Predicting Number of Purchasing Life Insurance

Using Markov Chain Method

Mohd Rahimie Bin Md Noor¹ and Zaidi mat Isa²

¹Mathematical Department, University Teknologi Mara Machang Kelantan;
²Actuarial Sceinces Department, University Kebangsaan Malaysia Bangi Selangor

¹Corresponding author

Abstract

This study describes the Markov chain approach applied in forecasting the life insurance buying patterns. This model using a sample purchase life insurance from General Assurance Berhad covering the years 2003-2006. Markov chain model built is kind of the first stage with a homogeneous time. This model uses the idea (stop-motion) to clarify the circumstances of the number and time of purchase. At the end of the study show how the Markov chain to be a good method in predicting. Therefore, this method should be extended in various fields.

Keywords: Markov-Chain; Stochastic Process; Homogeneous Time

INTRODUCTION

A company of course has the intention and goals to be achieved in the future. They will make every effort to increase profits from year to year. However, a company may not be able to maintain a balance of profits earned status without providing an optimistic strategy and plan. Normally, analysts use cash flow data to predict future profits earned by the company. However, there is a concept that should be addressed in the forecasting techniques of customer behavior and customer management. Forecasting technique is said to be synonymous with business because
they can help business competition strategy for gaining the advantage. The robustness strategy and strong financial resources can help attract more customers using the company's products. This condition must be maintained in order to maximize profits and put the company on solid performance clusters. The company is also able to predict the pattern of customer purchases of the product market. Management strategy to attract customers built taking into account customer behavior (Customer Life Value, CLV) as well as providing a satisfactory customer service. CLV concept is influenced by the management of the customer (Customer Relationship management, CRM). Dwyer (1989) in writing stating customer deposits will influence customer buying patterns. High savings rate that makes a person so obsessed to increase the number of purchases above requirements should.

Consequently, a model was constructed to predict the number of customers that remain with the insurance company and get insurance coverage. Construction of this model using the estimated probability of the customer to make purchases in the future. Customers will purchase the policy and receive the protection of all contractual agreements with insurance companies. Therefore, this study will use the model to find patterns insurance purchase behavior of customers.

In addition, the adaptive Markov chain model can be seen from the first stage of basic stationary Markov chain which will be explained in the literature review chapter. It also describes the tests to be done to validate the model built using the appropriate time period. Duration and interval should be set in the best possible because it affects the accuracy of the prediction.

Therefore, this model is used to predict the dynamics of the insurance purchase made by a customer using the transition probability matrix. The statistical test used to look for suitable model with the data collected is immobilization test or homogeneous.

**Insurance**

Prior studies have been made, we need to know and understand the fundamentals of insurance. Insurance is the transfer of risk by an individual or an organization known as the policyholder to the insurance company. In return, the insurance company receives payment in the form of premium. If the policyholders bear the loss, the insurance company will pay compensation for loss or damage. There are two types of insurance, namely life and general insurance (Takaful Association, 2003). The survey data used in this study is taken from the General Assurance Berhad. These data include the total number of customers in 2003, 2004, 2005 and 2006 for life insurance.

i. **Methodology**

Stochastic process is a collection of random variables, where \( t \) indexed in the set \( T \) (set parameters or set the time). The values taken by the so-called process conditions and form the state space (all conditions) for the process (AK Basu, 2003). Stochastic theory and the theory of probability can be used to describe the evolution of the model system over time. A discrete time stochastic process actually describes the relationship between pemboleubah (Winston, 1994). However, not known for certain ahead of time and is known for random variables. Discrete time stochastic process for explaining the relationship random variables. (Winston, 3rd). The relations of these variables is a condition that occurs in time \((0, 1, 2, \ldots)\)
Markov chain is a stochastic process that occurs in discrete time. Stochastic system in the Markov chain is said to be dynamic when the probability of achieving a particular situation depends on the previous period. In general, the stochastic process $\{X(t_u)\}_{u \in U}$ for discrete time.

Markov chains $\{t_0, t_1, \ldots; u \in U\}$ in the discrete time $S = \{1, 2, \ldots E\}$ can be expressed as follows:

$$P[X_{u+1} = j|X(t_0) = s_0, X(t_1) = s_1, \ldots, X(t_u) = i = i_{t-1}, \ldots, X_1 = i_1, X_0 = i_0]$$

$$= P(X_{u+1} = j|X(t_u) = i)$$

$$= q_{ij}(t_u, t_{u+1})$$

with and , then is the transition probability matrix. This is called the first stage of a stochastic process. In principle, the probability distribution of the Markov chain depends on the circumstances at the time the state has passed up the chain to the situation at the time. So, for a new statement results from the previous statement. It is also agreed by Limnios N. and G. Oprinson (2001) stating the expected future state probabilities can be calculated by taking into account the situation that has been traveled.

Markov chain is said to be time homogeneous or fixed (stationary) if it has a constant period of time. So, the probability of each of the processes occurring at the same time interval.

$$q_{ij}(t_u, t_{u+1}) = q_{ij} \forall t_u, t_{u+1}, i, j$$

Anderson and Goodman (1957) describes the use of maximum likelihood probability matrix in Markov chain data. Let $n_u(s_0, s_1, \ldots, s_k)$ is numbers of (times), correlation equations for each time period are shown below:

$$X(t_u) = s_0, X(t_{u+1}) = s_1, \ldots, X(t_{u+k}) = s_k$$

If we defined

$$n(s_0, s_1, \ldots, s_k) = \sum_{0 \leq u \leq T-k} n_u(s_0, s_1, \ldots, s_k)$$

$$N(s_0, s_1, \ldots, s_k) = \sum_{0 \leq u \leq T-k-1} n_u(s_0, s_1, \ldots, s_k)$$

as the number of time periods. While a sequence $(s_0, s_1, \ldots, s_k)$ the distribution of the events that occurred at any time in the sample data. $N$ is the number of time periods and the expected maximum likelihood of the transition matrix is:

$$\hat{q}_{ij}(t_u, t_{u+1}) = \frac{n_u(i, j)}{n_u(i)}$$

$i, j \in S$, $0 \leq u \leq T - 1$
If the Markov chain has a fixed time (stationary), the maximum probability is:

$$ \hat{q}_{ij} = \frac{n(i, j)}{N(i)}, \quad i, j \in s $$

Nature of the Markov chain assumption of future state obtained from the current state and the last state of a process. In addition, the Markov chain immobilization test was performed to obtain the appropriate time interval. Anderson and Goodman (1957) have used the chi-square test as a comparative hypothesis. The process is said to be stationary if the probability of a sequence of transitions occur in the same time period. Stationarity calculation using chi-square test $\chi^2(u)$:

$$ \chi^2(u) = \sum_{j=1}^{E} \sum_{i=1}^{E} \left[ \frac{n_{ij}(t_u, t_{u+1}) - n_i(t_u)}{N(i)} \right] \frac{n(i, j)}{N(i)} $$

Transition matrix for each Markov chain has a row E and column E. Degrees of freedom for the chi-square test was $E(E-1) - \sum r(i)$ which $r(i)$ the number of movements is not passable during the process.

If the Markov chain has no stationary point it may happen that the process occurs is seasonal. Different time periods used for each year of the transition matrix generated but must use the same time interval. Thus, the nature of the still not fit to be made use of divination but underlying factors influencing the processes occurring.

We can also find the Stationarity Markov (Markov specified) at each time interval used to see that process happen. We can increase the number of time intervals such as Markov chain generated by a high-level but still focus on the first stage of the method. Hypothesis for the Markov chain at each time interval are shown below:

$$ H^T(u) : q_{ij}(t_u, t_{u+1}) = q_{ij} $$

and the Chi-Square would be:

$$ X^2(u) = \sum_{u=0}^{T-1} \sum_{j=1}^{E} \sum_{i=1}^{E} \left[ \frac{n_{ij}(t_u, t_{u+1}) - n_i(t_u)}{N(i)} \right] \frac{n(i, j)}{N(i)} $$

Which the degree of freedoms $(T-1)[E(E-1) - \sum r(i)]$.

ii. Analysis problems

This calculation is continued until we get the number of insurance purchasing in 2006. The sum of the total purchase one, two and three insurance policies for each year are:
Predicting number of purchasing life insurance

\[ X^2(2003) = 1.4088 + 0.0017 + 0 + 0.25 + 0.675 + 3.2 = 5.5355 \]
\[ X^2(2004) = 20.2964 + 0.0620 + 0 + 0.1786 + 1.35 + 4.1667 = 26.0537 \]
\[ X^2(2005) = 2.6639 + 0.1450 + 24.4993 + 0.3130 + 0.9321 + 2.6667 = 31.22 \]
\[ X^2(2006) = 0.0819 + 0.1157 + 25.6507 + 1.5 + 1.5 + 4.5 = 33.3483 \]

Chi-square values only take readings in 2004, 2005, 2006 as the movement of customer behavior. However, data acquisition policy in 2003 is used to obtain the probability of 2004. Calculations are summarized in the table below:

<table>
<thead>
<tr>
<th>Intervals</th>
<th>( X^2 )</th>
<th>dof for ( \chi^2 )</th>
<th>( \chi^2 ) 95%</th>
<th>( \chi^2 ) 99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>26.5266</td>
<td>1</td>
<td>3.841</td>
<td>6.635</td>
</tr>
<tr>
<td>2005</td>
<td>31.22</td>
<td>1</td>
<td>3.841</td>
<td>6.635</td>
</tr>
<tr>
<td>2006</td>
<td>33.3483</td>
<td>1</td>
<td>3.841</td>
<td>6.635</td>
</tr>
<tr>
<td>Total</td>
<td>91.0949</td>
<td>3</td>
<td>11.523</td>
<td>19.905</td>
</tr>
</tbody>
</table>

All other values indicate less than 95% and 99%. Therefore, the hypothesis of homogeneous Markov chain has less time. Interval of time to explain the entire process of purchasing behavior can not be determined because of purchase behavior in each year is not fixed. However, alternative methods have been used to test the predictive capability of the model. This method uses the comparison of the cumulative expected value at the time of purchase will come with the actual value of purchase.

iii) **Real Model and Simulation Model**
Let \( n_m(t,1) \) is the number of registered customers and make a purchase insurance at the time interval \( t \) for \( m \) year. Than \( n_m(t,2) \) and \( n_m(t,3) \) is number of customers who buy two and three policy policy for the first time in the \( m \) year. Time \( 0 \) is the beginning of the existing data. Model predictions for the number of purchases during the period of insurance (\( cm \) year) is based on the cumulative number of customers \( c_m(t,i) \) who makes a purchase at the time \( t \). Here is the equation used in the model forecasts for the purchase of future policy

\[
\begin{align*}
    c_m(t+1,1) &= c_m(t,1)q^{1}_{m1} + n_m(t,1) \\
    c_m(t+1,2) &= c_m(t,1)q^{1}_{m2} + c_m(t,2)q^{1}_{22} + n_m(t,2) \\
    c_m(t+1,3) &= c_m(t,1)q^{1}_{m3} + c_m(t,2)q^{1}_{23} + c_m(t,3) + n_m(t,2)
\end{align*}
\]

Forecast at the time \( t \) of purchase of insurance also must satisfy the following equation

\[
C_m(t) = 0c_m(t,1) + 1c_m(t,2) + 2c_m(t,3)
\]

These formulas are derived from the assumption customers buy an insurance policy. The values obtained will be comparable with the actual model. However, the number of policies used in the actual model is two and number three purchases only and are derived from annual data. Results obtained between model predictions and actual models shown below:

Table 2: Number of customers who buy two and three supplemental insurance policy every year.

<table>
<thead>
<tr>
<th>Intervals</th>
<th>Year</th>
<th>Prediction Model</th>
<th>Actual Model</th>
<th>Accuracy Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2003</td>
<td>45.51</td>
<td>19</td>
<td>41.75</td>
</tr>
<tr>
<td>2</td>
<td>2004</td>
<td>66.44</td>
<td>65</td>
<td>97.83</td>
</tr>
<tr>
<td>3</td>
<td>2005</td>
<td>89.85</td>
<td>91</td>
<td>98.74</td>
</tr>
<tr>
<td>4</td>
<td>2006</td>
<td>145</td>
<td>113</td>
<td>77.93</td>
</tr>
</tbody>
</table>
Conclusion

We can expect similar values if using Markov test the second stage. At this stage, the second principle of the Markov chain is seen to narrow the scope of a process sample. So, this situation will become more relevant if smaller samples used. We also need to take into account the number of customers who buy this policy monthly numbers. Number of customers will make more accurate prediction models because we can see the changes that occur on a monthly basis.
References


Predicting number of purchasing life insurance


http://www.columbia.edu/ww2040/Fall03/lecture102303.pdf

Received: May 15, 2014