Robust Stability Analysis of Teleoperation by Delay-Dependent Neutral LMI Techniques

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Abstract
This paper studies the robust stability of teleoperation systems based on the 2-channel force-force (FF) control architecture, modelled by frequency domain or passivity techniques. More precisely, delay-dependent conditions are given as LMIs derived from Lyapunov-Krasovskii functionals. We show that the teleoperation systems are stable under specific LMI conditions. This stability is guaranteed in the presence of admissible uncertainty and neutral-type differential-delayed equations, that is, with time-varying delays in the states and their derivative. To evaluate the theoretical analysis, some numerical simulations are presented.

Keywords: Teleoperation, Robust Stability, Delay-dependent neutral systems, time-varying delay, LMI techniques

1 Introduction

A teleoperation system consists of a master, that is manipulated by a human operator, and a slave, which is designed to track the master movements in a
remote environment. For this, information is transmitted between master and slave via specific communication channels. The main concerns in the analysis and synthesis of these systems are its stability and performance, given in terms of tracking capabilities.

Teleoperation has challenged researchers in both control theory and robotics over the last few decades. The breakthrough was achieved in [1] where the concepts from scattering theory, passivity and network theory were used to derive a control law which guaranteed stability [2, 3, 7, 8, 16, 14, 15]. The robust stability criteria for teleoperation system with time-delay can be found in [4]. All of them are delay-independent analysis methods.

In this paper, we present a delay-dependent robust stability criteria for teleoperation system with admissible uncertainties, that can be described by neutral differential equations. The particularity of this neutral systems is the existence of delays in both the state vector and its derivative. It has been shown that these neutral systems are particularly sensitive to delays and can be easily destabilized [13, 9].

Motivated by the integral inequality and the Lyapunov-Krasovskii approach [10], we develop in this paper new results for robust stability of teleoperation systems for time varying delays. The results are expressed in terms of LMIs that can be easily solved using dedicated solvers [6], providing estimations of the maximum allowable delays.

The study is organized as follows. In Section 2 an overall description of teleoperation systems is introduced. In Section 3, we represent the system equations in state-space model and present a delay-dependent Lyapunov stability analysis by using LMI. In section 4, we extend the proposed criteria to the robust case. Section 5 presents simulation results that support the theoretical work and conclusion is drawn in Section 6.

2 Teleoperation modelling

A bilateral teleoperation system consists of the master and slave mechanical systems, often with control loops at each manipulator. Thus, the system can be represented by the block diagram of Fig. 1. The variables are $F_h$, the force applied by human operator, $F_e$, the contact force with the environment, $F_{mc}$ and $F_{sc}$, which are the forces computed by the local master/slave control algorithms. So the forces $F_m$ applied to the master and $F_s$ applied to the slave can be defined as:

$$
F_m = F_h - F_{mc},
$$
$$
F_s = F_e + F_{sc}.
$$

(1)
The blocks labelled Master, Slave in that figure represent the dynamics of the master and slave manipulators. This scheme also includes the signal flow through the communication channel, characterized by a transmission delay $\tau(t)$. In general, the exchanged variables among the blocks are velocity and force.

The master and slave dynamics can be described by

$$F_i = (M_i s + B_i) \nu_i, \ i = m, s$$

where $M_i$ and $B_i$ are the manipulator's inertia and damping coefficients, while $F_i$ and $\nu_i$ are the force and velocity. The subscripts $i = m$ and $i = s$ indicate the master and slave manipulators, respectively. Furthermore, a time-varying transmission delay in the communication channel is considered.

The study focuses on a specific control scheme belonging to two-channel architectures [12]: Force-Force control (FF). This scheme is derived from the general four-channel transparency-optimized controller [12]. In the Force-Force (FF) architecture the local controllers are given as follows:

$$F_{mc}(s) = \frac{K_m}{s} \nu_m(s) + C_2 F_s(s)e^{-\tau s} - C_6 F_e$$

$$F_{sc}(s) = -\frac{K_s}{s} \nu_s(s) + C_3 F_m(s)e^{-\tau s} - C_5 F_e$$

### 3 Delay-dependent stability for linear neutral systems

The goal of our design is to ensure the stability of the teleoperation system using the LMI framework, so that position and forces of master and slave track each other, even in the presence of the inherent time-varying delay.

Substituting (2) and (3) into (1) for nominal conditions ($F_e = 0$) and taking $\nu_m(t) = \dot{x}_m(t)$, $\dot{\nu}_m(t) = \nu_s(t)$ and $\dot{x}_m(t) = x_s(t)$, where $x_m$ is the master position, $x_s$ is the slave position, $\nu_m$ is the master velocity and $\nu_s$ is the slave
velocity gives

\[ f_m = M_m \dot{\nu}_m(t) + B_m \nu_m(t) \]
\[ = -K_m x_m(t) - C_2 M_s \dot{\nu}_s(t - \tau) - C_2 B_s \nu_s(t - \tau) + f_h(1 + C_6) \]  

and

\[ f_s = M_s \nu_s(t) + B_s \nu_s(t) \]
\[ = -K_s \nu_m(t) - C_3 M_m \dot{\nu}_m(t - \tau) - C_3 B_m \nu_m(t - \tau) \]  

The state-space vector \( x(t) \in \mathbb{R}^n \) is defined as:

\[ x(t) = \begin{bmatrix} x_m(t) & x_s(t) & \nu_m(t) & \nu_s(t) \end{bmatrix}^T \]

From \( x_s(t) = \nu_m(t) \), the closed-loop state equations of the system become

\[ \dot{x}(t) - C \dot{x}(t - \tau_d) = A_0 x(t) + A_1 x(t - \tau(t)) + B f_h(t) \]  

where

\[
A_0 = \begin{bmatrix}
0 & -K_m/M_m & 1 & 0 & 0 \\
-K_m/M_m & B_m/M_m & 0 & 0 & 0 \\
0 & 0 & 0 & -K_s/M_s & 1 \\
0 & 0 & 0 & -C_2 B_s/M_m & 0 \\
0 & 0 & 0 & 0 & -C_3 B_m/M_s \\
0 & 0 & 0 & 0 & -C_3 M_s/M_m \\
0 & 0 & 0 & 0 & -C_2 M_m/M_s \\
0 & 0 & 0 & 0 & -C_3 M_m/M_s \\
\end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/C_6/M_m \\ 0 \\ 0 \end{bmatrix},
\]

The relations between positions and velocities \( \dot{x}_m(t) = \nu_m(t) \), \( \dot{x}_s(t) = \nu_s(t) \) provide the first and third model equations in (7).

The delay parameter is assumed to be an unknown time-varying function that satisfies for all \( t \geq 0 \):

\[ 0 \leq \tau(t) \leq h_m, \quad \dot{\tau}(t) \leq d < 1. \]  

The bound \( d \) is strictly smaller than one to ensure causality: see [7]). The initial condition of system (7) is given by

\[ x(\theta) = \phi(\theta), \quad \theta \in [-\max[\tau_d, h_m], 0], \]  

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\( \phi(.) \) is a vector of differentiable functions of initial values (i.e., \( \phi \in C^1[-\max[\tau_d, h_m], 0] \)). Consider the operator \( D : C^1([-\tau_d, 0], \mathbb{R}^n) \rightarrow \mathbb{R}^n \) defined in [11] as

\[
D(x_t) = x(t) - Cx(t - \tau_d).
\]

**Definition 1.** (see [11]) This operator \( D \) is said to be stable if the zero solution of the homogeneous difference equation is uniformly asymptotically stable:

\[
D(x_t) = 0, \quad t \geq 0, \quad x_0 = \phi \in [\phi \in C^1[-\tau_d, 0] : D\phi = 0], \quad (10)
\]

**Remark 1.** A necessary condition for stability is that the system (7) with \( n \) state variables (6) must be Hurwitz stable for zero delay, that is, all the eigenvalues \( \lambda_i \) of \( \Lambda = (I_n \times n - C)^{-1}(A_0 + A_1) \) with an adequate selection of parameters verify:

\[
\text{real}(\lambda_i) < 0 \quad \forall i, i = 1...\text{rank}(\Lambda) \quad (11)
\]

**Theorem 1.** The teleoperation system (7) with state vector defined in (6) is stable for any delays \( \tau(t) \) and \( \tau_d \) of the communication channel, if there exist symmetric positive definite matrices \( P = P^T > 0, Q = Q^T > 0, R = R^T > 0 \) and \( W = W^T > 0 \), appropriately sized matrices \( N_i \) and \( T_i, i = 1, 2 \) such that the LMI shown in (12) holds

\[
\Omega = \begin{bmatrix}
\Omega_{11} & * & * & * & * \\
\Omega_{21} & \Omega_{22} & * & * & * \\
\Omega_{31} & \Omega_{32} & \Omega_{33} & * & * \\
C^T T_1^T & C^T T_2^T & 0 & -W & * \\
h_m N_1^T & 0 & h_m N_2^T & 0 & -h_m R
\end{bmatrix} < 0. \quad (12)
\]

where

\[
\Omega_{11} = T_1 A_0 + A_0^T T_1^T + Q + N_1 + N_1^T,
\]

\[
\Omega_{21} = P - T_1^T + T_2 A_0,
\]

\[
\Omega_{22} = -T_2 - T_2^T + h_m R + W,
\]

\[
\Omega_{31} = -N_1 + N_2 + A_1^T T_1^T,
\]

\[
\Omega_{32} = A_1^T T_2^T,
\]

\[
\Omega_{33} = -N_2 - N_2^T - (1 - d)Q,
\]
**Proof 1.** Consider the following Lyapunov-Krasovski functional:

\[
V(t) = x^T(t)Px(t) + \int_{t-\tau(t)}^{t} x^T(s)Qx(s)ds + \int_{t-h_m}^{t} \int_{t+\theta}^{t} \hat{x}^T(s)R\hat{x}(s)dsd\theta + \int_{t-\tau_d}^{t} \hat{x}^T(s)W\hat{x}(s)ds,
\]

(13)

It is clear that this functional is positive definite. Taking the derivative of this function we have that

\[
\dot{V}(t) = 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - (1 - \hat{\tau}(t))x^T(t - \tau(t))Qx(t - \tau(t))
\]

\[
+ h_m\dot{x}^T(t)R\dot{x}(t) - \int_{t-h_m}^{t} \hat{x}^T(s)R\hat{x}(s)ds + \hat{x}^T(t)W\hat{x}(t)
\]

\[-\hat{x}^T(t - \tau_d)W\hat{x}(t - \tau_d)
\]

(14)

From (8), it is clear that

\[
- \int_{t-h_m}^{t} \hat{x}^T(s)R\hat{x}(s)ds \leq - \int_{t-\tau(t)}^{t} \hat{x}^T(s)R\hat{x}(s)ds
\]

(15)

Applying the integral inequality of the Lemma in [10] to the term on the right hand side of (15) for any \(N_1, N_2\) yields the following integral inequality:

\[
- \int_{t-\tau(t)}^{t} \hat{x}^T(s)R\hat{x}(s)ds \leq + 2\xi^T(t) \begin{bmatrix} N_1 & -N_1 \\ N_2 & -N_2 \end{bmatrix} \xi(t)
\]

\[+ h_m\xi^T(t) \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} R^{-1} \begin{bmatrix} N_1^T & N_2^T \end{bmatrix} \xi(t)
\]

(16)

For any matrices \(T_1\) and \(T_2\) and nominal conditions \(f_h = 0\), the following relation is true:

\[2[x^T(t)T_1 + \hat{x}^T(t)T_2] \times [-\dot{x}(t) + C\dot{x}(t - \tau_0) + A_0x(t) + A_1x(t - \tau(t))] = 0\]

Then, adding this term on the left to \(\dot{V}(t)\) and substituting (15) and (16) into (14), we obtain

\[
\dot{V}(t) = 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - (1 - \hat{\tau}(t))x^T(t - \tau(t))Qx(t - \tau(t))
\]

\[
+ h_m\dot{x}^T(t)R\dot{x}(t) + 2\xi^T(t) \begin{bmatrix} N_1 & -N_1 \\ N_2 & -N_2 \end{bmatrix} \xi(t)
\]

\[+ h_m\xi^T(t) \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} R^{-1} \begin{bmatrix} N_1^T & N_2^T \end{bmatrix} \xi(t)
\]

\[+ 2[x^T(t)T_1 + \hat{x}^T(t)T_2] \times [-\dot{x}(t) + C\dot{x}(t - \tau_d) + A_0x(t) + A_1x(t - \tau(t))] + \hat{x}^T(t)W\hat{x}(t) - \hat{x}^T(t - \tau_d)W\hat{x}(t - \tau_d)
\]

(17)
Thus, if the following inequality holds:

$$\eta^T(t)\Gamma\eta(t) + h_m\xi^T(t) \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} R^{-1} \begin{bmatrix} N_1^T & N_2^T \end{bmatrix} \xi(t) < 0 \quad (18)$$

where

$$\eta(t)^T = \begin{bmatrix} x^T(t) & \dot{x}^T(t) & x^T(t - \tau(t)) & \dot{x}^T(t - \tau_d) \end{bmatrix},$$

$$\xi(t) = \begin{bmatrix} x(t) \\ x(t - \tau(t)) \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \Gamma_{11} & * & * & * \\ \Gamma_{21} & \Gamma_{22} & * & * \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & * \\ C^T T_1^T & C^T T_2^T & 0 & -W \end{bmatrix},$$

$$\Gamma_{11} = T_1 A_0 + A_0^T T_1^T + Q + N_1 + N_1^T,$$

$$\Gamma_{21} = P - T_1^T + T_2 A_0,$$

$$\Gamma_{22} = -T_2 - T_2^T + h_m R + W,$$

$$\Gamma_{31} = -N_1^T + N_2 + A_1^T T_1^T,$$

$$\Gamma_{32} = A_1 T_2^T,$$

$$\Gamma_{33} = -N_2 - N_2^T - (1 - d) Q.$$

Using the Schur complement we obtain $\Omega < 0$, which guarantees $\dot{V} < 0$, so the teleoperation system is asymptotically stable, which completes the proof.

4 Robust stability analysis

Friction forces and other interaction of the system with the environment may appear on the system as uncertainties. Thus, we now consider uncertain teleoperation system as follows

$$\dot{x}(t) - (C + \Delta C(t)))\dot{x}(t - \tau_d) = (A_0 + \Delta A_0(t))x(t) + (A_1 + \Delta A_1(t))x(t - \tau(t)) + B.f_h(t), \quad (19)$$

Where matrices $\Delta A_0$, $\Delta A_1$ and $\Delta C$ characterize the uncertainties in the system and satisfy the following assumption.

$$\begin{bmatrix} \Delta A_0(t) & \Delta A_1(t) & \Delta C(t) \end{bmatrix} = DF(t) \begin{bmatrix} E_0 & E_1 & E_2 \end{bmatrix} \quad (20)$$

Where $D, E_0, E_1$ and $E_2$ are constant matrices defining how the uncertainty enters in the model, $F(t) \in \mathbb{R}^{i \times j}$ (i, j integers) is an unknown real time-varying matrix function of uncertain parameters’ measurable elements such
that $F^T(t)F(t) \leq I$.

Before moving on, we introduce a lemma used to take uncertainties into account.

**Lemma 1.** [17] Let $D$, $E$ and $F(t)$ be real matrices of appropriate dimensions, with $\|F\| \leq I$. Then, for any scalar $\varepsilon > 0$, the following inequalities will be true:

$$DF(t)E + E^T F(t)^T D^T \leq \frac{1}{\varepsilon} DD^T + \varepsilon E^T E$$

(21)

**Theorem 2.** The uncertain teleoperation system (19) is stable for any delays $\tau(t)$ and $\tau_d$ of the communication channel and any uncertainty satisfying (20), if there exist symmetric positive definite matrices $P = P^T > 0, Q = Q^T > 0, R = R^T > 0$ and $W = W^T > 0$, appropriately sized matrices $N_i$ and $T_i$, $i = 1, 2$ and positive scalar $\varepsilon$ such that the LMI given in (22) holds

$$\begin{bmatrix}
\Psi_{11} & * & * & * & * & * \\
\Psi_{21} & \Psi_{22} & * & * & * & * \\
\Psi_{31} & \Psi_{32} & \Psi_{33} & * & * & * \\
C^T T_1^T + \varepsilon E_2^T E_0 & C^T T_2^T & \varepsilon E_2^T E_1 & -W + \varepsilon E_2^T E_2 & * & * \\
h_m N_1^T & 0 & h_m N_2^T & 0 & -h_m R & * \\
D^T T_1^T & D^T T_2^T & 0 & 0 & 0 & -\varepsilon I
\end{bmatrix} < 0$$

(22)

where

$$\begin{align*}
\Psi_{11} &= T_1 A_0 + A_0^T T_1^T + Q + N_1 + N_1^T + \varepsilon E_0^T E_0, \\
\Psi_{21} &= P - T_1^T + T_2 A_0, \\
\Psi_{22} &= -T_2 - T_2^T + h_m R + W, \\
\Psi_{31} &= -N_1^T + N_2 + A_1^T T_1^T + \varepsilon E_1^T E_0, \\
\Psi_{32} &= A_1^T T_2^T, \\
\Psi_{33} &= -N_2 - N_2^T - (1 - d)Q + \varepsilon E_1^T E_1,
\end{align*}$$

**Proof 2.** By replacing $A_0$, $A_1$ and $C$ with $A_0 + \Delta A_0$, $A_1 + \Delta A_1$ and $C + \Delta C$, respectively, in (12) we have

$$\Omega + \overline{DF(t)} \overline{E} + \overline{E}^T F^T(t) \overline{D}^T < 0,$$

(23)

where

$$\overline{D} = \begin{bmatrix}
D^T T_1^T & D^T T_2^T & 0 & 0 & 0
\end{bmatrix}^T, \quad \overline{E} = \begin{bmatrix}
E_0 & 0 & E_1 & E_2 & 0
\end{bmatrix}^T$$

According to Lemma 1, (23) holds if there exists $\varepsilon > 0$ such that

$$\Omega + \frac{1}{\varepsilon} \overline{D}^T + \varepsilon \overline{E}^T \overline{E} < 0$$

(24)
Thus, by the Schur complement, (24) is equivalent to (22) of the Theorem 2.

**Remark 2.** The method proposed in [5], do not allow us to check the robust stability with time-varying delay for teleoperation systems. In this paper, a delay-dependent robust stability criteria for teleoperation system with admissible uncertainties and time-varying delay is derived. The results are developed by using the integral inequality and Lyapunov-Krasovskii approaches [10] that have the potential to give less conservative results.

## 5 Numerical examples

### 5.1 Teleoperation system

To illustrate the validation of our proposed method, consider the teleoperation system with the following parameters, given in [5]:

\[ M_m = 10Kgm^2s^2/rad, \quad M_s = 12Kgm^2s^2/rad, \quad C_2 = 0.5, \quad C_3 = 0.1, \quad C_6 = 1.5, \]
\[ B_m = 2\delta_m M_m \omega_m, \quad B_s = 2\delta_s M_s \omega_s, \quad K_m = M_m \omega_m^2, \quad K_s = M_s \omega_s^2, \]

with \( \delta_m = \delta_s = 1 \) and \( \omega_m = \omega_s = 40 \text{rad/seg} \).

**Case 1: Constant time delay**

In [5], it was found that the teleoperation system was stable for a maximum delay \( \tau = 10.7 \). Using Theorem 1, we obtain the feasibility of the condition (12) even with a delay \( \tau = 10^6 \). It is obvious that our result is less conservative than that in [5].

The evolution of the state variables with \( F_h = 1, \quad F_e = 0 \) and the constant time delay \( \tau = 20 \) is shown in the following figures:

![Fig. 2. Evolution of Master position \( x_m \) (rad) in the FF scheme.](image)
Fig. 3. Evolution of Slave position $x_s$ (rad) in the FF scheme.

Fig. 4. Evolution of Master velocity $\nu_m$ (rad/sec) in the FF scheme.
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Fig. 5. Evolution of Slave velocity $\nu_s$ (rad/sec) in the FF scheme.

Case 2: Time-varying delay
The stability analysis is assessed applying Theorem 1 using the LMI Toolbox of the MATLAB. The inequality (12) is feasible for a maximum delay $h_m = 0.0865$ and $d = 0.9356$ with the following results:

$$P = 10^4 \begin{bmatrix} 0.2020 & 0.0006 & -0.0000 & -0.0000 \\ 0.0006 & 0.0001 & -0.0000 & -0.0000 \\ -0.0000 & -0.0000 & 1.4547 & 0.0042 \\ -0.0000 & -0.0000 & 0.0042 & 0.0009 \end{bmatrix},$$

$$Q = \begin{bmatrix} 4.6300 & -0.0834 & 0.0120 & 1.3490 \\ -0.0834 & 89.4375 & -1.4444 & 0.0278 \\ 0.0120 & -1.4444 & 32.1136 & -0.0673 \\ 1.3490 & 0.0278 & -0.0673 & 644.0083 \end{bmatrix},$$

$$R = \begin{bmatrix} 0.0569 & -0.0004 & 0.0017 & -0.0122 \\ -0.0004 & 0.0020 & 0.0124 & -0.0002 \\ 0.0017 & 0.0124 & 0.4097 & -0.0059 \\ -0.0122 & -0.0002 & -0.0059 & 0.0141 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.0033 & -0.0000 & 0.0000 & -0.0002 \\ -0.0000 & 0.0062 & 0.0002 & 0.0000 \\ 0.0000 & 0.0002 & 0.0239 & 0.0000 \\ -0.0002 & 0.0000 & 0.0000 & 0.0450 \end{bmatrix}.$$
5.2 Uncertain teleoperation system with time-varying delay

Now, let the system be subject to norm-bounded uncertainties of the form (20) with

\[
D = \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix},
E_0 = \begin{bmatrix}
0 & 0.1 & 0 & 0
\end{bmatrix},
E_1 = \begin{bmatrix}
0 & 0.1 & 0 & 0
\end{bmatrix},
E_2 = \begin{bmatrix}
0 & 0 & 0.1 & 0
\end{bmatrix},
F(t) = \sin(t),
\]

Using Theorem 2, we found that the uncertain teleoperation system is robustly stable for a maximum delay \( h_m = 0.099 \) and \( d = 0.918 \), with the following solution:

\[
P = 10^4 \begin{bmatrix}
0.3185 & 0.0025 & 0.0510 & 0.0003 \\
0.0025 & 0.0002 & 0.0004 & 0.0000 \\
0.0510 & 0.0004 & 2.3071 & 0.0076 \\
0.0003 & 0.0000 & 0.0076 & 0.0015
\end{bmatrix},
\]

\[
Q = \begin{bmatrix}
0.5419 & -1.6876 & -0.2686 & -1.5354 \\
-1.6876 & 134.9505 & 27.3111 & -13.8392 \\
-0.2686 & 27.3111 & 15.8727 & 25.7823 \\
-1.5354 & -13.8392 & 25.7823 & 989.5986
\end{bmatrix},
\]

\[
R = \begin{bmatrix}
0.0158 & 0.0148 & -0.1283 & 0.0013 \\
0.0148 & 0.0279 & -0.2390 & 0.0021 \\
-0.1283 & -0.2390 & 2.1111 & -0.0180 \\
0.0013 & 0.0021 & -0.0180 & 0.0002
\end{bmatrix},
\]

\[
W = \begin{bmatrix}
0.0005 & 0.0000 & -0.0001 & 0.0000 \\
0.0000 & 0.0097 & 0.1747 & 0.0005 \\
-0.0001 & 0.1747 & 7.4472 & -0.1607 \\
0.0000 & 0.0005 & -0.1607 & 0.0815
\end{bmatrix}
\]

It can be seen that the teleoperation system is robustly stable under time-varying delay and admissible uncertainties. The simulations demonstrate the validity of the methodology presented by this paper.

6 Conclusions

The problem of stabilizing an uncertain teleoperation system in the presence of time-varying delays in the communication channel is addressed. In this context, we have applied previous result for delay-dependent stability in terms
of LMI based on the theory of Lyapunov-Krasovskii functional for time-varying delays, that also take into account bound on the delay derivatives, to reduce conservativeness. Then, the results were extended to consider systems affected by norm-based bounded uncertainty. A numerical example was used to show the feasibility of the proposed methodology.

References


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