The Ordered Weighted Averaging Algorithm to
Multiple Attribute Decision Making within
Triangular Fuzzy Numbers

Qingjian Zhou
College of Science, Dalian Nationalities University, Dalian, 116600, China

Jia Jiao
College of Science, Dalian Nationalities University, Dalian, 116600, China

Abstract

In this paper, we propose an ordered weighted averaging algorithm to solve multiple attribute decision making with attribute values within triangular fuzzy numbers.

Keywords: Multiple attribute, Decision making, Triangular fuzzy number, OWA

1 Introduction

Multiple attribute decision making is characterized by a decision maker, who is called to rank all the alternatives as well as select the best. In many situations, attribute values are given in the form of triangular fuzzy numbers. Up to know a

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2 Corresponding author
lot of research has been done to it, see [1-3]. In this paper we will develop a new practical decision analysis method based the ordered weighted averaging algorithm to multiple attribute decision making within triangular fuzzy numbers.

2 Main Results

**Definition 1** $\tilde{M} = [l, m, u]$ is called a triangular fuzzy number, and its membership function is the following:

$$
\mu_{M}(x) = \begin{cases} 
\frac{x-l}{m-l}, & x \in [l, m]; \\
\frac{u-x}{u-m}, & x \in [m, u]; \\
0, & \text{others.}
\end{cases}
$$

Here, $0 < l < m < u$ are real numbers. Specially, when $l = m = u$, $\tilde{M}$ is degraded into a real number, as in [2,4]. Triangular fuzzy numbers can express uncertain information very well.

**Definition 2** $E(\tilde{M}) = \int_{-\infty}^{\infty} x \mu_{M}(x) dx = \int_{l}^{m} x \frac{x-l}{m-l} dx + \int_{m}^{u} x \frac{u-x}{u-m} dx$ is called the mean value of triangular number $\tilde{M}$. The mean value can well express the value of the corresponding triangular fuzzy number. So in this paper we will substitute the mean values of triangular fuzzy numbers for the triangular fuzzy numbers in the problems of multiple attribute with attribute values in the form of triangular fuzzy numbers.

**Definition 3** let $OWA : R^n \rightarrow R$, if $OWA_m(\alpha_1, \alpha_2, \cdots, \alpha_n) = \sum_{j=1}^{n} \omega_j b_j$, where $\omega=(\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weighted vector, $\omega_j \in [0,1]$, and $\sum_{j=1}^{n} \omega_j = 1$, $b_j$ is the $j$th number in the data $(\alpha_1, \alpha_2, \cdots, \alpha_n)$, $R$ is the set of all real numbers, then $OWA$ is called an ordered weighted averaging operator, or $OWA$ in short.

3 Algorithm Steps

**Step 1** Replace each triangular fuzzy number $\tilde{M} = [a_l, a_m, a_u]$ of the original decision matrix with its mean value $E(\tilde{M})$, so the original matrix is transformed into the matrix composed by their mean values $A = (a_{ij})$, here $a_{ij} = E(\tilde{M})$, 

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**Ordered weighted averaging algorithm**

**Step 2** Let $I_1, I_2$ represent the subscript sets of the benefit type and the cost benefit attributes. Standard $A=\{a_{ij}\}$ into standardization matrix $R=(r_j)$ as:

\[
    r_j = \frac{a_{ij}}{\max_i(a_{ij})}, i = 1,2\cdots n; j \in I_1
\]

\[
    r_j = \frac{\min_i(a_{ij})}{a_{ij}}, i = 1,2\cdots n; j \in I_2
\]

**Step 3** Obtain $\omega=(\omega_1, \omega_2, \cdots, \omega_n)$ and get overall value by OWA algorithm

\[
    z_i(\omega) = OWA_m(r_{i1}, r_{i2}, \cdots, r_{im}) = \sum_{j=1}^{m} \omega_j b_{ij}
\]

Here $b_{ij}$ is the $j$th number in the data $r_{i1}, r_{i2}, \cdots, r_{im}$.

**Step 4** Rank the alternatives and select the best by $z_i(\omega)$.

**4 Illustrative Example**

Consider the following example. We will evaluate four departments $x_i (i = 1, 2, 3, 4)$ in one university against four attributes:

- $u_1$: Teaching level, $u_2$: Research level,
- $u_3$: Operating cost, $u_4$: Satisfaction degree of students

And the evaluating results are shown in Table 1 with attribution values in the form of triangular fuzzy numbers, where Teaching level, Research level and Satisfaction degree of students are benefit attributes, and Operating cost is cost attribute. Which department is the best?

<table>
<thead>
<tr>
<th></th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>[7,8,10]</td>
<td>[5,6,8]</td>
<td>[7,8,9]</td>
<td>[5,7,8]</td>
</tr>
<tr>
<td>$x_2$</td>
<td>[5,8,10]</td>
<td>[4,5,7]</td>
<td>[4,6,7]</td>
<td>[6,8,10]</td>
</tr>
<tr>
<td>$x_3$</td>
<td>[6,7,9]</td>
<td>[6,7,8]</td>
<td>[7,8,9]</td>
<td>[6,8,9]</td>
</tr>
<tr>
<td>$x_4$</td>
<td>[6,8,9]</td>
<td>[6,7,8]</td>
<td>[6,8,10]</td>
<td>[5,7,9]</td>
</tr>
</tbody>
</table>

**Step 1** Replace each attribute value of triangular fuzzy number of $\tilde{M}=[a_l, a_m, a_u]$ in the original decision matrix in Table 1 with its mean value and get the matrix composed by the mean value,
Step 2  Calculate the standardization matrix by (1) and (2)

\[
R = (r_i) = \begin{pmatrix}
0.6522 & 1 & 1 & 0.6250 \\
1 & 0.8421 & 0.9412 & 1 \\
0.5739 & 0.7368 & 1 & 0.7188 \\
0.4174 & 0.7368 & 0.5 & 0.8750
\end{pmatrix}
\]

Step 3  Here we set \( \omega = (0.4, 0.3, 0.1, 0.2) \) based on the importance of the four attributes and calculate \( z_i(\omega) \) by (3)

\[
z_1(\omega) = 0.8929, z_2(\omega) = 0.9724, z_3(\omega) = 0.8222, z_4(\omega) = 0.7128,
\]

Step 4  Utilize \( z_i(\omega) \) to rank the alternatives: \( x_2 \succ x_1 \succ x_3 \succ x_4 \), and thus the best alternative is \( x_2 \), so the best department is the 2th department.

References


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