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An Analytical Comparison between Standard Johansen-Ledoit-Sornette (SJLS) Model and Generalized Johansen-Ledoit-Sornette (GJLS) Model

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Abstract

Economic bubbles can be defined as transient upward movements of prices above intrinsic value. The Standard Johansen-Ledoit-Sornette (SJLS) model and Generalized Johansen-Ledoit-Sornette (GJLS) models have been developed as flexible tools to detect bubble and forecasts the possible time of crash, tc. These models combines the economic theory of rational expectation bubbles with finite-time singular crash hazard rates, behavioural finance on imitation and herding of investors and traders as well as mathematical statistical physics of bifurcations and phase transitions. It has been employed successfully to a large variety of economic bubbles in many different markets. This study focused on the analytical differences between these two models to point out the best model to be used in forecasting time of crash and bubble detection. By doing so we are able to evaluate the differences and similarities of the methods and results in a practical way. The results appears that the two models are most appropriate to use for identify and predict financial bubbles and crash. But, the GJLS models selected as best model due to the limitation on the outputs of SJLS. The SJLS model only can detect and forecasts the financial bubble, but the GJLS models not only detect the time of crash but estimate the intrinsic value and the crash non-linearity as well. With the estimated intrinsic value, the unexplained problem which is differentiation between exponentially growing fundamental price and an exponentially growing bubble price are overcome. Moreover, the standard JLS model just describes the dynamics of the price during the bubble formation but the GJLS model can determines the dynamics of crash after the bubble by specifying how the price evolves towards the intrinsic value during crash.

Keywords: Economic bubbles, Forecasting. Intrinsic value, Crash non-linearity, Price dynamic

INTRODUCTION

Generally economic bubbles can be defined as positive acceleration of prices above intrinsic value [1]-[3]. A quick rise in the price of a continuous process also can be named as economic bubble [4]. Economic bubbles are one of the serious issue that give negative implications to the development of economy. This is because of bubble formation and dramatic bursts in financial markets [5]. According to [1], the rise in the asset price is depended on the euphoria that leads the investors to believe that there is a high probability that the bubble will continue to expand and lead to a high return, which remunerates them for the probability of crash. In this respect, [6] argues that the asset price increase was driven by irrational euphoria among individual investors. [5] also ascertain that the irrational behaviour of investors as investors who trade because of personal self satisfaction.
Many recent theories illustrate that economic bubbles still can be created even if there is no irrational investor because of positive feedback trading by noise traders, heterogeneous beliefs of investors together with a limitation on arbitrage and synchronization failures among rational traders. Researches done by [7]-[13] proved that the combined effects of heterogeneous beliefs and short-sales constrained may lead large movements in asset. In this kind of models which assume heterogeneous beliefs and short-sales, the asset prices are determined at equilibrium to the extent that they reflect the heterogeneous beliefs about payoffs, but short sales boundaries force the pessimistic investors disappear from the market, leaving only optimistic investors and thus magnified asset price levels. However, when short sales limitations no longer tie investors, then prices fall back downwards.

In another class of models, the role of “noise traders” in fostering positive feedback trading has been emphasized. The term “noise trader” was proposed first by [14] and [15] to portray irrational investors. These noise positive feedback traders purchase securities when prices increase and sell when prices fall. Due to this positive feedback mechanism, the divergence between the market price and the intrinsic value has been bloated [16]-[19]. The empirical evidences on this theory are mainly from the studies on momentum trading strategies. Stocks which performed poorly in the past will perform better in a long-term perspective (over the next three to five years) than stocks which performed well in the past [20]. In contrast, at intermediate horizon (three to twelve months), the stocks which performed well previously will still perform better [21].

Another activity preventing the expansion of bubbles has been argued to be the failure of synchronization of rational investors or traders [22]. In this mechanism, rational investors decide to ride the bubble to make profit on it, which has been confirmed by empirical studies on hedge funds (ideal rational investors) during the dot-com bubble [23]. They know that the market will eventually fall down when an adequate number of rational traders will sell out. However, the dispersion of their opinions on market timing and the consequent uncertainty of the synchronization of their sell-off are delaying this coming collapse, allowing the bubble to grow.

Apart from that, the factors that cause inflation have relationship to economic bubbles. The following diagram 1.1 portraits the linkage between inflation and economic bubbles.

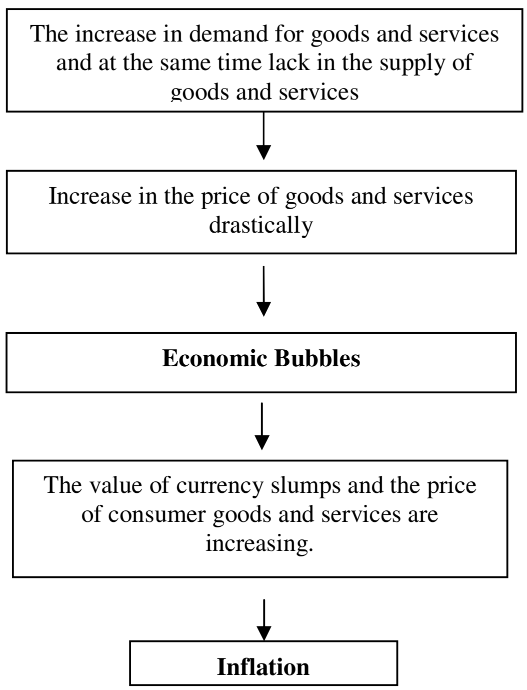

Diagram 1.1: The linkage between inflation and economic bubble

However, identifying the existence of economic bubbles remains an unsolved problem in standard econometric and financial economic methods [24], [25]. This is due to the fact that the intrinsic value is in general poorly constrained and it is impossible to differentiate between exponentially growing bubble prices. Diagnosing the bubble ex-ante could help to take several actions to stop from bubble bursting. But none of the theories mentioned above can diagnose bubble ex-ante. This may be due to the fact that all these theories cannot distinguish between intrinsic and bubble price and cannot give a price dynamics which leads to a crash. The Standard Johansen-Ledoit-Sornette (SJLS) model or Johansen-Ledoit-Sornette Model developed by Sornette and his colleagues has potential to describe the price dynamics during a bubble regime by analysing the cumulative human behaviour in new perspectives. It also has the ability to predict the most probable crash time after a bubble ex-ante. Generalized Johansen-Ledoit-Sornette (GJLS) Models have been developed as flexible tools to detect bubble and forecasts the possible time of crash, t_c by [26]. This study focused on the analytical differences between these two models to point out the best model to be used for forecasts time of crash and bubble detection.

BACKGROUND OF THE MODELS

The GJLS and SJLS models combines the economic theory of rational expectation bubbles with finite-time singular crash hazard rates, behavioural finance on imitation and herding of investors and traders as well as mathematical statistical physics of bifurcations and phase transitions.

Standard Johansen Leodit Sornette Model (SJLS): SJLS model has been employed successfully to a large variety of economic bubbles in many different markets such as the real estate market in Las Vegas [27], the U.K and the S&P 500 index anti-bubble in 2000-2003 [28], Dow Jones Industrial Average historical bubble [29], 2006-2008 oil bubble [30], the Chinese index bubble in 2009 [31], corporate bond spread [32], the polish stock market bubble [33], the western stock market [34] and the South African stock market bubble [35]. Furthermore, new experiments in ex-ante bubble identification and forecast has been submitted since November 2009 in the Financial Crisis Observatory at ETH Zurich [36]-[38]. The JLS model considers the faster-than-exponential briefly power law with finite-time singularity increase in asset prices tinted by accelerating oscillations as the main diagnostic of bubbles. It concludes a positive feedback loop of higher return expectations competing with negative feedback of crash expectations. The JLS model of financial bubbles and crashes is an extension of the rational expectation bubble model proposed by [39]. A crash in this model is seen as an event potentially terminating the run-up of a bubble. A financial bubble is modelled as a regime of accelerating or super-exponential power law growth punctuated by short-lived corrections organized according the symmetry of discrete scale invariance [40]. The super-exponential power law is argued to result from positive feedback resulting from noise trader decisions that tend to enhance deviations from fundamental valuation in an accelerating spiral.

The purely speculative asset that pays no dividends so that we do not take into account the interest rate, information asymmetry, risk aversion, and the market clearing condition. The rational expectations are simply corresponding to the familiar martingale hypothesis in this stylised framework:

$$E_t\left[p(t')\right] = p(t) \qquad \forall t' > t \tag{1}$$

where $p(t)$ denotes the price of the asset at time t and $E_t[.]$ indicates the expectation conditional on information revealed up to time t.

The cumulative distribution function (cdf) of the time of crash is called $Q(t)$, the probability density function (pdf) is $q(t) = \frac{dQ}{dt}$ and the hazard rate is $h(t) = \frac{q(t)}{1 - Q(t)}$. The hazard rate is the probability per unit of time that the crash will happen in the next instant if it has not happened yet.

In the JLS model, the stock market dynamics is described as

$$\frac{dp}{p} = \mu(t)dt - \kappa dj` \tag{2}$$

The term p is the stock market price and the term dj indicates a discontinuous jump such that $dj = 0$ before the crash and $dj = 1$ after the crash happens. The parameter κ determined the loss amplitude associated with the occurrence of a crash. The time-dependent drift $\mu(t)$ is chosen so that the price process satisfies the martingale condition, i.e

$$E_t[dp] = \mu(t)p(t)dt - \kappa p(t)h(t)dt = 0 \tag{3}$$

$$\mu(t) = \kappa h(t) \tag{4}$$

and the following equation is corresponding to the price;

$$\log\left[\frac{p(t)}{p(t_0)}\right] = \kappa\int_{t_0}^{t_1} h(t')dt' \tag{5}$$

This gives the logarithm of the price as the relevant observable. The higher the probability of a crash, the faster the price grow (conditional on having no crash) in order to obey the martingale condition. Intuitively, investors must be remunerated by a higher return in order to be induced to hold an asset that might crash. The sensitivity of the market reaction to news or external influences accelerate on the approach to this transition in a specific way characterized by a power law divergence at the critical time t_c of the form $F(t) = (t_c - t)^{-z}$, where z is called a critical exponent. This form amounts to the following property;

$$\frac{d\ln f}{d\ln(t_c - t)} = -z \tag{6}$$

(6) Is a constant, namely that the behaviours of the observable F become self-similar close to t_c. The symmetry of self similarity in the present context refers to the fact that the relative variations $d\ln F = \frac{dF}{F}$ of the observable with respect to relative variations $d\ln(t_c - t) = \frac{d(t_c - t)}{(t_c - t)}$ of the time-to-crash are independent of time t, as expressed by the constancy of the exponent z.

The crash hazard rate follow the same dependence, i.e.

$$h(t) = B'(t_c - t)^{m-1} \tag{7}$$

Where B' is a positive constant and t_c is the critical point or theoretical date of the bubble end. The term m must in the range of $0 < m < 1$ for an important economic reason's otherwise; the price would go infinity when approaching t_c (if the bubble has not crashed yet).

The first order expansion for the hazard rate is:

$$h(t) \approx B^{'}(t_c - t)^{m-1} + c^{'}(t_c - t)^{m-1}\cos\left[\omega \ln(t_c - t) + \phi^{'}\right] \tag{8}$$

The crash hazard rate now displays log-periodic oscillations. This can easily seen by taking the exponent z to be complex with a non-zero imaginary part, since the real part of $(t_c - t)^{-z+i\omega}$ is $(t_c - t)^{m-1}\cos[\omega \ln(t_c - t)]$. The evolution of the price before the crash and critical date is then given by:

$$\ln E[p(t)] \approx A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) + \phi) \tag{9}$$

where ϕ is another phase constant, $B = \frac{-\kappa B^{'}}{m}$, $C = \frac{-\kappa C^{'}}{\sqrt{m^2 + \omega^2}}$ and ω is log-angular frequency.

Generalised Johansen Leodit Sornette Model: The generalised Johansen Leodit Sornette Model is formed by inferring fundamental value of stock and crash non-linearity from bubble calibration. Derivation of GJLS Model that proposed by [26] as follows:
The price dynamics of an asset as

$$dp = \mu(t)pdt + \sigma(t)pdW - \kappa(p - p_1)^{\gamma} dj \tag{10}$$

where the $\mu(t)pdt + \sigma(t)pdW$ describes the statistical geometrical Brownian motion and the third term is the jump.

When the crash occurs at some time t^* (indicate $\int_{t^{*-}}^{t^{*+}} dj = 1$), the price drops abruptly by amplitude $\kappa\left(p\left(t^*\right) - p_1\right)^{\gamma}$. (11)

where $\kappa = \gamma = 1$, the price drops from $p(t^{*-})$ to $p\left(t^{*+}\right) = p_1$. The price changes from its value just before crash to a fixed well-defined valuation p_1.

Inferring no-arbitrage condition $E_t[dp] = 0$ to (10) leads to

$$\mu(t)p = k(p - p_1)^{\gamma} h(t) \tag{12}$$

Conditional on the absence of a crash, the dynamics of the expected price obeys the equation

$$dp = \mu(t)pdt = k(p - p_1)^{\gamma} h(t)dt \tag{13}$$

and the fundamental price must obey the condition $p_1 < \min p(t)$. For $\gamma = 1$, the solution is

$$\ln[p(t) - p_1] = F_{LPPL(t)} \tag{14}$$

where $F_{LPPL(t)}$ is given by the (9); For $\gamma \in (1,0)$, the solution is

$$(p - p_1)^{1-\gamma} = F_{LPPL(t)} \tag{15}$$

do not consider the case $\gamma > 1$ which would give an economically non-sensible behaviour, namely the price diverges in finite time before the crash hazard rate itself diverges.

In summary, [26] considered two models M_0 and M_1. In the following models, $F_{LPPL(t)}$ below is given by (9).

1. $M_0 : p_1 = 0, \gamma \in (0,1]$

   $p_{M_0} = (F_{LPPL}(t))^{\frac{1}{1-\gamma}}, \gamma \in (0,1)$ or $\exp(F_{LPPL}(t)), \gamma = 1$

2. $M_1 : p_1 \neq 0, \gamma \in (0,1]$

   $p_{M_1}(t) = p_1 + (F_{LPPL}(t))^{\frac{1}{1-\gamma}}, \gamma \in (0,1)$ or $p_1 + \exp(F_{LPPL}(t)), \gamma = 1$

RESULTS

The analytical results show that two models are suitable to for bubble diagnose and predict their crash. But GJLS model is selected as best model due to the limitation on the output of SJLS. The SJLS model only can predict the time of crash and detect financial bubble, but the GJLS models not only detect the time of crash but estimate the intrinsic value and the crash non-linearity as well.

Estimation of fundamental value by GJLS Models: The price drops from the price drops from $p(t^{*-})$ to $p(t^{*+}) = p_1$. The price changes from its value just before crash to a fixed well-defined valuation p_1. In the spirit of Fama's of the 19 october 1987 crash [42], if one interprets price of the asset after the crash as the exact price, i.e., the price discovery towards rational equilibrium without mispricing, the crash is nothing but an efficient assessment by investors of the fundamental value, once the panic ended. Hence, p_1 can be interpreted as intrinsic price which is discovered during the crash dynamics. Since p_1 is a fixed parameter, the GJLS model implies that we should measure the price dynamics in the frame moving with the fundamental price. In other words, p_1 is the fundamental price at the beginning t_1 of the time period over which the bubble develops. In order to compare in a consistent way the realized price to this fixed parameter, it is necessary to discount the asset price continuously by the rate of return of the fundamental price. If $p_{obs}(t)$ denotes the empirical price observed at time t, this means that the price $p(t)$ that enters in (1) is defined by

$$p(t) = p_{obs}(t) \prod_{s=t_1+1}^{t} \frac{1}{(1+r_f(s))^{\frac{1}{365}}}$$

where $r_f(s)$ is the annualized growth (risk free) rate of fundamental price.

With the estimated intrinsic value, the unexplained problem which is differentiation between exponentially growing bubble price are overcome. Moreover, the SJLS model just describes the dynamics of the price during the bubble formation but the GJLS model can determines the dynamics of crash after the bubble by specifying how the price evolves towards the intrinsic value during crash.

Description the dynamics of crash after the bubble by GJLS Models: The models provide a method to measure the amplitude of the crash that follows the bubble peak. Consider two types of drawdown after the peak: (i) $DD_{[i]}$ is the two months drop measured from the peak; (ii) $DD_{\max}$ is the peak-to-valley drawdown from the peak to the minimum of the asset price after the crash. We calculate the magnitude of the crash compared to the over-valued prices as follows. The ratio between the crash magnitude and over-valued prices is estimated as:

$$RC_i = \frac{DD_i}{p_{obs}(t_p) - p_1 \prod_{s=t_1+1}^{t} (1+r_f(s))^{\frac{1}{365}}}, i \in \{[i], \max\}$$

During the crashes, the hazard rate in (13) should be 1. Then comparing the definition of RC and (13), one can easily find that $\kappa = RC$ for the models whose $\gamma = 1, (M_0, M_1)$.

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