Prospects for the Use of Space Robots in the Neighbourhood of the Libration Points

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Abstract

This paper is devoted to the navigation tasks for space robots operating in the neighborhood of unstable libration points of the Earth-Sun system in the near-Earth space. The motion of robot in space is examined under a circular restricted three-body problem. The possibility of a space robot maneuvering in the near-Earth space with return to the neighborhood of a collinear libration point and stabilizing in the neighborhood of this point are shown.

Keywords: space robots, circular restricted three-body problem, Hill problem, libration point, controlled motion
1 Introduction

Space robotics is one of the most attractive areas of modern space. The main goal of space robotics is to create and develop complex mechanisms capable of operating in the harsh space environment.

Space robotics is essential for the overall ability, and hence the efficiency to work in space. It expands the functionality of spacecrafts significantly, and considerably helps astronauts in space or even takes their work entirely over in hazardous conditions. In general, space robotics opens new opportunities for the development of the traditional space associated with creation and development of fundamentally new types of spacecrafts, such as, constructing large (huge, extended) space stations that perform controlled maneuvers in the near-Earth space. This approach is particularly relevant in investigating other celestial bodies, for example, in solving comet and asteroid hazard problems.

Obviously, designing projects that involve the use of space robotics systems should include evaluation of such projects practical feasibility. This will necessarily involve the use of highly complex equations of celestial mechanics.

The so-called collinear libration points of the Sun-Earth system can play an important role in space maneuvering [1].

The motion of a space robot under the influence of the gravitational forces of two attracting bodies, such as the Earth and the Sun, can be described through a model of circular restricted three-body problem. According to this model, the movement of massless body \( p \) under the influence of gravitational fields of attraction between two massive bodies \( E \) and \( S \), revolving around their common center of mass, is investigated. The bodies are treated as material points. It is also assumed that the body of infinitesimal mass \( p \) does not affect the movement of the attracting centers of \( E \) and \( S \).

It is known that the equations of the three-body problem have five stationary solutions. In celestial mechanics, these solutions are called libration points or Lagrange points. Three of them \( L_1, L_2, L_3 \) – collinear libration points – are unstable equilibrium positions in a rotating frame, \( L_4 \) and \( L_5 \) – are stable and are called triangular due to their location in the configuration space [1].

Libration points are abstract concepts of the circular restricted three-body problem. However, their properties determine the qualitative character of the orbital motion of a space robot, which in reality occurs under the influence of many perturbing factors.

Investigation of orbital motion in the neighborhood of the collinear libration point of the Sun-Earth system has long been of practical importance for projects (ISEE-3, SOHO, WIND, ACE, Genesis, etc.) implemented by the National Aeronautics and Space Administration (NASA) and the European Space Agency (ESA).
2 Statement of the Problem

The development of astronautics has recently been facilitating implementation of projects related to the use of the neighborhoods of collinear libration points $L_1$ and $L_2$, and stabilizing a space robot in their neighborhood is becoming obvious in view of their properties [2, 3, 4, 5]. At the same time, the instability of collinear libration points can also be a positive factor for space maneuvering with a relatively small energy consumption [6, 7]. This requires creating control laws that can both stabilize orbital motion and facilitate efficient movement of a space robot in the near-Earth space.

In this regard, it is proposed to develop methods for remote control of space robotic systems in the near-Earth space with the use of the neighborhoods of collinear libration points. The specifics of the problem is determined by the following factors:

- the collinear libration point is unstable, i.e. if a space robot is moving along a halo-orbit around the collinear libration point and, having received a perturbation, can significantly leave the neighborhood,

- a large delay in control signal transmission from the control center to the orbit and back,

- functioning of a space craft in a non-deterministic environment, which, not predictable in nature, but whose nature of impact is determined, can generate a perturbation affecting the orbital motion.

It is rational to control such a space robot taking into account a time delay in signal transmission and a non-deterministic environment in the so called supervisory mode [7], when a person performs the role of a supervisor. Two types of telecontrol are proposed for this purpose:

- autonomous, through an information control complex located at the space robot itself,

- remote, through a human operator situated at the control center.

The instability of the collinear libration point is at the same time a positive factor and allows the space robot to carry out different maneuvers. Some of these maneuvers may last for a very long time without significant energy consumption by jet engines, using mostly the force of gravitational attraction to move, or possibly in the future – the light pressure with the use of such exotic control systems like a solar sail. To ensure a space robot stays long in the neighborhood of the libration point, we must first solve the problem of its stabilization. This, in turn, opens the way for the creation and use of special space robots to perform a large variety of tasks there, including robotics tasks, for example, the assembly of space stations.
The equations of motion of a space robot in a rotating frame when using Hill’s problem for solar potential can be represented in the following form [2]

\[
\begin{align*}
\dot{x}_1 &= x_2 + y_1, \\
\dot{x}_2 &= -x_1 + y_2, \\
\dot{x}_3 &= y_3, \\
\dot{y}_1 &= -\frac{3x_1}{\|x\|^3} + 2x_1 + y_2, \\
\dot{y}_2 &= -\frac{3x_2}{\|x\|^3} - x_2 - y_1, \\
\dot{y}_3 &= -\frac{3x_3}{\|x\|^3} - x_3,
\end{align*}
\]

(1)

where \( x = (x_1; x_2; x_3) \) is the coordinate vector of the space robot and \( y = (y_1; y_2; y_3) \) is the momentum vector. The center of mass of the Earth coincides with the origin of the coordinate system, while the \( Ox_1 \) axis is directed along the axis connecting the centers of mass of the Earth and the Sun. \( \| \cdot \| \) is the Euclidean norm of a vector. In the model adopted, the units of time and distance were chosen in such a way that the unit of distance is approximately \( 10^{-2} \) a. u. \( \approx 1.5 \times 10^6 \) km, and the unit of time is 58.0916 days (the year divided by \( 2\pi \)). Libration points \( L_1 \) and \( L_2 \) in the rotating frame are stationary and have coordinates \( x^* = (1; 0; 0), \ y^* = (0; 1; 0) \) and \( x^{**} = (-1; 0; 0), \ y^{**} = (0; -1; 0) \) respectively.

System (1) is Hamiltonian, where the Hamiltonian \( H \)

\[
H = \frac{1}{2} \|y\|^2 - \frac{3}{2} x_1^2 + \frac{\|x\|^2}{2} + x_2 y_1 - x_1 y_2.
\]

(2)

It is known that Hamiltonian (2) on motion trajectories retains its value. That is, it is an integral of system (1). This fact can be used to assess the accuracy of numerical integration examples.

### 4 Trajectory with a Return

We simulate the application of control action in the form of a small change in the speed of the space robot, located in the neighborhood of collinear libration point \( L_2 \). The results of numerical simulation of a space robot in the ecliptic plane (if we initially set \( x_3 = 0 \) and \( y_3 = 0 \)) are shown in Fig. 1.

Fig. 1 (left) shows that with a very small impact, a space robot performs a long maneuver in the near-Earth space – the motion along trajectories takes about a year. In Fig. 1 (right), a space robot is maneuvering with a return to the neighborhood of libration point \( L_1 \). This maneuver takes a longer period of time. The initial time and the method of application of control action are
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selected in such a way as to ensure the robot converges with the celestial body in question, whose trajectory in the figures below is indicated by dotted lines.

![Fig. 1. Trajectories with return to the neighborhood of collinear libration point L₁ (left: about 8 months, right: about 14 months)](image)

5 Motion in a Neighborhood of the Libration Point

The control equations of a space robot motion in a rotating frame can be represented in the following form [2, 3, 4, 5, 6]

\[
\begin{align*}
\dot{x}_1 &= x_2 + y_1, \\
\dot{x}_2 &= -x_1 + y_2, \\
\dot{x}_3 &= y_3, \\
y_1 &= -\frac{y_2}{||x||^3} + 2x_1 + y_2 + u_1, \\
y_2 &= -\frac{x_2}{||x||^3} - x_2 - y_1 + u_2, \\
y_3 &= -\frac{x_3}{||x||^3} - x_3 + u_3,
\end{align*}
\]

where \(u = (u_1; u_2; u_3)\) is the acceleration vector.

Here, the unit of acceleration is equal to \(5.93844 \times 10^{-5} \text{ m/s}^2 \approx 6.05552 \times 10^{-6} \times g\), where \(g\) is the standard acceleration of free fall for the Earth. The unit of speed in the adopted model is \(298.057 \text{ m/s} \approx 9.94211 \times 10^{-7} \) – the speed of light in vacuum.

For a space robot to stay longer in the neighborhood of collinear libration point \(L_1\), as was shown in [3, 5], it is necessary that its phase coordinates satisfy the following relation:

\[
d_1 = |b_1z| \leq \varepsilon, \tag{3}
\]

where \(b_1 = (\lambda_1^2 + 5; \frac{\lambda_2^2 - 3}{\lambda_1}; \frac{\lambda_3^2 - 3}{\lambda_1}; 2)\), \(z = (x_1 - 1; x_2; y_1; y_2 - 1)\) is a column vector, \(\lambda_1 = \sqrt{1 + 2\sqrt{7}}\), and \(\varepsilon\) is a sufficiently small real number.
Here is an example. Let us consider the orbital motion of a space robot subject to the value of input function (3). Let us assume that at the initial time, a space robot has the following coordinates in the phase space:

\[ x = (0.99; 0; 0), \quad y = (0; 1; 0). \]

Fig. 2. The trajectory of control motion in the neighborhood of collinear libration point \( L_1 \) (left: about 3 years, right: the value of function \( d_1 \) on trajectory (left))

6 Discussions

Let us that initially a space robot is in the neighborhood of the phase space of collinear libration points, and then performs a maneuver monitoring the near-Earth space. Here, we can use a small control action, which becomes effective due to the instability of the collinear libration point. In this case, instability is a positive factor, allowing to significantly alter the motion trajectory with low energy costs.

The validity of this statement is evident from the examples given in section 3. Obviously, this type of a space robot can be repeatedly used to investigate the near-Earth space. It should be noted that when performing such a maneuver, the problem of constructing a “return trajectory” \([6, 7]\) and retaining a space robot in the neighborhood of the collinear libration points \([2, 3, 4, 5]\) arises.

Therefore, developing methods of constructing stabilizing telecontrols, for example, based on input function (3), is a very important and prospective task when ensuring effective operation of robotic systems in the neighborhood of the collinear libration point of the Sun-Earth system for a long period of time.

Based on what has been investigated, it is important to note that it is possible to effect the necessary control when using such exotic systems like solar sail \([2, 5, 6]\). It can be applied due to the sufficiently small values of the control action.
Of course, a more accurate assessment of the possibility of a space robot performing the necessary maneuver can be obtained by taking into account other perturbations – influence of the orbital eccentricity of the Earth, Moon and other planets, and bodies. At the same time, this would involve considering a space robot as a rigid body (not a material point), whose movement is described through complex mathematical equations incorporating both translational and rotational motion.

References


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