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Uniform Regularity for the Landau-Lifshitz-Maxwell System without Dissipation

Jishan Fan

Department of Applied Mathematics
Nanjing Forestry University, Nanjing 210037, P.R. China

Tohru Ozawa[1]

Department of Applied Physics
Waseda University, Tokyo, 169-8555, Japan

Abstract

In this paper, we prove the existence of local solutions to the Cauchy problem for the 3D Landau-Lifshitz-Maxwell system without dissipation, where the local existence time and the corresponding Sobolev estimates are independent of the dielectric constant ϵ with $0 < \epsilon < 1$. Consequently, the limit as $\epsilon \to 0$ can be established.

Mathematics Subject Classifications: 35Q05, 35Q35

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[1]Corresponding author

1 Introduction

In this paper, we consider the following 3D Landau-Lifshitz-Maxwell system without dissipation:

$$\partial_t u = u \times (\Delta u + H), \tag{1.1}$$
$$\operatorname{rot} H = \epsilon \partial_t E + E, \tag{1.2}$$
$$-\operatorname{rot} E = \partial_t H + \partial_t u, \tag{1.3}$$
$$\operatorname{div} H + \operatorname{div} u = 0, \ \operatorname{div} E = 0, \tag{1.4}$$
$$(u, H, E)(\cdot, 0) = (u_0, H_0, E_0) \ \text{ in } \ \mathbb{R}^3. \tag{1.5}$$

Here $u : \mathbb{R}^3 \times \mathbb{R}_+ \to \mathbb{S}^2$ is a three-dimensional vector field, representing the magnetization, H is the magnetic field, E is the electric field, $\times$ denotes the cross product in $\mathbb{R}^3$, and ϵ is the dielectric constant.

When $H = 0$, the system (1.1) itself is also called the Schrödinger map equation, which has received many studies [1–8, 11–18, 20–31, 33, 34]. P. L. Sulem, C. Sulem and Bardos [33] showed the local existence of smooth solutions. Gustafson-Koo [13] proved the global existence of radial smooth solutions in two space dimensions. Smith [30] established conditional global regularity of 2D Schrödinger map. Fan and Ozawa [10] proved the following regularity criterion

$$\nabla u \in L^2(0, T; BMO(\mathbb{R}^d)) \tag{1.6}$$

in space dimensions $2 \leq d \leq 5$. Here BMO is the space of functions of bounded mean oscillation. When $\epsilon > 0$ or $\epsilon = 0$, we are unable to prove a similar regularity criterion for the full system (1.1)-(1.5).

The aim of this paper is to prove the existence and uniqueness of strong solutions with local existence time independent of ϵ. We will prove

Theorem 1.1. *Let $0 < \epsilon < 1$, let $u_0 : \mathbb{R}^3 \to \mathbb{S}^2$ satisfy $\nabla u_0 \in H^3$, and let $H_0, E_0 \in H^3$. Then there exists a small time $\tilde{T} > 0$ independent of $\epsilon > 0$ and a unique strong solution (u, H, E) to the initial value problem* (1.1)-(1.5) *such that*

$$\begin{aligned} &\nabla u \in L^\infty(0, \tilde{T}; H^3), \quad \partial_t u \in L^\infty(0, \tilde{T}; H^2), \\ &H \in L^\infty(0, \tilde{T}; H^3), \quad \partial_t H \in L^2(0, \tilde{T}; H^2), \\ &E \in L^2(0, \tilde{T}; H^3), \quad \sqrt{\epsilon} E \in L^\infty(0, \tilde{T}; H^3), \quad \epsilon \partial_t E \in L^2(0, \tilde{T}; H^2), \end{aligned} \tag{1.7}$$

with the corresponding norms that are uniformly bounded with respect to ϵ.

Remark 1.1. *By the standard argument, Theorem 1.1 implies the existence of local solutions to* (1.1)-(1.5) *with $\epsilon = 0$.*

Next, we consider the following 1D Landau-Lifshitz-Maxwell system:

$$\partial_t u = u \times (\Delta u + H) + \lambda u \times (u \times (\Delta u + H)), \tag{1.8}$$

with (1.2)-(1.5).

A similar argument implies

Theorem 1.2. *Let $0 < \epsilon, \lambda < 1$, let $u_0 : \mathbb{R} \to \mathbb{S}^2$ satisfy $\nabla u_0 \in H^1$, and let $H_0, E_0 \in H^1$. Then for any $T > 0$ independent of ϵ and λ there exists a unique strong solution (u, H, E) to the initial value problem* (1.8) *and* (1.2)-(1.5) *such that*

$$\begin{aligned} &\nabla u \in L^\infty(0,T;H^1),\ \partial_t u \in L^\infty(0,T;L^2),\\ &H \in L^\infty(0,T;H^1),\ \partial_t H \in L^2(0,T;L^2),\\ &E \in L^2(0,T;H^1),\ \sqrt{\epsilon}E \in L^\infty(0,T;H^1),\ \epsilon\partial_t E \in L^2(0,T;L^2), \end{aligned} \tag{1.9}$$

with the corresponding norms that are uniformly bounded with respect to ϵ and λ.

Remark 1.2. *By the standard argument, Theorem 1.2 implies the existence of local solutions to* (1.8) *with* (1.2)-(1.5) *with $\epsilon = \lambda = 0$. In higher dimension $d \geq 2$, we are unable to prove a similar result as $\lambda \to 0$.*

We will omit the proof of Theorem 1.2 since it is similar to that of Theorem 1.1 (see also [32]).

2 Proof of Theorem 1.1

This section is devoted to the proof of Theorem 1.1. Suppose that $\{u_n, H_n, E_n\}$ are the approximate solutions of the problem (1.1)-(1.5) constructed by Galerkin method. In the following, we will give a priori estimates for the approximate solutions $\{u_n, H_n, E_n\}$, then by a compactness argument we obtain the existence of a solution $\{u, H, E\}$. For simplicity of the presentation, we prove the estimates directly for $\{u, H, E\}$ satisfying (1.1)-(1.5).

Testing (1.1) by $-(\Delta u + H)$ and using the trivial identity $(a \times b) \cdot b = 0$, we see that

$$\frac{1}{2}\frac{d}{dt}\int |\nabla u|^2 dx = \int \partial_t u H dx. \tag{2.1}$$

Testing (1.2) and (1.3) by E and H and summing up, we find that

$$\frac{1}{2}\frac{d}{dt}\int (\epsilon|E|^2 + |H|^2) dx + \int |E|^2 dx = -\int \partial_t u H dx. \tag{2.2}$$

Summing up (2.1) and (2.2), we infer that

$$\frac{1}{2}\frac{d}{dt}\int (|\nabla u|^2 + \epsilon|E|^2 + |H|^2) dx + \int |E|^2 dx = 0. \tag{2.3}$$

To prove further estimates, we derive a new equation from (1.1)-(1.5). Applying ∂_t to (1.1) and using the formula $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$, the constraint $|u| = 1$, the equation (1.3), and the trivial identity $u \cdot \partial_i u = 0$, we have

$$\begin{aligned}
\partial_t^2 u &= \partial_t u \times (\Delta u + H) + u \times (\Delta \partial_t u + \partial_t H) \\
&= (u \times (\Delta u + H)) \times (\Delta u + H) + u \times \Delta[u \times (\Delta u + H)] - u \times (\operatorname{rot} E + \partial_t u) \\
&= (u \cdot (\Delta u + H))(\Delta u + H) - |\Delta u + H|^2 u + (u \cdot (\Delta u + H))\Delta u - (u \cdot \Delta u)(\Delta u + H) \\
&\quad + [u \cdot (\Delta^2 u + \Delta H)]u - (\Delta^2 u + \Delta H) + 2\sum_i [u \cdot (\partial_i \Delta u + \partial_i H)]\partial_i u \\
&\quad - u \times \operatorname{rot} E - (u \cdot (\Delta u + H))u + \Delta u + H. \qquad (2.4)
\end{aligned}$$

Now using $|u| = 1$ and the formulae

$$u \cdot \Delta u = -|\nabla u|^2, \qquad (2.5)$$

$$\begin{aligned}
0 &= \frac{1}{2}\partial_i \Delta |u|^2 = \partial_i (u \Delta u + |\nabla u|^2) \\
&= u \partial_i \Delta u + \partial_i u \Delta u + 2\sum_j \partial_j u \partial_i \partial_j u, \qquad (2.6)
\end{aligned}$$

$$\begin{aligned}
0 &= \frac{1}{2}\Delta^2 |u|^2 = \sum_i \Delta(u_i \Delta u_i + \nabla u_i \nabla u_i) \\
&= \sum_i [u_i \Delta^2 u_i + |\Delta u_i|^2 + 4\nabla u_i \nabla \Delta u_i + 2\sum_j (\partial_j \nabla u_i)^2],
\end{aligned}$$

we get

$$u \Delta^2 u = -|\Delta u|^2 - 4\sum_{i,j} \partial_j u_i \Delta \partial_j u_i - 2\sum_{i,j} (\partial_i \partial_j u)^2. \qquad (2.7)$$

Putting (2.5), (2.6) and (2.7) into (2.4), we arrive at

$$\begin{aligned}
\partial_t^2 u + \Delta^2 u &= (u \cdot H)(2\Delta u + H) - |\Delta u + H|^2 u - |\nabla u|^2 \Delta u + (u \cdot \Delta H)u - \Delta H \\
&\quad - u|\Delta u|^2 - 4u\sum_{i,j} \partial_j u_i \Delta \partial_j u_i - 2u\sum_{i,j} (\partial_i \partial_j u)^2 + 2\sum_i (u \partial_i H)\partial_i u \\
&\quad - 2\sum_i (\partial_i u \cdot \Delta u)\partial_i u - 4\sum_{i,j} (\partial_j u \partial_i \partial_j u)\partial_i u \\
&\quad - u \times \operatorname{rot} E + u|\nabla u|^2 - (u \cdot H)u + \Delta u + H \\
&= -4u\sum_{i,j} \partial_j u_i \Delta \partial_j u_i - 2u|\Delta u|^2 - 2u\sum_{i,j} (\partial_i \partial_j u)^2 \\
&\quad - |\nabla u|^2 \Delta u - 2\sum_i (\partial_i u \cdot \Delta u)\partial_i u - 4\sum_{i,j} (\partial_j u \partial_i \partial_j u)\partial_i u \\
&\quad + (u \cdot H)(2\Delta u + H) - |H|^2 u - 2(\Delta u \cdot H)u + (u \cdot \Delta H)u + 2\sum_i (u \partial_i H)\partial_i u \\
&\quad - u \times \operatorname{rot} E + u|\nabla u|^2 - (u \cdot H)u + \Delta u + H - \Delta H. \qquad (2.8)
\end{aligned}$$

The new equation (2.8) is similar to the biharmonic wave map introduced by Fan and Ozawa [9].

We will also use the following formula

$$0 = \Delta^2(u\partial_t u) = u\Delta^2\partial_t u + \partial_t u\Delta^2 u + C_1 D\partial_t u D^3 u + C_2 D^2\partial_t u D^2 u + C_3 D^3\partial_t u Du, \quad (2.9)$$

where C_1, C_2, C_3 are suitable constant tensors.

Testing (2.8) by $\Delta^2\partial_t u$ and using (2.9), we have

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int(|\Delta\partial_t u|^2 + |\Delta^2 u|^2)dx \\
&= 4\sum_{i,j}\int \partial_j u_i \Delta\partial_j u_i(\partial_t u\Delta^2 u + C_2 D^2\partial_t u D^2 u)dx - 4\sum_{i,j}\int C_1\partial_t u D[D^3 u\partial_j u_i\Delta\partial_j u_i]dx \\
&\quad -4\sum_{i,j}\int C_3 D^2\partial_t u D[Du\partial_j u_i\Delta\partial_j u_i]dx - 2\int u(|\Delta u|^2 + \sum_{i,j}(\partial_i\partial_j u)^2)\Delta^2\partial_t u dx \\
&\quad -\int[|\nabla u|^2\Delta u + 2\sum_i(\partial_i u\cdot\Delta u)\partial_i u + 4\sum_{i,j}(\partial_j u\partial_i\partial_j u)\partial_i u]\Delta^2\partial_t u dx \\
&\quad +\int(u\cdot H)(2\Delta u + H)\Delta^2\partial_t u dx - \int|H|^2 u\Delta^2\partial_t u dx - 2\int(H\Delta u)u\Delta^2\partial_t u dx \\
&\quad +\int(u\cdot\Delta H)u\cdot\Delta^2\partial_t u dx + 2\sum_i\int(u\partial_i H)\partial_i u\Delta^2\partial_t u dx - \int(u\times\operatorname{rot}E)\Delta^2\partial_t u dx \\
&\quad +\int|\nabla u|^2 u\Delta^2\partial_t u dx + \int(\Delta u + H - (u\cdot H)u)\Delta^2\partial_t u dx - \int\Delta H\Delta^2\partial_t u dx \\
&=: \sum_{i=1}^{14} I_i. \qquad (2.10)
\end{aligned}$$

Applying Δ to (1.2) and (1.3), testing by $-\Delta^2 E$ and $-\Delta^2 H$, and summing up, we obtain

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int(\epsilon|\nabla\Delta E|^2 + |\nabla\Delta H|^2)dx + \int|\nabla\Delta E|^2 dx \\
&= \int\Delta^2 H\Delta\partial_t u dx = \int\Delta H\Delta^2\partial_t u dx = -I_{14}. \qquad (2.11)
\end{aligned}$$

Summing up (2.3), (2.10) and (2.11), we get

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int(|\Delta\partial_t u|^2 + |\nabla u|^2 + |\Delta^2 u|^2 + \epsilon|E|^2 + \epsilon|\nabla\Delta E|^2 + |H|^2 + |\nabla\Delta H|^2)dx \\
&+\int(|E|^2 + |\nabla\Delta E|^2)dx = \sum_{i=1}^{13} I_i. \qquad (2.12)
\end{aligned}$$

In the following calculations, we will use the following bilinear product estimates due to Kato and Ponce [19]:

$$\|\Lambda^s(fg)\|_{L^p} \le C(\|\Lambda^s f\|_{L^{p_1}}\|g\|_{L^{q_1}} + \|f\|_{L^{p_2}}\|\Lambda^s g\|_{L^{q_2}}) \tag{2.13}$$

with $s > 0,\ \Lambda := (-\Delta)^{\frac{1}{2}}$ and $\frac{1}{p} = \frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{p_2} + \frac{1}{q_2}$.

We will use (1.1), Sobolev embedding theorem and (2.13) to bound I_i $(i = 1, \cdots, 13)$ and follows.

$$\begin{aligned} I_1 &\le C_1\|\nabla u\|_{L^\infty}\|\nabla\Delta u\|_{L^2}[(\|\Delta u\|_{L^\infty} + \|H\|_{L^\infty})\|\Delta^2 u\|_{L^2} + \|\Delta\partial_t u\|_{L^2}\|\Delta u\|_{L^\infty}] \\ &\le Cy^4. \end{aligned}$$

Here and later on C is independent of ϵ and we denote

$$y^2(t) := \int(|\Delta\partial_t u|^2 + |\nabla u|^2 + |\Delta^2 u|^2 + \epsilon|E|^2 + \epsilon|\nabla\Delta E|^2 + |H|^2 + |\nabla\Delta H|^2)dx.$$

Similarly,

$$\begin{aligned} I_2 &\le C\|\partial_t u\|_{L^\infty}(\|\Delta^2 u\|_{L^2}\|\nabla u\|_{L^\infty}\|\nabla\Delta u\|_{L^2} + \|D^3 u\|_{L^2}^2\|\nabla u\|_{L^\infty}) \\ &\le Cy^3\|\partial_t u\|_{L^\infty} \\ &\le Cy^4 \end{aligned}$$

by

$$\|\partial_t u\|_{L^\infty} \le C(\|\Delta u\|_{L^\infty} + \|H\|_{L^\infty}) \le Cy. \tag{2.14}$$

Moreover, we obtain the following series of estimates:

$$\begin{aligned} I_3 &\le C\|\Delta\partial_t u\|_{L^2}(\|\nabla u\|_{L^\infty}\|\Delta u\|_{L^\infty}\|\nabla\Delta u\|_{L^2} + \|\nabla u\|_{L^\infty}^2\|\Delta^2 u\|_{L^2}) \\ &\le Cy^4. \\ I_4 &= -2\int \Delta[u(|\Delta u|^2 + \sum_{i,j}(\partial_i\partial_j u)^2)]\Delta\partial_t u dx \\ &\le C(\|\Delta u\|_{L^6}^3 + \|\nabla\Delta u\|_{L^6}^2\|\Delta u\|_{L^6} + \|\nabla u\|_{L^6}\|\Delta u\|_{L^6}\|\nabla\Delta u\|_{L^6} \\ &\quad + \|\nabla\Delta u\|_{L^4}^2 + \|\Delta u\|_{L^\infty}\|\Delta^2 u\|_{L^2})\|\Delta\partial_t u\|_{L^2} \\ &\le Cy^4 + Cy^3. \\ I_5 &= -\int \Delta[|\nabla u|^2\Delta u + 2\sum_i(\partial_i u\cdot\Delta u)\partial_i u + 4\sum_{i,j}(\partial_j u\partial_i\partial_j u)\partial_i u]\Delta\partial_t u dx \\ &\le C(\|\Delta u\|_{L^6}^3 + \|\nabla u\|_{L^\infty}^2\|\Delta^2 u\|_{L^2} + \|\nabla u\|_{L^6}\|\Delta u\|_{L^6}\|\nabla\Delta u\|_{L^6})\|\Delta\partial_t u\|_{L^2} \\ &\le Cy^4. \end{aligned}$$

$$
\begin{aligned}
I_6 &= \int \Delta[(u\cdot H)(2\Delta u + H)]\Delta\partial_t u dx \\
&\le C(\|\Delta(u\cdot H)\|_{L^2}\|2\Delta u + H\|_{L^\infty} + \|H\|_{L^\infty}\|\Delta^2 u + \Delta H\|_{L^2})\|\Delta\partial_t u\|_{L^2} \\
&\le C[\|\Delta u\|_{L^2}\|H\|_{H^2}(\|\Delta u\|_{L^\infty} + \|H\|_{L^\infty}) + y^2]y \\
&\le Cy^4 + Cy^3. \\
I_7 &= -\int \Delta(u|H|^2)\cdot\Delta\partial_t u dx \\
&\le C(\|\Delta u\|_{L^2}\|H\|_{L^\infty}^2 + \|\Delta|H^2|\|_{L^2})\|\Delta\partial_t u\|_{L^2} \\
&\le C(y^3 + \|H\|_{H^2}^2)y \\
&\le Cy^4 + Cy^3. \\
I_8 &= -2\int \Delta[(H\cdot\Delta u)u]\Delta\partial_t u dx \\
&\le C(\|\Delta(H\cdot\Delta u)\|_{L^2} + \|H\cdot\Delta u\|_{L^\infty}\|\Delta u\|_{L^2})\|\Delta\partial_t u\|_{L^2} \\
&\le C(\|\Delta H\|_{L^2}\|\Delta u\|_{L^\infty} + \|H\|_{L^\infty}\|\Delta^2 u\|_{L^2} + y^3)y \\
&\le Cy^4 + Cy^3. \\
I_9 &= -\int (u\cdot\Delta H)(\partial_t u\Delta^2 u + C_1 D\partial_t u D^3 u + C_2 D^2\partial_t u D^2 u)dx \\
&\quad + C_3\int D^2\partial_t u\cdot D[Du(u\cdot\Delta H)]dx \\
&\le C\|\Delta H\|_{L^2}(\|\partial_t u\|_{L^\infty}\|\Delta^2 u\|_{L^2} + \|D\partial_t u\|_{L^6}\|D^3 u\|_{L^3} + \|D^2\partial_t u\|_{L^2}\|D^2 u\|_{L^\infty}) \\
&\quad + C\|\Delta\partial_t u\|_{L^2}(\|\Delta u\|_{L^\infty}\|\Delta H\|_{L^2} + \|\nabla u\|_{L^\infty}\|\nabla^3 H\|_{L^2} + \|\nabla u\|_{L^\infty}\|\Delta H\|_{L^2}) \\
&\le Cy^4 + Cy^3. \\
I_{10} &= 2\sum_i \int \Delta[(u\partial_i H)\partial_i u]\Delta\partial_t u dx \\
&\le C(\|\Delta(u\partial_i H)\|_{L^2}\|\nabla u\|_{L^\infty} + \|\nabla H\|_{L^\infty}\|\nabla\Delta u\|_{L^2})\|\Delta\partial_t u\|_{L^2} \\
&\le Cy^4 + Cy^3. \\
I_{11} &= -\int \Delta(u\times \operatorname{rot} E)\Delta\partial_t u dx \\
&\le C(\|\Delta u\|_{L^2}\|\operatorname{rot} E\|_{L^\infty} + \|\Delta\operatorname{rot} E\|_{L^2})\|\Delta\partial_t u\|_{L^2} \\
&\le \frac{1}{16}\int (E^2 + |\nabla\Delta E|^2)dx + Cy^4 + Cy^2. \\
I_{12} &= \int \Delta(|\nabla u|^2 u)\Delta\partial_t u dx \\
&\le C(\|\Delta|\nabla u|^2\|_{L^2} + \|\nabla u\|_{L^\infty}^2\|\Delta u\|_{L^2})\|\Delta\partial_t u\|_{L^2} \\
&\le Cy^4 + Cy^3. \\
I_{13} &= \int \Delta[\Delta u + H - (u\cdot H)u]\Delta\partial_t u dx \\
&\le C(\|\Delta^2 u\|_{L^2} + \|\Delta H\|_{L^2} + \|\Delta(u\cdot H)\|_{L^2} + \|H\|_{L^\infty}\|\Delta u\|_{L^2})\|\Delta\partial_t u\|_{L^2} \\
&\le Cy^3 + Cy^2.
\end{aligned}
$$

Inserting the above estimates into (2.12), we conclude that

$$\frac{dy^2}{dt} + \int_0^t \int |E|^2 + |\nabla \Delta E|^2 dxdt \leq Cy^4 + Cy^3 + Cy^2,$$

which leads to

$$y^2 + \int_0^T \int (|E|^2 + |\nabla \Delta E|^2) dxdt \leq C.$$

This completes the proof.

□

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