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Bayesian Estimation of Burr Type XII Distribution Based on General Progressive Type II Censoring

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Abstract

This article deals with the problem of estimating two shape parameters and the reliability function of Burr type XII distribution, on the basis of a general progressively type II censored sample using Bayesian viewpoints. However, the maximum likelihood and Bayes estimators do not exist in the explicit forms for the parameters. An approximation form due to Lindley (1980) is used for obtaining Bayes estimates under the squared error loss and linex loss functions. The root mean square errors of the estimates are computed. Comparisons are made between Bayes and the maximum likelihood estimates using Monte Carlo simulation study.

Keywords: Bayes estimation, Burr type XII distribution, General progressive censoring, Lindley's approximation, Linex loss

1 Introduction

The two parameter Burr type XII distribution (which is denoted by BrrXII (c, k)) was first introduced in the literature in Burr (1942). Its capacity of the various shapes often permits a good fit when used to describe biological, clinical or other experimental data. It has also been applied in areas of quality control, reliability, duration and failure time modelling.

The probability density function (p.d.f), cumulative distribution function (c.d.f) and reliability function of BurrXII(c, k) distribution are given, respectively, by

$$\begin{aligned} f(x) &= ckx^{c-1}(1+x^c)^{-(k+1)}, x>0, k>0, c>0, \\ F(x) &= 1-(1+x^c)^{-k} \quad \text{and} \quad R(t)=(1+t^c)^{-k}. \end{aligned} \tag{1.1}$$

In many lifetime studies, it is common that the lifetimes of test units may not be able to record exactly. For example, in the type II censoring, the test ceases after a predetermined number of failure in order to save time or cost. Furthermore, some test units may have to be removed at different stages in the study for various reasons. This would lead to progressive censoring. Progressively type II censored sampling is an important method for obtaining data in lifetime studies. Live units removed early can be readily used in other tests, thereby saving cost to the experimenter, and a compromise can be achieved between time consumption and the observation of some extreme values. These censoring occurs frequently in many applications, there are a lot of works on it. Some works can be found in Cohen (1963), Mann (1971), Thomas and Wilson (1972), Viveros and Balakrishnan (1994), Balakrishnan and Sandhu (1995), Balakrishnan and Aggarwala (2000), Kim (2006), Kim and Han (2009), Kim and Han (2010) and Kim et al. (2011).

Let us consider the following progressive type II censoring scheme which was generalized by Balakrishnan and Sandhu (1996). Suppose n randomly selected units were placed on a life test; the first r failure times, $X_1, \cdots, X_r$ are not observed; at time X_{r+1}, R_{r+1} units are removed from the test randomly; at time X_{r+2}, R_{r+2} units are removed from the test randomly. so on. Finally, at the time of mth failure, X_m, the experiment is terminated and the remaining R_m units are removed from the test. Therefore, $X_{r+1} \leq \cdots \leq X_m$ are the lifetimes of the completely observed units to fail and $R_{r+1}, \cdots, R_m$ are the number of units withdrawn from the test at these failure times. At $(i+1)$th failure, there are n_i units on test where

$$n_i = n - i - \sum_{j=r+1}^{i} R_j, \quad i = r+1, \cdots, m-1,$$

R_i's, m and r are prespecified integers which must satisfy the conditions: $0 \leq$

$r < m \leq n, 0 \leq R_i \leq n_{i-1} - 1$ for $i = r+1, \cdots, m-1$ with $n_r = n - r$ and $R_m = n_{m-1} - 1$.

In the next section, we deal with the problem of estimating the parameters c, k and the reliability function of BurrXII(c, k) under SEL and linex loss functions. The prior distribution for the parameter of the model has been taken as a natural conjugate prior.

The SEL and linex loss functions are the following forms, respectively:

$$L(\theta, \hat{\theta}) = (\theta - \hat{\theta})^2, \tag{1.2}$$

$$L(\theta, \hat{\theta}) = b(e^{a(\hat{\theta}-\theta)} - a(\hat{\theta} - \theta)) - 1), a \neq 0, b > 0 \tag{1.3}$$

where a and b are shape and scale parameters of the loss function, respectively. One of the most popular asymmetric loss function is the linex loss function which is introduced by Vraian (1975) and further properties of this loss function have been investigated by Zellner (1986). For a small values of a (near to zero), linex loss function is same as SEL and for choice of negative or positive values of a, it gives more weight to overestimation or underestimation. (For details see Zellner (1986))

2 Maximum Likelihood Estimation

Suppose that n randomly selected units from BurrXII(c,k) population, c and k are both unknown, are put on test under a general progressive type II censoring scheme and consider that $\mathbf{x} = (x_{r+1}, \cdots, x_m)$ is the observed sample.

The likelihood function of c and k based on general progressively type II censored sample can be written as

$$L(c, k|\mathbf{x}) = \frac{n!}{r!(n-r)!} \left(\prod_{j=r}^{m-1} n_j \right) [F(x_{r+1})]^r \prod_{i=r+1}^{m} f(x_i)[1 - F(x_i)]^{R_i}. \tag{2.1}$$

In accordance with (1.1) and (2.1), the log-likelihood function becomes proportional to

$$\begin{aligned} l(c, k|x) \;\propto\; & (m-r)logk + (m-r)logc + (c-1)\sum_{i=r+1}^{m} log(x_i) \\ & + rlog\left\{1 - exp(-klog(1 + x_{r+1}^c))\right\} \\ & - \sum_{i=r+1}^{m} [k(R_i + 1) + 1]log(1 + x_i^c). \end{aligned} \tag{2.2}$$

The maximum likelihood estimators (MLE) of k and c, denoted by $\hat{k}_M$ and

$\hat{c}_M$, can be derived by solving the equations, respectively,

$$\begin{aligned} L_k &= \frac{\partial l(k,c|\mathbf{x})}{\partial k} = \frac{m-r}{k} + \frac{r log(1+x_{r+1}^c)}{(1+x_{r+1}^c)^k - 1} - \sum_{i=r+1}^{m} (1+R_i) log(1+x_i^c), \\ L_c &= \frac{\partial l(k,c|\mathbf{x})}{\partial c} = \frac{m-r}{c} + \sum_{i=r+1}^{m} log(x_i) - \sum_{i=r+1}^{m} (k(1+R_i)+1) \frac{x_i^c log(x_i)}{1+x_i^c} \\ &\quad + kr \frac{x_{r+1}^c log(x_{r+1})}{(1+x_{r+1}^c)[(1+x_{r+1}^c)^k - 1]}. \end{aligned} \tag{2.3}$$

The MLE of k and c can not be solved explicitly. Therefore, the solutions could be obtained by using Newton-Raphson method. For a given t, the MLE of the reliability function in (1.1), $\hat{R}_M$, can be obtained by replacing k and c by $\hat{k}_M$ and $\hat{c}_M$, respectively.

The asymptotic variance-covariances of the MLE for parameters k and c are given by the elements of the inverse of the Fisher information matrix

$$I_{ij} = -E\left(\frac{\partial^2 l(k,c|\mathbf{x})}{\partial k \partial c}\right), \quad i,j = 1,2.$$

But, the exact mathematical expression for the above expectation is very difficult to obtain. Therefore, we will take the approximate asymptotic variance-covariance matrix for the MLE.

The asymptotic variance-covariance matrix is given by

$$\hat{\Sigma} = \begin{bmatrix} -\frac{\partial^2 l(k,c|\mathbf{x})}{\partial k^2} & -\frac{\partial^2 l(k,c|\mathbf{x})}{\partial k \partial c} \\ & -\frac{\partial^2 l(k,c|\mathbf{x})}{\partial c^2} \end{bmatrix}^{-1}_{k=\hat{k}, c=\hat{c}} = \begin{bmatrix} \hat{\sigma}_k^2 & \hat{\sigma}_{ck} \\ & \hat{\sigma}_c^2 \end{bmatrix} \tag{2.4}$$

with

$$\frac{\partial^2 l(k,c|\mathbf{x})}{\partial k^2} = -\frac{m-r}{k^2} - \frac{r(1+x_{r+1}^c)^k [log(1+x_{r+1}^c)]^2}{[(1+x_{r+1}^c)^k - 1]^2}, \tag{2.5}$$

$$\begin{aligned} \frac{\partial^2 l(k,c|\mathbf{x})}{\partial c^2} &= -\frac{m-r}{c^2} - \sum_{i=r+1}^{m} (k(1+R_i)+1) \frac{x_i^c [log(x_i)]^2}{(1+x_i^c)^2} \\ &\quad - \frac{kr x_{r+1}^c [log(x_{r+1})]^2 \left[1 - (1+x_{r+1}^c)^k (1 - k x_{r+1}^c)\right]}{\left(1+x_{r+1}^c\right)^2 \left[(1+x_{r+1}^c)^k - 1\right]^2}, \end{aligned} \tag{2.6}$$

$$\begin{aligned} \frac{\partial^2 l(k,c|\mathbf{x})}{\partial k \partial c} &= -\sum_{i=r+1}^{m} (1+R_i) \frac{x_i^c log(x_i)}{1+x_i^c} - r x_{r+1}^c log(x_{r+1}) \\ &\quad \times \frac{1 - (1+x_{r+1}^c)^k [1 - k log(1+x_{r+1}^c)]}{(1+x_{r+1}^c)\left[(1+x_{r+1}^c)^k - 1\right]^2}. \end{aligned} \tag{2.7}$$

The asymptotic normality of the MLE can be used to compute the approximate confidence intervals for the parameters k and c where

$$\hat{k}_M \pm z_{\frac{a}{2}}\sqrt{\hat{\sigma}_k^2} \text{ and } \hat{c}_M \pm z_{\frac{a}{2}}\sqrt{\hat{\sigma}_c^2} \tag{2.8}$$

where $z_{a/2}$ is a standard normal variate.

3 Bayes Estimation

Let $\mathbf{x} = (x_{r+1}, x_{r+2}, \cdots, x_m)$, $r \geq 1$, be a general progressively type II censored sample from Burr type XII distribution with parameters k and c. Since k and c are both unknown, a bivariate prior density function for a natural choice of the prior distributions of k and c is assumed as following form

$$\pi(c,k) = \pi_1(k|c)\pi_2(c) \tag{3.1}$$

where

$$\pi_1(k|c) = \frac{c^\alpha}{\Gamma(\alpha)}k^{\alpha-1}e^{-ck} \text{ and } \pi_2(c) = \beta e^{-\beta c}.$$

By combining (2.1) and (3.1), the joint posterior density of k and c is given by

$$\begin{aligned}\pi(c,k|\mathbf{x}) \quad &\propto \quad k^{m-r+\alpha-1}c^{m-r+\alpha}\left(\prod_{i=r+1}^{m}\frac{x_i^{c-1}}{1+x_i^c}\right)\left[1-exp\left(-klog(1+x_{r+1}^c)\right)\right]^r \\ &\times \quad exp\left(-k\sum_{i=r+1}^{m}(1+R_i)log(1+x_i^c) - ck - c\beta\right).\end{aligned} \tag{3.2}$$

Under SEL in (1.2), Bayes estimator of a function $U = U(c,k)$, $\hat{U}_S$, is the posterior expectation given as

$$\hat{U}_S = E(U(c,k)|X) = \frac{\int_0^\infty\int_0^\infty U(c,k)\pi(c,k|x)dkdc}{\int_0^\infty\int_0^\infty \pi(c,k|x)dkdc}. \tag{3.3}$$

Under linex loss function in (1.3), Bayes estimator of a function $U(c,k)$, $\hat{U}_L$, is given by the following form

$$\hat{U}_L = -\frac{1}{a}logE(e^{-aU}|\mathbf{X}), \quad a \neq 0 \tag{3.4}$$

where $E(\cdot|\mathbf{X})$ denotes the posterior expectation. In general, the ratio of the two integrals given by (3.3) and (3.4) can not be obtained in a closed form. Therefore, in such cases, we want to use a numerical integration technique such as Lindley's approximation which Lindley (1980) developed approximate procedures for a evaluation of the ratio of two integrals.

In a two parameter case, $\theta = (\theta_1, \theta_2)$, Lindley approximation leads to

$$\begin{aligned}
\hat{U}_B &= E[U(\theta_1,\theta_2)|X] \\
&= U(\theta_1,\theta_2) + \frac{1}{2}\Bigg[\sum_{i=1}^{2}\sum_{j=1}^{2} U_{ij}\tau_{ij} + Q_{30}(U_1\tau_{11} + U_2\tau_{12})\tau_{11} \\
&+ Q_{21}(3U_1\tau_{11}\tau_{12} + U_2(\tau_{11}\tau_{22} + 2\tau_{12}^2)) + Q_{12}(3U_2\tau_{22}\tau_{21} \\
&+ U_1(\tau_{11}\tau_{22} + 2\tau_{21}^2)) + Q_{03}(U_2\tau_{22} + U_1\tau_{21})\tau_{22}\Bigg]
\end{aligned} \tag{3.5}$$

where

$$U_{ij} = \frac{\partial^2 U}{\partial\theta_i \partial\theta_j}, \quad i,j = 1,2, \quad U_i = \frac{\partial U}{\partial \theta_i}, \quad i = 1,2, \quad Q_{ps} = \frac{\partial^{p+s} Q}{\partial\theta_1^p \partial\theta_2^s}$$

for $p, s = 0, \cdots, 3$ and $p + s = 3$. τ_{ij} is the (i, j)th element in the inverse matrix $Q^* = \{Q_{ij}\}$, such that $Q_{ij} = -\partial^2 Q/\partial\theta_i\partial\theta_j$, $i, j = 1, 2$.

Now, we apply Lindley's approximation in (3.5) into our case where $(\theta_1, \theta_2) = (c, k)$, $U(\theta_1, \theta_2) = U(c, k)$ and $Q = log\pi(c, k|\mathbf{x})$.

Therefore, the elements τ_{ij} is obtained as

$$\tau_{11} = -\frac{I}{D}, \quad \tau_{12} = \tau_{21} = \frac{H}{D}, \quad \tau_{22} = -\frac{G}{D} \tag{3.6}$$

where

$$\begin{aligned}
D &= GI - H^2, \\
G &= -\frac{m-r+\alpha}{c^2} - \sum_{i=r+1}^{m} \frac{(1+k+kR_i)x_i^c q_i^2}{t_i^2} \\
&\quad -\frac{kr(q_{r+1})^2 x_{r+1}^c[1-(1-kx_{r+1}^c)(t_{r+1})^k]}{(t_{r+1})^2[(t_{r+1})^k - 1]^2}, \\
H &= -1 - \sum_{i=r+1}^{m} \frac{(1+R_i)x_i^c(q_i)}{t_i} \\
&\quad -\frac{r(q_{r+1})x_{r+1}^c[1-(1-klog(t_{r+1}))(t_{r+1})^k]}{(t_{r+1})[(t_{r+1})^k - 1]^2}, \\
I &= -\frac{m-r+\alpha-1}{k^2} - \frac{r[log(t_{r+1})]^2(t_{r+1})^k}{[(t_{r+1})^k - 1]^2},
\end{aligned}$$

$t_i = 1 + x_i^c$ and $q_i = logx_i$.

Furthermore, the values of Q_{ps} can be obtained as follows for $p, s = 0, 1, 2, 3,$

$$
\begin{aligned}
Q_{30} &= \frac{2(m-r+\alpha)}{c^3} + \sum_{i=r+1}^{m} \frac{q_i^3(1+k+kR_i)(t_i-1)(t_i-2)}{t_i^3} \\
&\quad + \frac{A}{t_r^3[t_r^k-1]^3}, \\
Q_{21} &= -\sum_{i=r+1}^{m} \frac{q_i^2(1+R_i)(t_i-1)}{t_i^2} + \frac{B}{t_r^2(t_r^k-1)^3}, \\
Q_{12} &= \frac{rq_r logt_r(t_r-1)t_r^{k-1}(2-2t_r^k+klogt_r(1+t_r^k))}{(t_r^k-1)^3}, \\
Q_{03} &= \frac{2(m-r+\alpha-1)}{k^3} + \frac{r(logt_r)^3 t_r^k(1+t_r^k)}{(t_r^k-1)^3}
\end{aligned} \tag{3.7}
$$

where

$$
\begin{aligned}
A &= krq_r^3(t_r-1)(t_r^k-1)^2 + k^2(t_r-1)^2 t_r^k(1+t_r^k) \\
&\quad -(t_r-1)(t_r^k-1)(-1+t_r^k+3kt_r^k), \\
B &= rq_r^2(t_r-1)(1-t_r^k)(1-t_r^k+klogt_r t_r^k) + k(t_r-1)t_r^k(2-2t_r^k \\
&\quad +klogt_r(1+t_r^k))).
\end{aligned}
$$

It follows from (3.5), (3.6) and (3.7) that Bayes estimator of the function $U(c, k)$ relative to SEL is given by

$$\hat{U}_S = U(c,k) - \frac{W}{2D} + \frac{\Psi_1}{2D^2}U_1 + \frac{\Psi_2}{2D^2}U_2 \tag{3.8}$$

where

$$
\begin{aligned}
W &= U_{11}I - H(U_{12}+U_{21}) + U_{22}G, \\
\Psi_1 &= Q_{30}I^2 + Q_{12}(GI+2H^2) - 3Q_{21}HI - Q_{03}GH, \\
\Psi_2 &= -Q_{30}IH + Q_{21}(GI+2H^2) - 3Q_{12}GH + Q_{03}G^2.
\end{aligned}
$$

From (3.8), we can deduce the values of Bayes estimates under SEL of various parameters in what follows.
(I) If $U(c, k) = c$, then

$$\hat{c}_S = c + \frac{\Psi_1}{2D^2}. \tag{3.9}$$

(II) If $U(c, k) = k$, then

$$\hat{k}_S = k + \frac{\Psi_2}{2D^2}. \tag{3.10}$$

(III) If $U(c,k) = R(t)$, then

$$\hat{R}_S = R(t)\left\{1 - \frac{\Phi_1}{2D} - \frac{\Phi_2}{2D^2}\right\} \tag{3.11}$$

where

$$\begin{aligned}\Phi_1 &= \frac{kt^c(kt^c-1)(logt)^2}{(1+t^c)^2}I + \frac{2t^c logt(1+logR(t))}{1+t^c}H + \frac{(logR(t))^2}{k^2}G,\\ \Phi_2 &= -\frac{kt^c logt}{1+t^c}\Psi_1 + \frac{logR(t)}{k}\Psi_2.\end{aligned}$$

With same argument, we can obtain Bayes estimators of the parameters c, k and the reliability function $R(t)$ under the linex loss function. They are given by the following forms:
(I) If $U(c,k) = e^{-ac}$, then

$$\hat{c}_L = c - \frac{1}{a}log\left[1 - \frac{a^2 I}{2D} - \frac{a\Psi_1}{2D^2}\right]. \tag{3.12}$$

(II) If $U(c,k) = e^{-ak}$, then

$$\hat{k}_L = k - \frac{1}{a}log\left[1 - \frac{a^2 G}{2D} - \frac{a\Psi_2}{2D^2}\right]. \tag{3.13}$$

(III) If $U(c,k) = e^{-aR(t)}$, then

$$\hat{R}_L = R(t) - \frac{1}{a}log\left\{1 - aR(t)\left[\frac{R(t)\zeta_1}{2D} + \frac{\zeta_2}{2D^2}\right]\right\} \tag{3.14}$$

where

$$\begin{aligned}\zeta_1 &= kt^c t_c^{-2}[akt^c + R(t)^{-1}(1-kt^c)](logt)^2 I - 2t^c t_c^{-1}(logt)[R(t)^{-1}\\ &\quad +(-a+R(t)^{-1})(logR(t))]H + (a - R(t)^{-1})\frac{(logR(t))^2}{k^2}G,\\ \zeta_2 &= kt^c t_c^{-1}(logt)\Psi_1 - \frac{logR(t)}{k}\Psi_2\end{aligned}$$

and $t_c = (1+t^c)$. All functions of the right-hand side of (3.9)-(3.14) are to be evaluated at the posterior mode $(\hat{c}^*, \hat{k}^*)$.

4 Simulation study and Comparisons

In this section, a simulation study is conducted in order to access the performance of the Bayes estimate with MLE for different sample sizes and censoring schemes.

Applying the algorithms of Balakrishnan and Aggarwala (2000), the following steps are used to generate a general progressively type II censored sample from BurrXII(c, k) distribution.
(1) Generate V_m from Beta distribution with parameters $n - r$ and $r + 1$.
(2) Independently generate Z_{r+i} from $U(0, 1)$ for $i = 1, \cdots, m - r - 1$.
(3) Set $V_{r+i} = Z_{r+i}^{\frac{1}{a_{r+1}}}$, $a_{r+i} = i + \sum_{j=m-i+1}^{m} R_j$, $i = 1, \cdots, m - r - 1$.
(4) Set $U_{r+i} = 1 - V_{m-i+1}V_{m-i+2} \cdots V_m$, $i = 1, \cdots, m - r$.
(5) For given c and k, we generated a censored sample of size m from Burr type XII distribution using the inverse cdf as follow

$$X_i = F^{-1}(U_i), \quad i = r + 1, \cdots, m.$$

This is the desired general progressively type II censored sample of size m from BurrXII(c, k).
(6) The MLEs, $\hat{c}_M$ and $\hat{k}_M$ of the parameters c and k were obtained by iteratively solving (2.3). Substituting $\hat{c}_M$ and $\hat{k}_M$ into (1.1), we obtained the MLE of the reliability function, $\hat{R}_M$ for some t. Bayes estimates with respect to SEL and linex loss functions of the parameters c, k and the reliability function $R(t)$ are computed using (3.9)-(3.13) and (3.14), respectively.
(7) Above steps are repeated 1,000 times and computed the averaged value and root mean squared error (RMSE) for different censoring schemes as follow

$$RMSE = \sqrt{\frac{1}{1000}\sum_{i=1}^{1000}(h(\theta) - h_i(\hat{\theta}))^2}$$

where $h(\theta)$ is true value and $h_i(\hat{\theta})$ is the ith estimate of $h(\theta)$ evaluated at $\hat{\theta}$.

To compare the performance of Bayes estimate with MLE for different sizes and censoring schemes, we generate a general progressively type II censored sample of size m with given censoring scheme $(R_{r+1}, R_{r+2}, \cdots, R_m)$ from Burr type XII distribution with parameters $c = 3$ and $k = 1$ (as true value) for fixed $r = 1$. For simplicity, we denote the censoring scheme, for example, by $(3 * 0, 1, 0, 1)$ which represents the censoring scheme $(-, R_2 = 0, R_3 = 0, R_4 = 0, R_5 = 1, R_6 = 0, R_7 = 1)$. It is assumed that the joint prior density for c and k is given by (3.1) with $\alpha = 0.1$ and $\beta = 0.1$ as improper prior.

For given progressive censoring scheme (-,0,2,0,3,0,2,0,3,0,2,0,0,3,0) and various a, the MLEs and Bayes estimates under SEL and linex functions of the parameters c and k are computed and presented in Table 1.

From Table 1, it is seen that if a goes to a negative value, Bayes estimates of the parameters c and k relative to linex loss function tend to give more weight to overestimation. Otherwise, it gives more weight to underestimation. For small a, linex loss function is almost symmetric and not far from SEL,

Table 1: Estimated mean of MLEs and Bayes estimators of c and k for various a.

a	$\hat{c}_M$	$\hat{c}_S$	$\hat{c}_L$	$\hat{k}_M$	$\hat{k}_S$	$\hat{k}_L$
-5	3.2805	2.9734	3.2226	1.0990	0.8612	0.9395
-3			3.2175			0.9192
-1			3.1254			0.9132
0.01			2.9715			0.8843
1			2.7795			0.8359
3			2.5724			0.7872
5			2.5328			0.7507

that is, asymmetric Bayes estimates relative linex are almost same as Bayes estimates under SEL.

Table 2 and 3 display the averaged values of MLE and Bayes estiamtes relative to SEL and linex loss functions of the parameters and its corresponding RMSE for different censoring schemes with fixed $r = 1$, respectively. The RMSEs are calculated as the average of the root mean squared deviation as mentioned above. From Table 2 and 3, it is seen that the Bayes estimates to be better than the MLE and the Bayes estimates relative to linex loss function preformed better than the others in the sense of comparing the RMSE of the estimates. As the effective sample proportion m/n increases, the RMSE of the estimates decreases.

Table 4 provides the averaged values of MLE and Bayes estimates of the reliability function $R(t)$ for some t and its corresponding RMSE for different sample sizes and censoring schemes. The true value of the reliability function $R(t)$ with $c = 3$ and $k = 1$ is $R(0.7) = 0.7446$. We observe that the Bayes estimates are better than MLE, and the Bayes estimates with respect to linex loss function are better than the others in terms of RMSE. As sample increases, the RMSE decreases which is the case in our computer simulation.

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Table 2: Estimated mean of MLEs and Bayes estimates of c with $a = 1$.

n	m (effective sample size)	$R_{(r+1)}, \cdots, R_m$ Censoring Schemes	$\alpha = 0.1$, $\beta = 0.1$ $\hat{C_M}$	$\hat{C_S}$	$\hat{C_L}$
30	15	(0*6,15,0*7)	3.2603	2.9663	2.7808
	(50%)		0.7546	0.5870	0.5484
		(1*6,2,1*7)	3.2859	2.9644	2.7712
			0.7857	0.5979	0.5591
	21	(0*19,9)	3.2134	3.0199	2.8493
	(70%)		0.6784	0.5797	0.5295
		(0*9,9,0*10)	3.1845	3.0279	2.8651
			0.6465	0.5686	0.5161
		(0*5,1*9,0*6)	3.1862	3.0284	2.8648
			0.6480	0.5696	0.5169
40	20	(0*18,20)	3.2524	2.9178	2.7648
	(50%)		0.7062	0.5184	0.5137
		(0*9,20,0*9)	3.1812	2.9601	2.8242
			0.5986	0.4900	0.4741
		(1*9,2,1*9)	3.1977	2.9649	2.8233
			0.6222	0.5021	0.4835
	28	(0*26,12)	3.1613	3.0240	2.8967
	(70%)		0.5768	0.5157	0.4782
		(0*13,12,0*13)	3.1414	3.0280	2.9063
			0.5537	0.5050	0.4666
		(0*4,3,0*5,3,0*5,	3.1424	3.0319	2.9095
		3,0*5,3,0*4)	0.5542	0.5072	0.4679
50	25	(0*23,25)	3.2058	2.9441	2.8192
	(50%)		0.6188	0.4804	0.4716
		(0*11,25,0*12)	3.1488	2.9797	2.8712
			0.5312	0.4526	0.4358
		(0*3,5,0*3,5,0*3,5,	3.1604	2.9799	2.8668
		0*4,5,0*3,5,0*3)	0.5475	0.4613	0.4439
	35	(0*33,15)	3.1320	3.0205	2.9204
	(70%)		0.4981	0.4497	0.4242
		(0*16,15,0*17)	3.1147	3.0235	2.9285
			0.4790	0.4407	0.4147
		(3,0*10,2,1,3,	3.1169	3.0322	2.9354
		0*10,3,0*7,1,2)	0.4817	0.4465	0.4180

Table 3: Estimated mean of MLEs and Bayes estimates of k with $a = 1$.

n	m (effective sample size)	$R_{(r+1)}, \cdots, R_m$ Censoring Schemes	$\alpha = 0.1$, $\beta = 0.1$		
			$\hat{k}_M$	$\hat{k}_S$	$\hat{k}_L$
30	15	(0*6,15,0*7)	1.0976	0.9927	0.9636
	(50%)		0.3612	0.2093	0.2001
		(1*6,2,1*7)	1.1032	0.9906	0.9617
			0.3931	0.2061	0.1975
	21	(0*19,9)	1.0509	1.0088	0.9857
	(70%)		0.2617	0.1912	0.1837
		(0*9,9,0*10)	1.0500	1.0131	0.9896
			0.2579	0.1957	0.1874
		(0*5,1*9,0*6)	1.0499	1.0130	0.9894
			0.2583	0.1956	0.1873
40	20	(0*18,20)	1.0874	0.9843	0.9623
	(50%)		0.3115	0.1810	0.1760
		(0*9,20,0*9)	1.0693	0.9947	0.9721
			0.2791	0.1893	0.1827
		(1*9,2,1*9)	1.0700	0.9941	0.9715
			0.2807	0.1880	0.1816
	28	(0*26,12)	1.0228	0.9959	0.9787
	(70%)		0.2106	0.1707	0.1668
		(0*13,12,0*13)	1.0225	0.9988	0.9813
			0.2084	0.1729	0.1686
		(0*4,3,0*5,3,0*5,	1.0222	0.9992	0.9815
		3,0*5,3,0*4)	0.2089	0.1734	0.1691
50	25	(0*23,25)	1.0572	0.9805	0.9626
	(50%)		0.2641	0.1750	0.1712
		(0*11,25,0*12)	1.0441	0.9894	0.9711
			0.2403	0.1815	0.1768
		(0*3,5,0*3,5,0*3,5,	1.0447	0.9883	0.9700
		0*4,5,0*3,5,0*3)	0.2422	0.1805	0.1759
	35	(0*33,15)	1.0219	1.0001	0.9861
	(70%)		0.1826	0.1543	0.1511
		(0*16,15,0*17)	1.0218	1.0026	0.9883
			0.1817	0.1564	0.1529
		(3,0*10,2,1,3,	1.0207	1.0033	0.9888
		0*10,3,0*7,1,2)	0.1824	0.1575	0.1539

Table 4: Estimated mean of MLEs and Bayes estimates of the reliability function with $a = 1$.

n	m (effective sample size)	$R_{(r+1)}, \cdots, R_m$ Censoring Schemes	$\alpha = 0.1$, $\beta = 0.1$		
			$\hat{R_M}$	$\hat{R_S}$	$\hat{R_L}$
30	15	(0*13,15)	0.7450	0.7382	0.7453
	(50%)		0.0747	0.0550	0.0526
		(1*6,2,1*7)	0.7435	0.7371	0.7440
			0.0755	0.0586	0.0563
	21	(0*19,9)	0.7467	0.7319	0.7368
	(70%)		0.0680	0.0587	0.0567
		(0*9,9,0*10)	0.7448	0.7317	0.7367
			0.0687	0.0591	0.0571
		(0*5,1*9,0*6)	0.7449	0.7316	0.7366
			0.0687	0.0592	0.0572
40	20	(0*18,20)	0.7449	0.7396	0.7450
	(50%)		0.0617	0.0497	0.0480
		(0*9,20,0*9)	0.7426	0.7392	0.7445
			0.0628	0.0523	0.0508
		(1*9,2,1*9)	0.7436	0.7386	0.7438
			0.0631	0.0527	0.0511
	28	(0*26,12)	0.7490	0.7376	0.7412
	(70%)		0.0596	0.0528	0.0517
		(0*13,12,0*13)	0.7477	0.7374	0.7410
			0.0599	0.0529	0.0518
		(0*4,3,0*5,3,0*5,	0.7478	0.7372	0.7408
		3,0*5,3,0*4)	0.0602	0.0535	0.0523
50	25	(0*23,25)	0.7472	0.7426	0.7470
	(50%)		0.0569	0.0476	0.0465
		(0*11,25,0*12)	0.7453	0.7422	0.7464
			0.0579	0.0499	0.0489
		(0*3,5,0*3,5,0*3,5,	0.7459	0.7419	0.7460
		0*4,5,0*3,5,0*3)	0.0581	0.0501	0.0490
	35	(0*33,15)	0.7478	0.7387	0.7416
	(70%)		0.0518	0.0471	0.0463
		(0*16,15,0*17)	0.7466	0.7384	0.7414
			0.0522	0.0474	0.0465
		(3,0*10,2,1,3,	0.7469	0.7381	0.7410
		0*10,3,0*7,1,2)	0.0529	0.0482	0.0474