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*Bi*hypergraphs and *G*-Designs with Broken Chromatic Spectrum: Results and Problems

Mario Gionfriddo and Vitaly Voloshin

Catania University, Troy University

Abstract

Characterization of *bihypergraphs* with the gaps in their chromatic spectrum, especially those derived from G-designs, is a very interesting open problem with few known facts until today. We survey the results about P_3-designs and P_4-designs and provide a list of currently open problems.

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1 Introduction

A *mixed hypergraph* is a triple $\mathcal{H}$=(X,$\mathcal{C}$,$\mathcal{D}$), where X is the vertex set, and $\mathcal{C}$, $\mathcal{D}$ are the families of nonempty subsets of X, the $\mathcal{C}$-edges and the $\mathcal{D}$-edges respectively. A proper k-colouring of $\mathcal{H}$ is a surjection φ of X onto a *set of colours* {1,2,...,k} such that each $\mathcal{C}$-edge has at least two vertices assigned a common colour, and each $\mathcal{D}$-edge has at a least two vertices assigned distinct colours. If $\mathcal{C}$=$\mathcal{D}$, then $\mathcal{H}$ is called a *bihypergraph*. A mixed hypergraph $\mathcal{H}$ is called *k-colourable* if there exists a colouring of $\mathcal{H}$ that uses at most k colours. The subsets $A_1, ..., A_k$ of X whose elements are coloured respectively by the colours 1,...,k are called the *colour classes*. The minimum number of colours over all proper colourings of $\mathcal{H}$ is called the *lower chromatic number* of $\mathcal{H}$ and is denoted by $\chi(\mathcal{H})$; the maximum number of colours over all proper colourings of $\mathcal{H}$ is called the *upper chromatic number* of $\mathcal{H}$ and is denoted by $\overline{\chi}(\mathcal{H})$.

Every proper colouring of a mixed hypergraph $\mathcal{H}$ partitions the vertex set X into a set of colour classes; such partitions are called *feasible*. If $|X| = n$, and r_i is the number of feasible partitions of X into i colour classes for each $i \in \{1,2,...,n\}$, then the vector $R(\mathcal{H})=[r_1,r_2,...,r_n]$ is called the *chromatic spectrum* of $\mathcal{H}$. Obviously, $r_i=0$ for every index $i<\chi(\mathcal{H})$ or $i>\overline{\chi}(\mathcal{H})$; however, it is possible that $r_i=0$ for some indices i such that $\chi(\mathcal{H})<i<\overline{\chi}(\mathcal{H})$; in these cases we say that the chromatic spectrum of $\mathcal{H}$ *has gaps* or it is *broken*. The number of the consecutive zeroes in a gap is called the *length* of the gap, and a gap of length one is called a *hole*. The minimum and the maximum indices between $\chi(\mathcal{H})$ and $\overline{\chi}(\mathcal{H})$, if there exist, for which $r_i=0$ are respectively called *leftmost* and *rightmost hole* of the chromatic spectrum. Obviously, if $\chi(\mathcal{H})=\overline{\chi}(\mathcal{H})$, then $\mathcal{H}$ admits only colourings with a fixed number of colours: in this case, $\mathcal{H}$ is said to be *monocolourable*. Further, if the only nonzero components in the chromatic spectrum have indices $\chi(\mathcal{H})$ and $\overline{\chi}(\mathcal{H})$ where $\chi(\mathcal{H})<\overline{\chi}(\mathcal{H})$, then $\mathcal{H}$ is said to be *bicolourable*. A mixed hypergraph $\mathcal{H}$ that does not admit any proper colouring is called *uncolourable*. The basic concepts of mixed hypergraph colorings were introduced in [20],[21].

If λK_v is the complete multigraph on v vertices where every edge is repeated λ times and G is a graph with k vertices, then λK_v is said to be *G-decomposable* if it is the union of edge-disjoint subgraphs of K_v each of them isomorphic G. In these cases, we write $\lambda K_v \rightarrow G$ and we say that λK_v admits a *G-decomposition* that is a pair $\Sigma=(X,\mathcal{B})$, where X is the vertex set of K_v and $\mathcal{B}$ is the edge-disjoint decomposition of λK_v into copies of G: the items of $\mathcal{B}$ are usually called *blocks* and Σ is called a *G-design* of order v, block-size k and index λ. A P_k-design is a G-design having order v, block-size k, where G is the path having vertices $x_1,x_2,...,x_k$ and edges the pairs $\{x_i,x_{i+1}\}$ for every $i \in \{1,2,...,k-1\}$. In what follows we will consider $P_3(v)$-designs and $P_3(4)$-designs. It is well known that:

T1) *There exists a P_3-design if and only if $v \equiv$ 0 or 1 mod 4,* $v \geq 4$.

T2) *There exists a P_4-design if and only if $v \equiv$ 0 or 1 mod 3,* $v \geq 4$.

A k-colouring $f: X \rightarrow \{1,...,k\}$ of a G-design $\Sigma = (X,\mathcal{B})$ is $\{r\}$-regular if $|f(E)| = r$ for every block $E \in \mathcal{B}$ [2]. We say that f is an $\{r\}$-*regular equicolouring* if it is $\{r\}$-*regular* and for every block $E \in \mathcal{B}$ the frequencies of any r colours in the block differ by at most 1. For example, in a $\{2\}$-regular equicolouring of a $\mathbf{P}_4$-design each block has two vertices coloured with a colour and the other two vertices coloured with a different colour, while in a $\{2\}$-regular equicolouring of a P_5-design each block has three vertices coloured with a colour and the other two vertices coloured with a different colour [1].

In what follows, a path P_k having vertices in the order $x_1, x_2, ..., x_k$ will be denoted by $< x_1, x_2, ..., x_k >$. Further, to facilitate the symbolism, the set of integers $\{a, a+1, ..., b\}$ will be denoted by $[a, b]$.

2 About P_3-designs

A P_3-design can be considered as a 3-uniform hypergraph. Therefore, we can define mixed P_3-designs and also *bi-P_3-designs*, which have the property that in every block there are exactly two vertices assigned the same colour. These systems are called BP_3-*designs* (designs with bicoloured blocks).

The following constructions will be used to obtain the results described in this paper [11],[12]:

Theorem 2.1 *Starting from a* $P_3(v)$-*design with* $n \equiv 0 \pmod 4$, *it is possible to construct a* $P_3(v+1)$-*design containing it.*

Proof. - Let $(X, \mathcal{B})$ be a $P_3(v)$-design, for any admissible v, and let z be a vertex not belonging to X. Considering a 1-factor F defined on X and the block-set $\mathcal{C}=\{<x,z,y>, \text{ with } (x,y)\in F\}$, it is immediate to see that the pair $(X\cup\{z\}, \mathcal{B}\cup\mathcal{C})$ is a $P_3(v+1)$-design. □

Theorem 2.2 *Starting from a* $P_3(v)$-*design with* $v \equiv 0$ *or* $v \equiv 1 \pmod 4$, *it is possible to construct a* $P_3(v+4)$-*design containing it.*

Proof. - It is similar to the previous one.

Theorem 2.3 *Starting from h pairwise vertex-disjoint* $P_3(v)$-*designs, where* $v \equiv 0 \pmod 4$ *and* $h \in \mathbb{N}$, $h>1$, *it is possible to construct a* $P_3(hv)$-*design containing them.*

Proof. - Let $(X_0, \mathcal{B}_0)$, $(X_1, \mathcal{B}_1)$, ..., $(X_{h-1}, \mathcal{B}_{h-1})$ be h pairwise vertex-disjoint $P_3(v)$-designs, where $X_i=\{x_{vi}, x_{vi+1}, ..., x_{v(i+1)-1}\}$, $i\in\{0,1,...,h\text{-}1\}$. For every $j\in\{0,1,...,h\text{-}1\}$, let F_j be a 1-factor of X_j and consider the block-set $\mathcal{B}=\{<x, x_{vi+k}, y>\}$, where $(x,y)\in F_j$, $i, j\in\{0,1,...,h\text{-}1\}$, $i\neq j$ and $k\in\{0,1,...,v\text{-}1\}$, then the G-design having vertex set $X_0\cup X_1\cup ...\cup X_{h-1}$ and block set $\mathcal{B}_0\cup\mathcal{B}_1\cup...\cup\mathcal{B}_{h-1}\cup\mathcal{B}$ is clearly a $P_3(hv)$-design. □

It is not difficult to prove the following Theorem about BP_3-*designs*, which gives two useful bounds [11],[12]:

Theorem 2.4 - *If* $\mathcal{H}$ *is a colourable* $BP_3(v)$-*design, then*

$$2 \leq \chi(\mathcal{H}) \leq \overline{\chi}(\mathcal{H}) \leq \lceil \tfrac{v}{2} \rceil,$$

where the bounds are the best possible.

To see that the bounds are the best possible, we can consider the following case. Let $\Sigma = (X, \mathcal{B})$ be the BP$_3$-design of order $v = 8$ defined in Z_8 having for blocks:

(1) $< 0, 1, 2 >, < 0, 2, 3 >, < 0, 3, 2 >,$
(2) $< 4, 5, 6 >, < 4, 6, 7 >, < 4, 7, 5 >,$
(3) $< 0, 4, 1 >, < 0, 5, 1 >, < 0, 6, 1 >, < 0, 7, 1 >,$
(4) $< 2, 4, 3 >, < 2, 5, 3 >, < 2, 6, 3 >, < 2, 7, 3 > .$

If $A = \{0, 2, 4, 6\}$, $B = \{1, 3, 5, 7\}$, then A and B define the colour classes of a 2-colouring of Σ. If $A = \{0, 1\}$, $B = \{2, 3\}$, $C = \{4, 6\}$,$D = \{5, 7\}$, these are the colour classes of a 4-colouring of Σ. Observe that Σ contains two subsystems of order 4. The same construction can be given in general with Σ of order $4h$, containing h subsystems of order 4, admitting a 2-colouring and a $2h$-colouring and some holes in its chromatic spectrum.

Using similar techniques it is possible to prove the following results about BP$_3(v)$-designs having a broken chromatic spectrum with only two positive distinct values: $\chi(\Sigma)$ and $\overline{\chi}(\Sigma)$ [11],[12].

Theorem 2.5 - *For every* $v \in \mathbb{N}$ *such that* $v \geq 4$ *and* $v \equiv 0$ or $v \equiv 1$ (mod 4), *there exists a* BP$_3(v)$-*design* $\mathcal{H}$ *which admits only* $\chi(\mathcal{H})$-*colourings and* $\overline{\chi}(\mathcal{H})$-*colourings.*

Theorem 2.6 - *For every* $k \in \mathbb{N}$, *there exist bicolourable* BP$_3(v)$-*designs having chromatic spectrum with a gap of length not less than* k, *i.e. of type* $[0,...,0,r_i,0,...,0,r_j,0,...,0]$, *where* $j \geq i+k+1$ *and* $r_i,r_j>0$.

If we consider the system Σ defined in Z_8 with the blocks described above, then we observe that Σ has no 3-colourings; therefore there is a hole in its chromatic spectrum, which is therefore *broken*.

Using similar techniques it is possible to prove the following results.

Theorem 2.7 - *If* Σ *is a* k-*colourable* BP$_3(v)$-*design where* $v \equiv 0$ (mod 4), *then a* BP$_3(v+1)$-*design* Σ' *containing* Σ *can be uncolourable*, k-*colourable or* $(k+1)$-*colourable.*

The following theorems give results about *uncolourable* and *monocolourable* systems.

Theorem 2.8 - *There exists an uncolourable* BP$_3(12)$-*design.*

The following blocks define a system verifying this property.

$<x_{4i},x_{4i+1},x_{4i+2}>$, $<x_{4i},x_{4i+2},x_{4i+3}>$, $<x_{4i},x_{4i+3}, x_{4i+1}>$ $(i\in\{0,1,2\})$
$<x_0,x_4,x_1>$, $<x_2,x_4,x_3>$, $<x_0,x_5,x_2>$, $<x_1,x_5,x_3>$
$<x_0,x_6,x_3>$, $<x_1,x_6,x_2>$, $<x_0,x_7,x_1>$, $<x_2,x_7,x_3>$
$<x_0,x_i,x_1>$, $<x_2,x_i,x_3>$, $<x_4,x_i,x_5>$, $<x_6,x_i,x_7>$ $(i\in\{8,9,10,11\})$.

Starting from such uncolourable system it is possible to prove that there exist infinitely many $\mathrm{BP}_3(v)$-*designs* containing it:

Theorem 2.9 - *For every* $v\in\mathbb{N}$ *such that* $v\geq 12$ *and* $v\equiv 0$ or $v\equiv 1$ (mod 4), *there exists an uncolourable* $\mathrm{BP}_3(v)$-*design.*

Theorem 2.10 - *For every* $v\in\mathbb{N}$ *such that* $v\equiv 0$ or $v\equiv 1$ (mod 4), *there exists a monocolourable* BP_3 (v)-*design having chromatic spectrum* $[0,r_2,0,...,0]$, *where* $r_2>0$.

Theorem 2.11 - *For every* $v\in\mathbb{N}$ *such that* $v\geq 8$ *and* $v\equiv 0$ or $v\equiv 1$ (mod 4), *there exists a monocolourable* $\mathrm{BP}_3(n)$-*design having chromatic spectrum* $[0,0,r_3,0,...,0]$, *where* $r_3>0$.

It is possible also to prove that there are monocolourable BP_3-designs in which the only one nonzero value occupies particular positions in the chromatic spectrum, depending on the order v [12].

Theorem 2.12 - *For every* $v\in\mathbb{N}$ *such that* $v\geq 8$ *and* $v\equiv 0$ (mod 4), *there exist monocolourable* $\mathrm{BP}_3(v)$-*designs having chromatic spectrum of the type*:
1) $[0,...,0, r_{\frac{v-4}{2}},0,...,0]$, *where* $r_{\frac{v-4}{2}}>0$, *or*
2) $[0,...,0,r_{\frac{v-2}{2}},0,...,0]$, *where* $r_{\frac{v-2}{2}}>0$, *or*
3) $[0,...,0,r_{\frac{v}{2}},0,...,0]$, *where* $r_{\frac{v}{2}}>0$.

Theorem 2.13 - *For every* $v\in\mathbb{N}$ *such that* $v\geq 9$ *and* $v\equiv 1$ (mod 4), *there exist monocolourable* $\mathrm{BP}_3(v)$-*designs having chromatic spectrum of the type*:
1) $[0,...,0,r_{\frac{v-1}{2}},0,...,0]$, *where* $r_{\frac{v-1}{2}}>0$, *or*
2) $[0,...,0,r_{\frac{v-3}{2}},0,...,0]$, *where* $r_{\frac{v-3}{2}}>0$.

The main general result on monocolourable BP_3-designs is [12]:

Theorem 2.14 - *For every admissible* $v\in\mathbb{N}$ *and for every* $k\in\mathbb{N}$ *such that* $2\leq k\leq\lceil\frac{v}{2}\rceil$, *there exists a monocolourable* $\mathrm{BP}_3(v)$-*design having chromatic spectrum* $[0,...,0,r_k,0,...,0]$, *where* $r_k>0$ *and with the possible exceptions of* $v\equiv 1$

(mod 4) *and* $k=\frac{n+1}{2}$.

With the following theorem, we point out that it is possible to construct BP_3-designs with *regular* gaps in the chromatic spectrum [12].

Theorem 2.15 - *For every* $v\in\mathbb{N}$ *such that* $v\equiv 0$ (mod 4) *there exist* $BP_3(v)$-*designs having chromatic spectrum* $[0,r_2,0,r_4,0,...,0,r_{\frac{v}{2}},0,...,0]$, *where* $r_i>0$.
For every $v\in\mathbb{N}$ *such that* $v\equiv 1$ (mod 4) *there exist* $BP_3(v)$-*designs having chromatic spectrum* $[0,r_2,0,r_4,0,...,0,r_{\frac{v-1}{2}},r_{\frac{v+1}{2}},0,...,0]$, *where* $r_i>0$.

We conclude this section by the following interesting results.

Lucia Gionfriddo was the first who studied the BP_3-designs with *letfmost hole* and *rightmost hole* [11]. She defined *letfmost hole* and *rightmost hole* respectively the minimum and the maximum values of the index i for which $\chi(\mathcal{H})<i<\overline{\chi}(\mathcal{H})$ and $r_i=0$.

Theorem 2.16 - *In the chromatic spectrum of a* $BP_3(v)$-*design, the leftmost hole is possible at* 3 *and* $v\geq 8$, *the rightmost hole is possible at* $i=\lceil\frac{v}{2}\rceil$-1 *and* $v\geq 8$.

Theorem 2.17 - *For every* $v\in\mathbb{N}$ *such that* $v\geq 8$ *and* $v\equiv 0$ or $v\equiv 1$ (mod 4), *the following holds:*

1. *There exists a* $BP_3(v)$-*design whose chromatic spectrum has leftmost hole and rightmost hole both equal to* 3.
2. *There exists a* $BP_3(v)$-*design whose chromatic spectrum has leftmost hole and rightmost hole both equal to* $\lceil\frac{v}{2}\rceil$-1.

3 About P_4-designs

The first theorem stating that there is a P_4-design with a gap was proved in [1]:

Theorem 3.1 - *There exists a* $\mathbf{P}_4(10)$-*design which has* $\{2\}$-*regular* k-*equicolouring only for* $k=2$ *and* 4.

Proof. Let $\Sigma=(Z,\mathcal{B})$ be a $P_4(10)$-design on $Z=X\cup Y$, where $X=[0,3]$, $Y=[4,9]$, and $\mathcal{B}$ comprises blocks of the design Π_4 on X, blocks of the design Π_6 on Y and blocks of $\mathcal{B}_1$ given below that constitute a P_4-decomposition $\mathcal{D}_1^{4,6}$ of $K_{4,6}$ with bipartition X,Y:

Π_1
(0,1,2,3)
(1,3,0,2)

Π_2
$(4,5,7,8)$
$(4,6,7,9)$
$(4,8,5,9)$
$(5,6,8,9)$
$(6,9,4,7)$

Π_3	
$(0,4,1,5)$	$(0,7,1,8)$
$(1,6,0,5)$	$(1,9,0,8)$
$(2,4,3,5)$	$(2,7,3,8)$
$(3,6,2,5)$	$(3,9,2,8)$

A $\{2\}$-regular 2-equicolouring of Σ can be obtained by partitioning Z into the 2 colour classes $\mathbf{A} = \{0,2,4,6,8\}$, $\mathbf{B} = \{1,3,5,7,9\}$. A $\{2\}$-regular 4-equicolouring of Σ can be obtained considering the 4 colour classes $\mathbf{A} = \{0,1\}$, $\mathbf{B} = \{2,3\}$, $\mathbf{C} = \{4,5,6\}$, $\mathbf{D} = \{7,8,9\}$. By a bit of reflection, one can see that there are no $\{2\}$-regular 3-equicolourings of Σ and, therefore, it has a chromatic spectrum with a gap at 3. □

Other results about $P_4(v)$-designs are the following [1].

Theorem 3.2 - *A $\{2\}$-regular 2-equicolourable $\mathbf{P}_4(v)$-design exists for each $v \equiv 0$ or 1 (mod 3).*

Theorem 3.3 - *There exist $\{2\}$-regular $2h$-equicolourable $\mathbf{P}_4(6h)$-designs.*

Theorem 3.4 - *For every admissible $v \geq 6$, there exists $\mathbf{P}_4(v)$-design which is not $\{2\}$-regular equicolourable.*

Theorem 3.5 1) *For every $p, h \in \mathbb{N}$, with $h \geq p$, there exists a monocolourable $\mathbf{P}_4(6h)$-design having a $\{2\}$-regular $2p$-equicolouring.*

2) *For every $p, h \in \mathbb{N}$, with $h \geq p$, there exists a multicolourable $\mathbf{P}_4(6h+4)$-design having only two $\{2\}$-regular k-equicolourings, for $k = 2p$ and $2p+2$. Thus all these $\mathbf{P}_4(6h+4)$-designs have a gap in $2p+1$.*

From the previous theorem it follows that Σ *admits only $2p$-colourings and $(2p+2)$-colourings* and therefore it has a gap in the chromatic spectrum.

4 *Open problems*

In this section we formulate some open problems on the subject of this paper.

1. Prove or disprove the existence of uncolourable C_4-designs or uncolourable P_4-designs.
2. Prove or disprove the existence of monocolorability of C_4-designs or P_4-designs.

3. Prove or disprove the existence of bicolorability of C_4-designs or P_4-designs.

4. Prove or disprove the existence of C_4-designs or P_4-designs with regular gaps.

5. Prove or disprove the existence of C_4-designs or P_4-designs with extremal gaps.

6. Prove or disprove the existence of BP_3-designs having simultaneously the only leftmost and rightmost hole.

7. Prove or disprove that in 2-regular colourings (not necessarily equicolourings) of C_4-designs there are gaps in the chromatic spectrum.

8. Prove or disprove that in 3-regular colourings (not necessarily equicolourings) of C_4-designs there are gaps in the chromatic spectrum.

9. Find new classes of G-designs with gaps in the chromatic spectrum.

10. Prove or disprove the previous questions for octogonal quadrangle systems [3][4][5].

11. Prove or disprove the previous questions for balanced P_k-systems [6].

12. Prove or disprove the previous questions for balanced 4-kite systems [15].

13. Prove or disprove the previous questions for dodecagon quadrangle systems [13].

14. Prove or disprove the previous questions for C_4-systems [14][17].

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