270
	WLSM	1.2884	1.0413	0.1562	0.0554	0.5079
	MLM	1.7462	0.9953	0.2628	0.0486	0.6099
	MOM	1.7593	0.9968	0.2417	0.0485	0.5978
n=20	LSM	1.3346	1.0406	0.0957	0.0274	0.3901
	WLSM	1.3711	1.0228	0.0861	0.0267	0.3604
	MLM	1.6106	0.9990	0.0976	0.0240	0.3658
	MOM	1.6218	0.9996	0.0969	0.0240	0.3684
n=30	LSM	1.3653	1.0326	0.0676	0.0185	0.3245
	WLSM	1.4098	1.0162	0.0569	0.0181	0.2888
	MLM	1.5716	1.0008	0.0558	0.0163	0.2779
	MOM	1.5792	1.0010	0.0569	0.0164	0.2821
n=40	LSM	1.3799	1.0284	0.0516	0.0139	0.2840
	WLSM	1.4325	1.0133	0.0454	0.0137	0.2526
	MLM	1.5522	1.0016	0.0401	0.0123	0.2349
	MOM	1.5583	1.0018	0.0413	0.0124	0.2390
n=50	LSM	1.3945	1.0218	0.0433	0.0112	0.2570
	WLSM	1.4442	1.0083	0.0369	0.0110	0.2261
	MLM	1.5420	0.9989	0.0311	0.0101	0.2072
	MOM	1.5470	0.9990	0.0327	0.0101	0.2121
n=60	LSM	1.4029	1.0219	0.0367	0.0093	0.2364
	WLSM	1.4554	1.0085	0.0310	0.0093	0.2057
	MLM	1.5354	1.0009	0.0254	0.0082	0.1867
	MOM	1.5397	1.0009	0.0273	0.0083	0.1929
n=70	LSM	1.4132	1.0207	0.0319	0.0077	0.2181
	WLSM	1.4624	1.0084	0.0268	0.0078	0.1900
	MLM	1.5325	1.0019	0.0214	0.0071	0.1719
	MOM	1.5365	1.0020	0.0229	0.0071	0.1769
n=80	LSM	1.4158	1.0192	0.0277	0.0069	0.2050
	WLSM	1.4660	1.0079	0.0236	0.0068	0.1779
	MLM	1.5273	1.0016	0.0185	0.0062	0.1595
	MOM	1.5317	1.0018	0.0201	0.0062	0.1653
n=90	LSM	1.4233	1.0171	0.0244	0.0061	0.1917
	WLSM	1.4680	1.0066	0.0204	0.0062	0.1662
	MLM	1.5240	1.0013	0.0158	0.0056	0.1483
	MOM	1.5272	1.0013	0.0175	0.0056	0.1545
n=100	LSM	1.4276	1.0155	0.0240	0.0054	0.1868
	WLSM	1.4711	1.0052	0.0200	0.0054	0.1619
	MLM	1.5209	1.0007	0.0150	0.0050	0.1427
	MOM	1.5239	1.0008	0.0158	0.0050	0.1460

Table 3. Simulation results of the parameter estimation, $c = 2.5$, $\delta = 1$

Sample size	Method	$\overline{c}$	$\overline{\delta}$	s_c^2	s_δ^2	*RMSE*
n=5	LSM	2.2223	1.0423	1.5070	0.0387	1.2745
	WLSM	2.1367	1.0329	1.3909	0.0383	1.2498
	MLM	3.5748	0.9844	3.7999	0.0346	2.2336
	MOM	3.5226	0.9830	3.7840	0.0345	2.2054
n=10	LSM	2.1727	1.0325	0.5197	0.0197	0.8047
	WLSM	2.1559	1.0212	0.4450	0.0196	0.7637
	MLM	2.9158	0.9951	0.7474	0.0178	0.9685
	MOM	2.8902	0.9942	0.7499	0.0178	0.9591
n=20	LSM	2.2304	1.0203	0.2744	0.0098	0.5976
	WLSM	2.2878	1.0099	0.2433	0.0098	0.5460
	MLM	2.6860	0.9960	0.2778	0.0089	0.5668
	MOM	2.6736	0.9956	0.2741	0.0089	0.5596
n=30	LSM	2.2736	1.0151	0.1894	0.0064	0.4973
	WLSM	2.3494	1.0056	0.1656	0.0064	0.4412
	MLM	2.6212	0.9962	0.1611	0.0059	0.4263
	MOM	2.6132	0.9959	0.1604	0.0059	0.4233
n=40	LSM	2.3064	1.0130	0.1397	0.0047	0.4266
	WLSM	2.3809	1.0046	0.1242	0.0047	0.3782
	MLM	2.5889	0.9978	0.1111	0.0043	0.3511
	MOM	2.5817	0.9975	0.1109	0.0043	0.3491
n=50	LSM	2.3228	1.0145	0.1193	0.0040	0.3935
	WLSM	2.4116	1.0065	0.0997	0.0039	0.3339
	MLM	2.5787	1.0003	0.0834	0.0036	0.3053
	MOM	2.5736	1.0001	0.0839	0.0036	0.3048
n=60	LSM	2.3335	1.0127	0.0972	0.0031	0.3580
	WLSM	2.4207	1.0050	0.0860	0.0031	0.3089
	MLM	2.5563	1.0002	0.0682	0.0028	0.2724
	MOM	2.5526	1.0001	0.0686	0.0028	0.2723
n=70	LSM	2.3552	1.0113	0.0882	0.0028	0.3348
	WLSM	2.4372	1.0044	0.0715	0.0029	0.2799
	MLM	2.5546	1.0002	0.0575	0.0026	0.2510
	MOM	2.5509	1.0001	0.0575	0.0026	0.2504
n=80	LSM	2.3646	1.0085	0.0758	0.0023	0.3106
	WLSM	2.4432	1.0018	0.0666	0.0024	0.2687
	MLM	2.5452	0.9985	0.0505	0.0021	0.2337
	MOM	2.5419	0.9983	0.0508	0.0021	0.2338
n=90	LSM	2.3680	1.0089	0.0700	0.0022	0.2995
	WLSM	2.4467	1.0023	0.0583	0.0022	0.2518
	MLM	2.5366	0.9994	0.0451	0.0020	0.2201
	MOM	2.5341	0.9993	0.0453	0.0020	0.2202
n=100	LSM	2.3805	1.0082	0.0646	0.0019	0.2844
	WLSM	2.4580	1.0021	0.0539	0.0019	0.2399
	MLM	2.5381	0.9993	0.0410	0.0018	0.2104
	MOM	2.5359	0.9992	0.0412	0.0018	0.2104

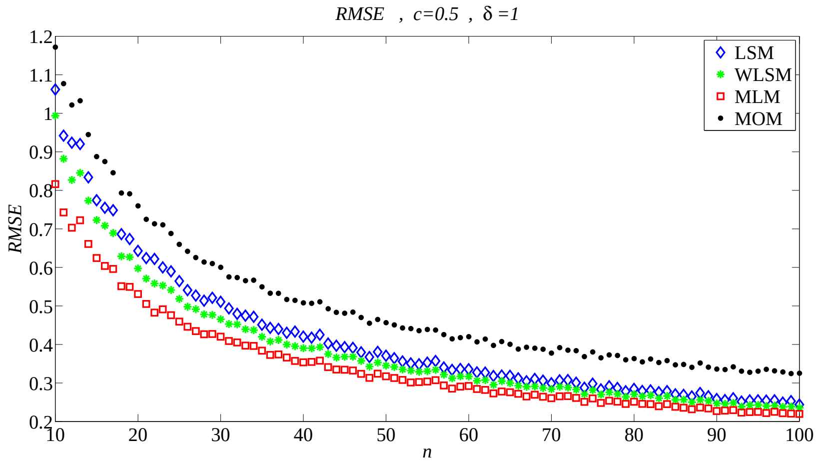

Figure 1. Root mean square error, $c = 0.5$, $\delta = 1$

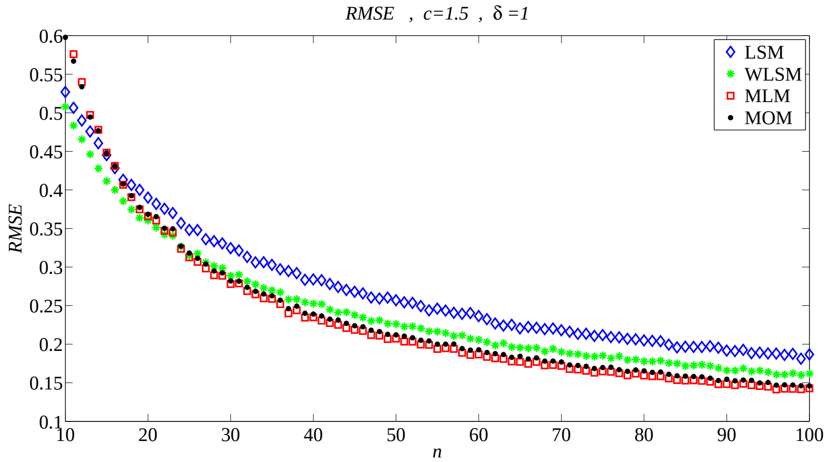

Figure 2. Root mean square error, $c = 1.5$, $\delta = 1$

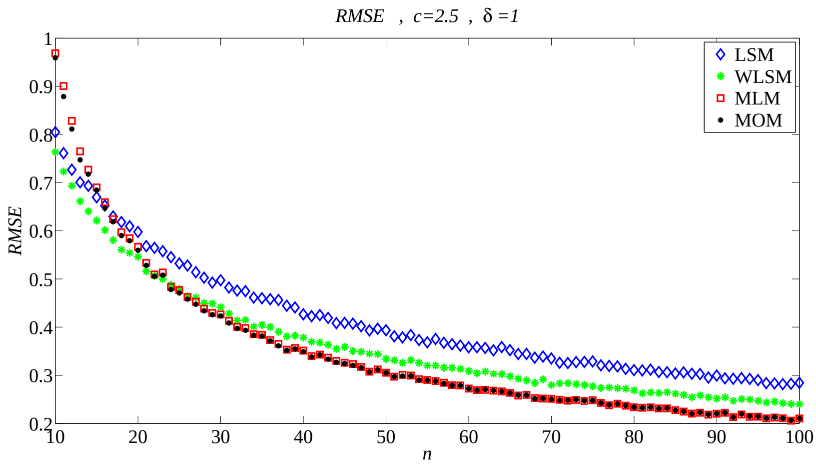

Figure 3. Root mean square error, $c = 2.5$, $\delta = 1$

5 Conclusions

This paper describes the four methods for estimating the Weibull distribution parameters: the least square method (LSM), the weighted least square method (WLSM), the maximum likelihood method (MLM) and the method of moments (MOM). The performance of these methods is compared using the Monte Carlo simulation. The efficiency of the methods is compared based on the *RMSE* criterion and the sample size n. As the sample size n increases the values of the *RMSE* of all methods decrease and hence the estimation precision of the parameters increases.

It is evident that the MLM and the MOM perform better than the LSM and the WLSM when the sample size is middle or large enough. Only for very small sample sizes the WLSM and the LSM outperform the MLM and the MOM. The WLSM performs better than the LSM. Both these methods are good methods due to their simplicity. For very small sample sizes we recommend the WLSM. For middle or large sample sizes the WLSM is useful alternative to the MLM or the MOM in the situation, when the simple computing is preferred.

The MOM provides very similar estimates to the ones obtained by the MLM. There is one complication by using the MOM. This method needs to use the gamma function. However the gamma function can be easily obtained by using the software Matlab. The performance of the MLM is more often better than the MOM. The MLM is the most popular for its efficiency, good properties and it is simpler to compute than the MOM. Therefore we recommend the MLM to estimate the Weibull distribution parameters.

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