Operations on Q-Fuzzy Soft Set

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Abstract

The concept of Q-fuzzy soft set is further extended to include the operations of union, intersection, AND and OR using De Morgan’s Law. Definitions and propositions on these operations are introduced.

Keywords: Intersection, Multi Q-fuzzy set, Q-fuzzy set, Soft set, Union

1 Introduction

After Molodtsov [1] introduced soft set theory, many studies extended his concept to vague soft set theory [2-12] and applications in genetic algorithms [13-14]. In this paper we introduce operations on a Q-fuzzy soft set as an extension of our studies on Q-fuzzy sets [15-18].

2 Preliminaries

In this section we present the basic definitions of soft set theory and Q-fuzzy soft set required in this paper.

Definition 2.1 [1]. A pair \((F, E)\) is called a soft set (over \(U\)), if and only if \(F\) is a mapping of \(E\) into the set of all subsets of the set \(U\). In other words, the soft set is a parameterized family of subsets of the set \(U\).

Definition 2.1.[18] Let \(U\) be a universal set, \(E\) be a set of parameters, and \(Q\) be a non-empty set. Let \(M^QF(U)\) denote the power set of all multi Q-fuzzy subset of
U with dimension \( k = 1 \). Let \( A \subseteq E \). A pair \((F_\varrho, A)\) is called a Q-fuzzy soft set (in short \( QF-S\) set) over \( U \) where \( F_\varrho \) is a mapping given by

\[
F_\varrho : A \rightarrow M^k QF(U) \quad \text{such that} \quad F_\varrho(x) = \emptyset \quad \text{if} \quad x \notin A.
\]

Here a Q-fuzzy soft set can be represented by the set of ordered pairs

\[
(F_\varrho, A) = \{(x, F_\varrho(x)) : x \in U, F_\varrho(x) \in M^k QF(U)\}.
\]

Note that the set of all Q-fuzzy soft set over \( U \) will be denoted by \( QFS(U) \).

### 3 Union and intersection of q-fuzzy soft set

In this section we introduce the union, intersection, AND and OR operations on a Q-fuzzy soft sets.

**Definition 3.1.** Let \((F_\varrho, A), (G_\varrho, B) \in QFS(U)\). The union of two Q-fuzzy soft sets \((F_\varrho, A)\) and \((G_\varrho, B)\) is the Q-fuzzy soft set \((H_\varrho, C)\), written as

\[
(F_\varrho, A) \cup (G_\varrho, B) = (H_\varrho, C),
\]

where \( C = A \cup B \) for all \( e \in C \) and

\[
H_\varrho(e) = \begin{cases} 
F_\varrho(e) & \text{if } e \in A - B; \\
G_\varrho(e) & \text{if } e \in B - A; \\
F_\varrho(e) \cup G_\varrho(e) & \text{if } e \in A \cap B.
\end{cases}
\]

**Definition 3.2.** Let \((F_\varrho, A), (G_\varrho, B) \in QFS(U)\). The intersection of two Q-fuzzy soft sets \((F_\varrho, A)\) and \((G_\varrho, B)\) is the Q-fuzzy soft set \((H_\varrho, C)\) where \( C = A \cap B \) and for all \( e \in C \), \( (H_\varrho, C) = \{e, \min(\mu_{q_{i_0}}(x, q), \mu_{q_{i_0}}(x, q)) : x \in U, q \in Q\} \) and \( i = 1, 2, \ldots, k \).

Now we give propositions on the union and intersection of Q-fuzzy soft sets.

**Proposition 3.1.** Let \((F_\varrho, A), (G_\varrho, B), (H_\varrho, C) \in QFS(U)\). Then

1. \((F_\varrho, A) \cup (\Phi, A) = (F_\varrho, A)\). 
2. \((F_\varrho, A) \cup (U, A) = (U, A)\). 
3. \((F_\varrho, A) \cup (F_\varrho, A) = (F_\varrho, A)\). 
4. \((F_\varrho, A) \cup (G_\varrho, B) = (G_\varrho, B) \cup (F_\varrho, A)\). 
5. \((F_\varrho, A) \cup ((G_\varrho, B) \cup (H_\varrho, C)) = ((F_\varrho, A) \cup (G_\varrho, B)) \cup (H_\varrho, C)\).

**Proof.** The proof can be easily obtained from Definition 3.1.

**Proposition 3.2.** Let \((F_\varrho, A), (G_\varrho, B), (H_\varrho, C) \in QFS(U)\). Then
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(1) $(F_Q, A) \cap (\Phi, A) = (\Phi, A)$. 
(2) $(F_Q, A) \cap (U, A) = (F_Q, A)$. 
(3) $(F_Q, A) \cap (F_Q, A) = (F_Q, A)$. 
(4) $(F_Q, A) \cap (G_Q, B) = (G_Q, B) \cap (F_Q, A)$. 
(5) $(F_Q, A) \cap ((G_Q, B) \cap (H_Q, C)) = ((F_Q, A) \cap (G_Q, B)) \cap (H_Q, C)$. 

Proof. Result follows trivially from Definition 3.2.

**Proposition 3.3.** (De Morgan’s Law) Let $(F_Q, A), (G_Q, B), (H_Q, C) \in QFS(U)$. Then

(1) $((F_Q, A) \cap (G_Q, B))^C = (F_Q^C, A^C) \cup (G_Q^C, B)^C$. 
(2) $((F_Q, A) \cup (G_Q, B))^C = (F_Q^C, A^C) \cap (G_Q^C, B)^C$. 

Proof. The proof are straight forward by using the properties of a multi $Q$-fuzzy set.

**Proposition 3.4.** Let $(F_Q, A), (G_Q, B), (H_Q, C) \in QFS(U)$. Then

(1) $(F_Q, A) \cap ((G_Q, B) \cup (H_Q, C)) = ((F_Q, A) \cap (G_Q, B)) \cup (F_Q, A) \cap (H_Q, C))$. 
(2) $(F_Q, A) \cup ((G_Q, B) \cap (H_Q, C)) = ((F_Q, A) \cup (G_Q, B)) \cap (F_Q, A) \cup (H_Q, C))$. 

Proof. The proof are straight forward by using properties of a $Q$-fuzzy soft set.

**Definition 3.3.** If $(F_Q, A)$ and $(G_Q, B)$ are two $Q$-fuzzy soft sets, then $(F_Q, A)$ AND $(G_Q, B)$ is the Q-fuzzy soft set denoted by $(F_Q, A) \wedge (G_Q, B)$ and defined by $(F_Q, A) \wedge (G_Q, B) = (H_Q, A \times B)$, where $H_Q(\alpha, \beta) = F_Q(\alpha) \cap G_Q(\beta)$ for all $\alpha \in A$ and $\beta \in B$, is the operation of intersection of two Q-fuzzy sets.

**Definition 3.4.** If $(F_Q, A)$ and $(G_Q, B)$ are two $Q$-fuzzy soft sets, then $(F_Q, A)$ OR $(G_Q, B)$ is the Q-fuzzy soft set denoted by $(F_Q, A) \vee (G_Q, B)$ and defined by $(F_Q, A) \vee (G_Q, B) = (H_Q, A \times B)$, where $H_Q(\alpha, \beta) = F_Q(\alpha) \cup G_Q(\beta)$ for all $\alpha \in A$ and $\beta \in B$, is the operation of union of two Q-fuzzy sets.

4 Conclusion

In this paper we have defined the operations of union, intersection, OR and AND on $Q$-fuzzy soft sets.

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References

http://dx.doi.org/10.1016/s0898-1221(99)00056-5


http://dx.doi.org/10.1155/2012/208489


http://dx.doi.org/10.12988/ams.2013.310576

http://dx.doi.org/10.12988/ams.2013.310575

http://dx.doi.org/10.12732/ijpam.v93i3.5


http://dx.doi.org/10.12732/ijpam.v93i4.3


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