On the Transmission of the Notion of Norms

Having Matrix Values in a Peripheral European Country: Portugal

Catarina Cosme
Universidade de Trás-os-Montes e Alto Douro
Vila Real, Portugal

Cecília Costa
Universidade de Trás-os-Montes e Alto Douro, Vila Real, Portugal
CIDTFF - Research Centre of Didactics and Technology in Education of Trainers (LabDCT of UTAD)

Dedicated to José Vitória
on the occasion of his 75th birthday.

Abstract

In this paper we show some aspects related to the first evidences of the transmission of the notion of norms with matrix values in Portugal (through Lourenço Marques, Mozambique) between the end of the 1960s
and the beginning of the 1980s. We will refer the first contacts with the

Mathematics Subject Classification: 00A35; 01A60; 15-03

Keywords: Norms with matrix values, Mathematical education; History of

1 Introduction

This paper intends to contribute to the research on the history of Mathematics,
of its teaching and research, by searching for relations that can be established
between the knowledge of the historical development of a mathematical concept,
by the professor, the research and the teaching of that concept.

In our research, the case study was the methodological approach that we
considered most convenient, as we intend to understand, describe, illustrate and
show evidences of how the introduction and transmission of the concept of norm
having matrix values took place in a peripheral European country, where several
factors are involved [2], [29]. In our case, a young scholarship holder travelled
abroad to carry out several studies that would lead to the conclusion of his PhD
acquired knowledge on norms with matrix values. Later, he introduced these new
concepts in his classes of numerical analysis I (JV1) of the degree in Mathematics
of the University of Lourenço Marques and later in a master’s degree of the
University of Coimbra (JV5). He also published results of his research on this topic:
“Matricial norms and lambda-matrices” [32], “Matricial norms and the roots of
polynomials” [33], “Matricial norms and the differences between the zeros of
determinants with polynomial elements” [34], “Latent roots of lambda-matrices,
Kronecker sums and matricial norms” [35], “Matricial norms: Some applications
and open questions” [36], “Upper bounds for the latent roots of lambda-matrices” [37], “A survey on vectorial norms of matrices and some open
questions” [38].
On the transmission of the notion of norms

The process of data collecting was done within a period of two years. It began in January 2009 until February 2011. During the process, two types of instruments were used [2]: both interviews and documental analysis (master’s degree dissertations, PhD dissertations and scientific articles, among others).

This paper is organized in two sections. In the first section, we briefly refer the origin of the concept of norms with matrix values in central Europe, followed by the reporting of the first steps of this concept in Portugal, describing the outer influences and the inner repercussions.

2 Origin of the concept of matrix norm in central Europe

The concept of vectorial norm was presented for the first time by Leonid Kantorovich (Russian, 1912-1986) in the article “The method of successive approximations for functional equations” [18] published in Acta Math in 1939. In this paper, the researcher generalized the concept of norm to functions with values in a semi-ordered vectorial space. A few years later, the concept of norm was extended to several areas of Mathematics. In the field of functional analysis, it was studied by Leonid Kantorovich, Boris Vulikh (Russian, 1913-1978) and Aron Pinsker (Russian, 1906-1986) and the work later presented in the book “Functional analysis in semi-ordered spaces” [19] published in 1950 [16], [21].

In relation to the field of numerical analysis, the first studies were presented six years later in 1956 by Johann Schroeder (German, 1925-2007) with the publication of the paper “The iteration in a more general notion of distance” [27]. In 1961, in the same field, Alexander Ostrowski (Ukrainian, 1893-1986) contributed to the progress of this topic with the publication of the paper “Metrical properties of operator matrices and matrices partitioned into blocks” [23]. “Generalized norms of matrices and the location of the Spectrum” which was presented one year later [9] by Miroslav Fiedler (Czech, 1926) and Vlastimil Ptak (Czech, 1925-1999) [16], [21].

François Robert (French, 1939), Friedrich Bauer (German, 1924) and Emeric Deutsch (Romanian, 1929) are still a reference to researchers who currently study/work on the theme of norms.

3 The first steps of matrix norms in Portugal

3.1 The first contact with the theme

José Vitória (JV2; E6), born on 28th June 1939 in Lisbon, but having always lived in Abrantes, Portugal, graduated in Mathematical Sciences at the University of Coimbra in June 1962. After his military service in Angola he began teaching at the University of Lourenço Marques in Mozambique (at that time a Portuguese colony). In 1968 he went to France to develop his studies to undertake his PhD at
the Scientific and Medical University of Grenoble. It was then that he first contacted with the topic of norms with matrix values.

He attended this University for three years, until 1971. He was supervised by Nöel Gastinel (French, 1925-1984) a mathematician and teacher at the Scientific and Medical University of Grenoble. This researcher obtained his PhD in 1960 with the dissertation “Matrices du second degré et normes générales en analyse numérique linéaire. Le théorème de Stone Weirstrass” [10] under the orientation of Jean Kuntzmann (French, 1912-1992). Throughout his career Nöel Gastinel directed seven more PhD dissertations, namely the one of Pierre Pouzet, “Analyse numérique: étude, en vue de leur traitement numérique, d'équations intégrales et intégralodifférentielles de type Volterra pour des problèmes de conditions initiales” [24], the dissertation of François Robert, “Étude et utilisation de normes vectorielles en Analyse numérique linéaire” [25], the dissertation of Jean-Claude Miellou, “Opérateurs para monotones” [22], the dissertation of Claude Brezinski, “Méthodes d'accélérations de la convergence en analyse numérique” [4], the dissertation of Françoise Chatelin, “Méthodes numériques pour le calcul de valeurs et vecteurs propres d'un opérateur linéaire” [5], the dissertation of Jean Della-Dora, “Sur quelques algorithmes de recherche de valeurs propres” [8] and the dissertation of Jacques Baranger, “Quelques résultats en optimisation convexe” [3]. In his curriculum we also find the publication of several books, including, “Analyse numérique linéaire” [13] and several scientific papers, such as: “Sur décomposition de normes générales et procédés itératifs” [11], “Propriétés de certains ensembles normés de matrices” [12], “Sur l'extension de normes sur des algèbres de matrices” [14] and “Sur le calcul des produits de matrices” [15]. Thus, he conducted researches in numerical linear algebra and, in particular, about norms with vector values and norms with matrix values.

Nöel Gastinel assigned as a first work to José Vitória, the study of a paper by Alston S. Householder (American, 1904-1993) entitled “Norms and the localization of roots of matrices” [17] of 1968 with the objective of elaborating a mini dissertation and also suggested that he carried out a research in the Computing Reviews about norms and to prepare a dossier with publications on the topics of the last ten years on the subject (E6).

Some time before José Vitória’s arrival in France, the mathematician François Robert, professor at the Institute National Polytechnique de Grenoble, has finished his PhD dissertation [25] on norms with vectorial values in numerical analysis. Through the elaboration of his mini dissertation, later entitled “Norms of vectors and matrices” [31], José Vitória established correspondence (E6) with several researchers interested in the topic, such as François Robert, the Romanian Emeric Deutsch (working in America) and the American George Barker.

In the correspondence between José Vitória and François Robert (JV3; JV4), the first informs the latter that using the lower bound of a matrix he couldn’t achieve the result he obtained in the upper bound, questioning him about that problem. Furthermore, he informs about his intention of elaborating a summary of
Deutsh’s dissertation, presenting a similar chart to the one Robert had elaborated for the vectorial norms to the case of matricial norms. Robert answered José Vitória saying he was interested in receiving that summary and contested the base result of the lower bound sending him the correction.

These letters show that norms with vector values and norms with matrix values were studied at that time by several researchers who taught in different universities and countries. They were familiar with the works of one another and shared their knowledge with the aim of obtaining new results.

3.2 The return to the University of Lourenço Marques and the discipline of Numerical Analysis I

After the end of his research scholarship in Grenoble, in 1971, José Vitória returned to Mozambique and to the University of Lourenço Marques (JV2), where he taught several disciplines in several courses at the Department of Mathematics, including Numerical Analysis I of the 4th year of the Mathematics degree (scientific section) (JV2). It was during his lectureship of this discipline that José Vitória introduced the extension of the concept of norm, and ceased speaking only about vector and matrix norms, and also started using the concepts of norm of vectors and matrices in the case of norms with scalar values and vectorial norms when norms have vector or matrix value (JV5).

In this discipline the following contents were taught (JV5): basis matrices, rule of the product of two basis matrices of $M_{nn}$ (set of square complex matrices of order n) and its applications, norms of vectors and matrices, elementary properties and norms of matrices, comparison of the norms of Holder, definition of “geometric” norms, geometric norms of matrices and norms of matrices of $M_{nn}$.

During this period, he influenced at least two students of this subject, Raquel Valença and Elvira Coimbra to study norms with vector and matrix values. The first, after her graduation in Mathematics, started a PhD at Oxford University and did not return to Mozambique. Following José Vitória’s suggestion, Raquel Valença established contact with another PhD student in Oxford, Fernanda Oliveira and her supervisor Leslie Fox (English, 1918-1992) specialist in numerical analysis, more specifically in interval analysis. Raquel Valença was also supervised by Leslie Fox and completed her PhD dissertation on interval analysis. She published papers about matricial norms of matrices having interval entries. One of them is an extension of a conjecture that José Vitória presented in a congress in Barcelona in 1980 where he presented results about norms of matrix values, but with matrices with no null entries and left an unsolved problem about what would happen if the entries were intervals [36]. The study published by Raquel Valença in the journal Linear Algebra and its Applications is the development and proof of that conjecture. This was the end of her contribution about norms with matrix values and intervals, because she passed away. During
her short life, she lectured at the University of Minho, in Braga, Portugal. She was the author of the book “Numerical analysis” of the *Universidade Aberta* [30], participated in several meetings and had an active role in the development of the Department of Mathematics at her University.

After her graduation, Elvira Coimbra left Mozambique and went to Lisbon where she became a member of the staff of the Department of Mathematics of the Faculty of Sciences and Technology of the New University of Lisbon. She studied the dissertation of François Robert which included a first part based on the functional analysis and extended it to Yosida space. In 1979, she presented her PhD dissertation “Approximations on v-metric spaces” [6] at the New University of Lisbon, in Portugal, under the supervision of António César Freitas. João de Deus Marques, lecturer at the same university, was influenced by Elvira Coimbra to undertake the studies of norms with matrix values. He is currently studying norms with vectorial values associated to vectorial inner products of infinite dimension.

### 3.3 The return to the University of Coimbra and the master’s degree in Linear Algebra and Applications of 1982

Later, in 1977, José Vitória returned to Portugal and joined the Department of Mathematics of the Faculty of Sciences and Technology of the University of Coimbra as an assistant professor.

In 1982, in this department a master’s degree in Linear Algebra and Applications was initiated. It was created by the administrative decree n. 187/82 of 13th February [7].

This master’s degree aimed to promote the acquisition of knowledge at an advanced level and to the initiation of research. The *numerus clausus* was 8, the holders of a Mathematics degree (scientific specialization) and the holders of a Mathematics degree (educational), Pure Mathematics, Applied Mathematics and Mathematical Sciences. This course was organized by the professor and mathematician Graciano de Oliveira, at the time responsible for the research group of Linear Algebra of the University of Coimbra.

The discipline of Matricial Norms was part of the study plan of this master’s degree course. A new discipline with new contents in Portugal. It was José Vitória who created and taught this discipline. He also established its goals and programmatic contents, bibliography and evaluation process. It had duration of 15 hours. The syllabus was organized in two chapters. In the first chapter, norms (scalar) of vectors and matrices were worked up and in the second chapter norms (vectorial) of vectors and matrices. The evaluation of the discipline was made bearing in mind three items: dossier of exercises, elaboration of a memoir (scientific theme or dissertation’s project) and an oral discussion of the dossier, memoir and the taught contents. The teacher presented all the information about
On the transmission of the notion of norms

how the discipline functioned and the work to be carried out in numerated work sheets which were posted (JV6).

Eight students attended this subject: Armando Duarte da Silva Gonçalves, Margarida Maria Fernandes Saraiva, Maria Helena Seabra de Almeida, Maria Teresa dos Reis Pedroso de Lima, Maria Celeste de Almeida Gouveia, Joaquim Machado, Maria Madalena de Almeida Correia Gomes Martins and Rui Manuel Pires Almeida.

The first four of the eight students used norms with matrix values in the studies leading to their master degree dissertation. Armando Gonçalves studied the compactness and convexity in the class of the best approximations in vectorial norm [16]. Margarida Saraiva dedicated herself to the study of the product of Kronecker and adjoint operator, $A_dX = AX -XA$ [26]. Maria Helena Almeida did her research about cones and matricial norms [1] and Teresa Pedroso de Lima worked on equations of linear differences with matrix coefficients [20]. Later, some of them, for professional or personal reasons, eventually abandoned the study of the topic.

In some cases, José Vitória supported other students in the study and application of the norms with vectorial values to research work which they were developing: this was the case of Maria Madalena de Almeida Correia Gomes Martins, a lecturer of the Department of Mathematics of the University of Coimbra when she elaborated her PhD “Relaxation methods for systems of linear equations” [21], in 1986.

4 Conclusion

In the family tree image (Figure n.1), we synthesize what we have presented in this paper where we recorded who we consider to be the main disseminator and impeller of the topic about norms with matrix values in Portugal, his direct and secondary disciples and the way that influence was processed.

Figure n.1: Family tree of the researchers of the topic in Portugal
The transmission was mainly conducted through two disciplines, introducing the concept in the lectured contents. First, in the beginning of the 1970s, through the discipline of Numerical Analysis I in the degree of Mathematics from the University of Lourenço Marques, in Mozambique (when it was still a Portuguese colony) and later on, in the beginning of the 1980s, creating a specific discipline: Matrix Norms lectured in the master degree course in Linear Algebra held at the University of Coimbra, in Portugal.

The image shows that José Vitória also has secondary disciples, i.e. researchers who were influenced to study this topic by his direct followers. As secondary disciples, we indicate João de Deus Marques and Maria Manuela Vivaldo. The first because he worked in partnership with Elvira Coimbra and was influenced to undertake the study of norms with matrix values and Maria Manuela Vivaldo who was influenced by Teresa Pedroso de Lima when supervising her master degree dissertation [28]. Currently, none of them study norms, but they use them as tools to reach their research goals.

We cannot say that this was the only path of the transmission and development of this new topic “norms with matrix values” in Portugal, but we are convinced that it was the first.

As in other moments in the history of Mathematics, the process identified here to the transmission of new mathematic knowledge in peripheral countries (to our knowledge) was sending a young researcher in Mathematics to a foreign university to undertake a PhD. Being in contact with other mathematicians with advanced knowledge that he met and used in his research, later in his country and university, he transmitted what he had learned including these topics in subjects that he taught, giving seminars and supervising other researchers.

**Acknowledgments.** We would like to thank Professor José Vitória from the University of Coimbra, Portugal, for making his personal files available and authorizing their use for historical, scientific and didactic purposes.

We thank all the referred professors and researchers and who were available to participate in this study.

**References**


A. Gonçalves, *Convexity and compactness of the set of best approximation with vectorial norm*, (Portuguese) Master dissertation, Faculty of Sciences and Technology of the University of Coimbra, Coimbra, Portugal, 1985.


[34] J. Vitória, Matricial norms and the differences between the zeros of determinants with polynomial elements, *Linear Algebra Applications*, 28 (1979), 279-283. MR0549440 (80k:15014)


**Sites:**
Mathematics Genealogy Project on http://genealogy.math.ndsu.nodak.edu/

**Sources:**
Personal files of José Vitória:

JV1 – Notes of the theoretical classes of the subject: Numerical Analysis I taught by José Vitória in Lourenço Marques, 1971

JV2 - Curriculum Vitae of José Vitória
Jv3 – Letter from José Vitória to François Robert in April 1970
Jv4 – Letter from François Robert to José Vitória on 15/04/70
Jv5 – Summary sheets of the subject “Matrix norms” of the master’s degree in Linear Algebra taught by José Vitória at the University of Coimbra, 1982
Jv6 – Worksheets on the subject “Matrix norms” of the master’s degree in Linear Algebra taught by José Vitória at the University of Coimbra, 1982

Interviews:
E1 – Interview with Armando Gonçalves on 9th December 2009
E2 - Interview with Maria Almeida on 9th December 2009
E3 – Interview with Maria Pedroso Lima on 15th January 2010
E4 - Interview with Maria Vivaldo on 15th January 2010
E5 - Interview with Maria Martins on 22nd January 2010
E6 - Interview with José Vitória on 2nd December 2009

Received: February 19, 2014