Stochastic Analysis of Manpower Levels Affecting Business with Varying Recruitment Rates

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Abstract

In this paper we consider a business organization in fluctuating condition of availability of Manpower, Business and recruitment with a special emphasis given to a new and prevailing idea of frequent changes taking place in Manpower. The different states have been discussed under the assumption that changes from availability to shortage and shortage to availability occur in exponential times with different parameters. An expression for rate of crisis under steady state \( C_\infty \) is derived and steady state cost has also been worked by assuming different costs for the parameters under different conditions.
Keywords: Manpower Planning, Crisis State, Steady State Probabilities and Setting Clock Back to Zero setup

1. INTRODUCTION

Nowadays we find that labour has become a buyer market as well as seller market. Any company normally runs on commercial basis wishes to keep only the optimum level of any resources needed to meet company’s requirement at any time during the course of the business and manpower is not an exception. This is spelt in the sense that a company does not want to keep manpower more than what is required. Hence, recruitment is done when the business is busy and shed manpower when the business is lean. Equally true with the labour. The workers have the option to switch over to other jobs because of better working condition, better emolument, and proximity to their living place or other reasons. Under such situations the company may be there but manpower may not be available. If skilled labourers and technically qualified persons leave the business the seriousness is much felt and the company has to hire paying heavy price or pay overtime to employees.

Approach to manpower problems have been treated in very many different ways as early as 1947 by Vajda [8] and others. Models in manpower planning has been discussed in depth in Bartholomew [1], Grinold and Marshal [2] and Vajda [8]. The method to compute wastages (Resignation, dismissal and death) and promotion intensities which produce the proportions corresponding to some desire planning proposals have been studied by Lesson [3]. Markov models are designed for wastages and promotion in manpower system by Vassilou [9], V. Subramaniam [7] in his thesis has made an attempt to provide optimal policy for recruitment training, promotion, and wastages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement scheme. For application of Markov chains in a manpower system with efficiency and seniority and Stochastic structures of graded size in manpower planning systems one may refer to Setlhare [6]. For a system involving manpower, money and machine one may refer to C. Mohan and R. Ramanarayanan [5]. For the study of Semi Markov Models for Manpower planning one may refer to the paper by Sally Meclean [4]. Rao and Talwalker (1990) introduce the concept of setting the clock back to zero property.

In this model we consider three characteristics namely manpower, business and recruitment and derive a formula for steady state rate of crisis and the steady state probabilities. The different states have been discussed under the assumption that changes from availability to shortage and shortage to availability occur in exponential times with different parameters. An expression for rate of crisis under steady state $(C_\infty)$ is derived and steady state
cost has also been worked by assuming different costs for the parameters under different conditions.

2. Model Description

2.1 Assumptions

1. There are two levels of business namely
   (i) Business is fully available and the period is exponentially distributed with parameter ‘a’
   (ii) Business is Lean or nil and the period is exponentially distributed with parameter ‘b’

2. There are two levels of manpower namely manpower is full and manpower is nil. The time during which the manpower remains continuously full and becomes nil has exponential distribution (Departure distribution) and the time required to complete full recruitment from nil level has exponential distribution (Recruitment distribution).

3. The parameter of the departure distribution is \( \lambda_{101} \) when the manpower is full, business is nil. The parameter of the same \( \lambda_{101} \) changes to \( \lambda_{102} \) in an exponential time with parameter ‘\( \beta \)’. It is \( \lambda_{111} \) when the manpower is full and business is full. It changes to \( \lambda_{112} \) in an exponential time with parameter ‘\( \beta \)’.

4. The parameter of the recruitment distribution is \( \mu_{001} \) when the manpower is nil and business is nil. It changes to \( \mu_{002} \) in an exponential time with parameter ‘\( \alpha \)’

5. When there is no manpower, there is no business.

2.2 System Analysis

The Stochastic Process \( X(t) \) describing the state of the system is a continuous time Markov chain with 6 points states space as given below in the order where the first co-ordinate indicates the level of manpower, the second co-ordinate indicates the level of business and the third co-ordinate indicates the rate level of recruitment / departure with case may be (i) When the manpower is nil, the third co-ordinate indicates the recruitment rate level (ii) When the manpower is full the third co-ordinate indicates the departure rate level.

\[
S = \{ (0,0,1)(0,0,2) (1,0,1) (1,0,2) (1,1,1) (1,1,2) \} - (1)
\]
The Infinitesimal generator $Q$ of this continuous time Markov chain of the state space is given below which is a matrix of order 6

$$Q = \begin{pmatrix}
M / B / R & (0,0,1) & (0,0,2) & (1,0,1) & (1,0,2) & (1,1,1) & (1,1,2) \\
(0,0,1) & -\varepsilon_1 & \alpha & \mu_{001} & 0 & 0 & 0 \\
(0,0,2) & 0 & -\mu_{002} & \mu_{002} & 0 & 0 & 0 \\
(1,0,1) & \lambda_{101} & 0 & -\varepsilon_2 & \beta & b & 0 \\
(1,0,2) & \lambda_{102} & 0 & 0 & -\varepsilon_3 & 0 & b \\
(1,1,1) & \lambda_{111} & 0 & a & 0 & -\varepsilon_4 & \beta \\
(1,1,2) & \lambda_{112} & 0 & 0 & a & 0 & -\varepsilon_5 \\
\end{pmatrix}$$

Here

$$\varepsilon_1 = (\alpha + \mu_{001}), \quad \varepsilon_2 = (b + \beta + \lambda_{101}), \quad \varepsilon_3 = (b + \lambda_{102}),$$

$$\varepsilon_4 = (a + \beta + \lambda_{111}), \quad \varepsilon_5 = (a + \lambda_{112}).$$

Let $\overline{\pi} = (\pi_{001}, \pi_{002}, \pi_{101}, \pi_{102}, \pi_{111}, \pi_{112})$ be the steady state probability vector of the matrix $Q$, then

$$\overline{\pi} Q = 0 \quad \text{and} \quad \overline{\pi} e = 1 \quad - (2)$$

Using (2), we can get this steady state probabilities

$$\pi_{102} = \frac{-\beta \pi_{111} + \varepsilon_4 \pi_{112}}{b} \quad - (3)$$
K.Hari Kumar, R.Ramanarayanan and P.Sekar

Using the equation-(2), we get

\[ \pi_{112} = \frac{\beta (\varepsilon_3 + \varepsilon_4)}{(ab - \varepsilon_3 \varepsilon_5)} \frac{1}{c} \]  

- (9)

**Here** 
\[ c = \frac{(\varepsilon_3 \varepsilon_4 - ab)(\mu_{002} + \alpha)}{b(\alpha + \mu_{001})\mu_{002}} + \frac{(\varepsilon_4 - \beta)}{b} + \frac{\beta(\varepsilon_3 + \varepsilon_4)(\varepsilon_5 - 1)}{(ab - \varepsilon_3 \varepsilon_5)} + 1 \]

The Crisis rate in the steady state is given by

\[ C_\infty = \lambda_{111}\pi_{111} + \lambda_{412}\pi_{112} \]  

- (10)

Using the steady state probabilities, we get the rate of crisis

\[ C_\infty = \frac{1}{c} \left[ \frac{\lambda_{111}(a\lambda_{102} + b\lambda_{112} + \lambda_{102}\lambda_{111}) + \beta\lambda_{111}(a + b + \beta\lambda_{102}\lambda_{111})}{a\lambda_{112} + b\lambda_{112} + \lambda_{102}\lambda_{111}} \right] \]  

- (11)

**2.3 Numerical Illustrations**

**Case (i)**
If we assume the values $\alpha = 2$, $\beta = 4$, $a = 5$, $b = 10$, $\mu_{001} = 3$, $\mu_{002} = 5$

Then we get the steady state probability vector

$$
\pi_{001} = 0.5073 \quad \pi_{102} = 0.0340
$$

$$
\pi_{002} = 0.2029 \quad \pi_{111} = 0.0818
$$

$$
\pi_{101} = 0.1227 \quad \pi_{112} = 0.0513
$$

And the crisis rate is

$$C_{cr} = 0.9014$$

Case (ii)

If we assume the fixed values for the $a = 4$, $b=12$, $\mu_{001} = 3$, $\mu_{002} = 5$, $\lambda_{010} = 6$, $\lambda_{102} = 8$, $\lambda_{101} = 6$, $\lambda_{111} = 4$, $\lambda_{112} = 5$ and varying the $\alpha$ and $\beta$ with be different values and we get the different crisis rates.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$B$</th>
<th>$C_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>2.2962</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>2.3343</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>2.3622</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>2.3834</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>2.4001</td>
</tr>
</tbody>
</table>

When the values of $\alpha$ and $\beta$ increase, simultaneously the crisis rate also gets increased.

3. Steady State Costs and Numerical Illustration

The steady state cost are determined by using the formula
Stochastic analysis of manpower levels

\[ C_{jk} = \pi_{jk} \left( c_{MP}^j + c_{B}^j + c_{R}^k \right) \]

Here \( c_{MP}^i \) stands for cost of Manpower at the states \( i = 0 \) or \( 1 \), \( c_{B}^j \) stands for cost of Business at the states \( j = 0 \) or \( 1 \)

\( c_{R}^k \) stands for the rate level of Recruitment Cost / Departure Cost at the states \( k = 1 \) or \( 2 \)

We assume the costs and arrive the expected total cost

\[ c_{MP}^0 = 15, \ c_{MP}^1 = 25, \ c_{B}^0 = 30, \ c_{B}^1 = 20, \ c_{R}^1 = 30 \ and \ c_{R}^2 = 35 \]

<table>
<thead>
<tr>
<th>Steady State Probability</th>
<th>Steady State Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{001} ) = 0.5073</td>
<td>27.9015</td>
</tr>
<tr>
<td>( \pi_{002} ) = 0.2029</td>
<td>12.174</td>
</tr>
<tr>
<td>( \pi_{101} ) = 0.1227</td>
<td>7.9755</td>
</tr>
<tr>
<td>( \pi_{102} ) = 0.0340</td>
<td>2.04</td>
</tr>
<tr>
<td>( \pi_{111} ) = 0.0818</td>
<td>6.135</td>
</tr>
<tr>
<td>( \pi_{112} ) = 0.0513</td>
<td>4.104</td>
</tr>
<tr>
<td><strong>Total Expected Cost</strong></td>
<td><strong>60.33</strong></td>
</tr>
</tbody>
</table>

We observe that the steady cost decreases, when there is full business, when departure/recruitment rate increases. Also, when there is no business the steady cost increases when departure/recruitment rate increases.

It has been observed that when there is full business and departure rate increases, the steady state cost decreases.
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References


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