Joint Two-Echelon Inventory Model for Optimal Inventory Decisions and Shipment Policies under Non-Linear Price Dependent Demand

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Abstract

In supply chain management, achieving effective coordination among manufacturers and retailers has become a pertinent research issue. Supply chain coordination is a joint decision policy achieved by the manufacturer and a retailer. This work proposes a joint two-level supply chain model with a single-manufacturer and a single-retailer. The novelty of the proposed model is that the demand is expressed as non-linear function of the retailer’s unit selling price. Ordering/setup costs, carrying costs and transportation costs are considered in the development of the model. The objective of the model is to demonstrate the optimality of replenishment quantities, shipment frequency and the total relevant
costs under non-linear price dependent demand. The model is solved using a computer programme written in MATLAB, as per the optimality criterion derived for the decision variables. Further, the sensitivity analysis is carried out to show the influence of the model parameters.

**Keywords:** Supply chain coordination, Non-linear function; Price dependent demand; Total relevant cost; Selling price

1. Introduction

The fierce competition resulting due to globalization has increased attention of all parties in the business on the effective Supply Chain Management (SCM). The objective of SCM is to create the most value not only to the company but to whole supply chain network, which includes the end user. SCM cannot be left to chance. The efficiency of the supply chain does not arise within each link in the chain. All the stakeholders of the supply chain realized that the competition is no longer a contest among echelons of the supply chain. Each supply chain is competing with other related supply chains. Hence, no echelon of the supply chain can function in isolation. Coordination has become a primary key for effective Supply Chain Management.

Supply chains are based on centralized and decentralized decision-making processes. A unique decision-maker manages the whole supply chain in case of a centralized decision-making process. The objective of this process is to minimize/maximize the total supply chain cost/profit. In the decentralized decision-making process, multiple decision-makers who have conflicting objectives involve in a supply chain and each decision-maker tends to optimize his/her own performance leading to an inefficient supply chain system. A proper coordination mechanism should be adopted to address this inefficiency. Such coordination mechanisms include flexibility contracts in quantity (Tsay, [3]), the payback/return policies (Emmons & Gilbert, [8]), the payback agreements (Eppen&Iyer, [6]), and revenue sharing contracts (Giannoccaro&Pontrandolfo, [10]). The supply chain model developed in this work assumes a centralized decision-making process as a coordination mechanism.

Malone and Crowston [32] defined coordination as the process of managing dependencies among activities. The fundamental assumption underlying this emphasis is that this long-term relationship enhances the profitability of both the supplier and the retailer. The essence is that the total supply chain profit after coordination must be greater than the sum of un-coordinated individual profits, since the profit margin at the chain level might not necessarily be optimal for each firm. This means that in the long run, each participant in the supply chain would expect larger profit than that it would attain without coordination. Lambert and Cooper [16] suggested integrating business processes and the key members of the supply chain as the structure of business processes within and among the companies.
Joint two-echelon inventory model

is vital to create maximum profitability and competitiveness. A natural means of achieving coordination is the integration of inventory decision models of players in the supply chain (e.g., Goyal and Gupta, [25]).

The motivation is to work on joint inventory optimization of the supply chain, as integration identifies interdependencies between the echelons. Hence, it paves ways to mutually define goals and to share the risks and rewards. This work is mainly focussed on the development of two-echelon inventory models for a coordinated supply chains. This research is presented as follows. Section 2 is presenting literature review of the problem considered. Section 3 comprises of the features and assumptions, notations used and the development of the mathematical model. In Section 4, a numerical investigation is presented. Section 5 is narrating the conclusions of the work and indicating future scope of the work.

2. Preliminary notes

It is more challenging, but highly essential to design and operate a supply chain in such a way that the total system-wide costs are minimized in addition to maintaining system wise service levels. Each member of the supply chain has its own state of information and incentive. Hence, no member has the possibility of optimizing entire supply chain performance. In a centralized supply chain, information is public to all echelons and information flow is smooth, and there is a central planner who makes all the decisions. Optimal decisions which maximize the system-wide supply-chain profit can be made. In order to improve supply chain efficiency, the supply chain entities can form a partnership via setting an appropriate contract. A contract that results in decisions by individual parties that maximize the profit of the entire supply chain and leaves each member of the chain satisfied is called a coordinating contract. In the literature, various policies for supply chain coordination have been introduced.

Jaber and Osman [17] proposed a centralized model for a two-level supply chain having supplier and retailer to coordinate their orders for the minimization of local costs as well as supply chain cost. In the proposed model, the permissible delay is adopted as a means of trade credit for coordinating the order quantity. Coordination plays a vital role in managing the supply chain for optimizing entire supply chain performance. Inventory can be considered as the key driver that invites larger degree of coordination among echelons within a supply chain. Some researchers showed that coordination could be achieved by integrating lot-sizing models (e.g., Goyal & Gupta, [25]). The benefits of coordinating the inventories of supply chain by using common replenishment time periods was studied and analyzed by Viswanathan and Piplani [30]. Esmaeil et al. [15] modelled the seller-buyer relationship in the environment of demand sensitivity to both price and promotion under non-cooperative and cooperative situations.

Panda [26] showed that revenue and cost sharing contract is superior to conventional revenue sharing contract for effective coordination. Further, the two-
echelon model developed concluded that the range of cost sharing fraction which is leading to win-win situation is independent of the format of cost structure of retailer. Naini et al. [24] considered a decentralized supply chain comprising of three echelons, which is associated with procurement, production and selling one type of product in separate and independent markets. They have determined inventing, manufacturing and pricing policies simultaneously. A coordination framework for aligning the inventory decisions in decentralized supply chains (ASCEND) was presented by Piplani and Fu [21]. Their research work indicated that cost sharing and service level contracts can realize the value of coordination by each partner in the supply chain network.

Sarmah, Acharya and Goyal [27] reviewed literature pertaining to buyer-vendor coordination models having quantity discount as a coordination mechanism when the environment is deterministic. They have classified the various models and identified critical issues. Thematical models developed by Sarmah, Acharya and Goyal [28] for coordinating two-stage supply chain under asymmetric information environment by credit option. This work involves the development of two-echelon inventory models under non-linear price dependent demand. The literature related to the development of inventory models in a multi-echelon supply chain network is presented. Yadavalli et al. [34] considered a two commodity inventory system under continuous review with maximum storage capacity for the commodity and computed the limiting probability distribution for the joint inventory levels. Premvrat and Padmanabhan [33] developed an inventory model under inflation in respect of items depending on stock consumption rate. Nurfadhllina and Saiful [18] presented an approach to maximize the profits by implementing the fair profit sharing mechanism using stochastic programming method. The conditions of demand rate dependent on stock-level and the holding cost dependent on storage time are discussed by Alfares [9]. He determined that the optimal order quantity and the cycle time decreases by increasing the holding cost. Ashok and Ravi [20] modelled backlog fill rate in a lost sale recapture model as a function of the proportion of rebate in relation to price to satisfy some portion of the unfulfilled demand. An optimal ordering policy in an inventory model having two warehouses is discussed by Kaur et al. [11] in case of non-instantaneous deteriorating items under stock dependent demand. Jinshi and Jiazhen [35] integrated the Black-Scholes rule and contract option and developed a model to optimize the supplier-led supply chain. This model optimizes the supply chain in addition to following Black Schools rule. Chang et al. [4] assumed the demand rate of the deteriorating items dependent on on-display stock, unit selling price and the limited amount of shelf space. Further, formulated mathematical models and derived the algorithms for finding optimal solutions. Kasthuri and Seshiah [23] presented a mathematical model in a multi-item inventory model having no shortages to minimize total cost by adopting Kuhn-Trucker conditions. Singh [31] developed an EOQ model with respect to deteriorating items with time-dependent quadratic demand and also with variable
deteriorating under permissible delay in payment. The impact of revenue-sharing contract mechanism is investigated by Saha et al. [29], when the demand rate is stock-of time-price sensitive.

Nagaraju et al. [5] developed a model for a two-level supply chain, when the demand is dependent on selling price under wholesale price index and consumer price index. Chen et al. [7] studied coordination mechanism in a two-level supply chain by considering the lead time and price-dependent demand. They proposed risk and profit sharing agreement, which is flexible for both the parties of supply chain. A two-echelon supply chain with price dependent demand is modelled by Syam Sundar et al. [14]. They determined optimal replenishment quantity, annual relevant cost and inventory ratio for coordinated and non-coordinated supply chain. Lau and Lau [1] applied different demand-curve functions in an inventory/pricing model instead of linear demand function. They noticed that there is a substantial and unpredictable change over a very minute change in the demand by a multi-echelon inventory/pricing model. Clark [2] summarized the studies of the multi-echelon inventory theory and suggested to make use of the theory both for establishing integrated inventories and scheduling factory production with dynamic and stochastic demands for a product. An economic cost model to quantity supply chain costs was developed by Chiuadamrong and Wajcharapornjindia [19] to enable companies to quantify hidden benefits and their savings along with realising the true costs. Forghani, Mirzazadeh and Rafiee [13] presented a single period inventory problem under a price-dependent model and suggested three functions for representing the demand rate as a function of selling price. Their research revealed a significance improvement in the system costs after the consideration of price revision. An inventory model with stock dependent demand was developed by Rathod and Bhathawaala [12]. In their model, they considered that holding cost is a step function of storage time. Vasanthi and Sesaiah [22] formulated a multi-item inventory model having shortages and demand dependent on unit cost in addition to the constraints storage space and set up cost. Despite the existence of the extensive research in inventory coordination domain, there is a very wide opportunity to work further. The research works are pronouncing much about the variability of demand as a key parameter that influences supply chain profitability. In this work, two-echelon supply chain is modelled under non-linear demand environment. This research attempt aimed at evaluating the optimal values of decision variables like replenishment quantity, shipment frequency and the total relevant cost with coordination.

3. Mathematical Model Development

The development of a mathematical model with suitable assumptions and notation are summarized in the current section as follows.
3.1 Features and Assumptions
The following features and assumptions are used for the development of the model as follows.

- Non-linear price dependent demand
- Production rate is infinite
- Instantaneous replenishment rate
- Shortages are not allowed
- Replenishment batch size at the manufacturer is multiple integer of replenishment quantity at the retailer

3.2 Notation:
For the convenience of model development, the following notation is used throughout the model development.

\[ D = \alpha - \beta (P_R)^\gamma \text{ where } \beta > 0, \alpha >> \beta \text{ and } 0 < \gamma < 1 \]

\[ S_m \] Setup cost incurred per setup at the manufacturer (in INR/setup)

\[ S_R \] Ordering cost incurred per order at the retailer (in INR/order)

\[ C_m \] Unit cost at the manufacturer (in INR/unit)

\[ C_R \] Unit cost at the retailer (in INR/unit)

\[ P_R \] Unit selling price at the retailer (in INR/unit)

\[ \tau_m \] Manufacturer’s transportation cost for shipping a shipment to the retailer (in INR/shipment)

\[ \tau_R \] Retailer’s transportation cost for receiving a shipment from the manufacturer (in INR/shipment)

\[ k \] Carrying charge of the inventory (in INR/INR/year)

\[ q \] Replenishment quantity at the retailer in units (decision variable)

\[ \lambda \] Number of shipments from the manufacturer to retailer (decision variable, a positive integer)

\[ Q \] Manufacturer’s replenishment batch size in units

\[ \psi_R(q) \] Annual total relevant cost of the retailer expressed in terms of \( q \)

\[ \psi_m(Q) \] Annual total relevant cost of the manufacturer expressed in terms of \( Q \)

\[ \psi_m(\lambda, q) \] Annual total relevant cost of the manufacturer expressed in terms of \( \lambda \) and \( q \)
\( \psi_s(\lambda, q) \)  
Annual total relevant cost of the supply chain expressed in terms of \( \lambda \) and \( q \)

### 3.3 Model Formulation:

#### 3.3.1 Retailer Point:

Annual ordering cost of the retailer = \( \left( \alpha - \beta \left( \frac{P_R}{q} \right)^\gamma \right) S_R \)

Annual transportation cost of the retailer = \( \left( \frac{\alpha - \beta \left( \frac{P_R}{q} \right)^\gamma}{\tau_R} \right) \)

Annual carrying cost of the retailer = \( \frac{q}{2} C_{Rk} \)

Annual total relevant cost of the retailer is expressed as the sum of annual ordering cost, transportation cost and carrying cost.

\[
\psi_R(q) = \left( \frac{\alpha - \beta \left( \frac{P_R}{q} \right)^\gamma}{q} \right) (S_R + \tau_R) + \frac{q}{2} C_{Rk} \quad (1)
\]

#### 3.3.2 Manufacturer point:

Annual setup cost of the manufacturer = \( \left( \frac{\alpha - \beta \left( \frac{P_R}{q} \right)^\gamma}{\lambda q} \right) S_m \)

Annual transportation cost of the manufacturer = \( \left( \frac{\alpha - \beta \left( \frac{P_R}{q} \right)^\gamma}{q} \right) \tau_m \)

Annual carrying cost of the manufacturer = \( \frac{(\lambda - 1)q}{2} C_{mk} \quad (Since \quad Q = \lambda q) \)

Annual total relevant cost of the manufacturer is expressed as the sum of the annual ordering cost, transportation cost and carrying cost.

\[
\psi_m(\lambda, q) = \left( \frac{\alpha - \beta \left( \frac{P_R}{q} \right)^\gamma}{q} \right) \left( \frac{S_m}{\lambda} + \tau_m \right) + \frac{(\lambda - 1)q}{2} C_{mk} \quad (2)
\]

#### 3.3.3 Integrated Supply Chain:

In an integrated two-echelon supply chain, both the manufacturer and retailer agree to cooperate and follow the joint optimal policy. The annual total relevant
cost of the retailer and manufacturer coordinated supply chain, \( \psi_s(\lambda, q_R) \) with non-linear price dependent demand is expressed as

\[
\psi_s(\lambda, q) = \frac{\alpha - \beta (P_R)^\gamma}{q} \left(S_R + \frac{S_m}{\lambda} + \tau_R + \tau_m\right) + \frac{q}{2}k(C_R + (\lambda - 1)C_m)
\]  

**Proposition 1:** For given value of \( \lambda \), the expression representing the annual total relevant cost of the supply chain is convex in terms of \( q \). The optimal replenishment quantity of \( q^* \) is obtained by taking the first order and second order derivative of the annual total relevant cost function as given by Eq. (4).

\[
q^* = \left( \frac{2(\alpha - \beta (P_R)^\gamma)}{k(C_R + (\lambda - 1)C_m)} \left(\frac{S_R + \frac{S_m}{\lambda} + \tau_R + \tau_m}{1.5}\right) \right)^{\frac{1}{2}}
\]  

**Proof:** Taking the first order and second order partial derivatives of eq. (3) with respect to \( q \), and equating the first order derivative to zero, we have

\[
\frac{\partial}{\partial q} \psi_s(\lambda, q) = 0
\]

\[
-\frac{\alpha - \beta (P_R)^\gamma}{q^2} \left(S_R + \frac{S_m}{\lambda} + \tau_R + \tau_m\right) + \frac{k}{2}(C_R + (\lambda - 1)C_m) = 0
\]

\[
\frac{\alpha - \beta (P_R)^\gamma}{q^2} \left(S_R + \frac{S_m}{\lambda} + \tau_R + \tau_m\right) = \frac{k}{2}(C_R + (\lambda - 1)C_m)
\]

\[
q = \left( \frac{2(\alpha - \beta (P_R)^\gamma)}{k(C_R + (\lambda - 1)C_m)} \left(\frac{S_R + \frac{S_m}{\lambda} + \tau_R + \tau_m}{1.5}\right) \right)^{\frac{1}{2}}
\]

\[
\frac{\partial^2}{\partial q^2} \psi_s(\lambda, q) = \frac{2(\alpha - \beta (P_R)^\gamma)}{q^3} \left(S_R + \frac{S_m}{\lambda} + \tau_R + \tau_m\right)
\]
Note: From Eq. (9), the principal minor of the Hessian matrix
\[ H(\lambda, q) = \frac{\partial^2}{\partial q^2} \left( \psi_S(\lambda, q) \right) > 0 \] for all values of \( \alpha, \beta, \gamma, \lambda, q \) and other model parameters. Hence \( q \) becomes optimum. \( \psi_S(\lambda, q) \) is strictly said to be convex.

**Proposition 2:** For given value of \( q \), the optimal value of \( \lambda \), \( \lambda^* \) always satisfies the following condition:
\[
\lambda^* (\lambda^* - 1) \leq \frac{2(\alpha - \beta(P_R)^*) S_m}{q^2 C_m I} \leq \lambda^* (\lambda^* + 1)
\] (10)

**Proof:** For given value of \( q \), the optimal value of \( \lambda \), \( \lambda^* \) always satisfies the following expressions given below.
\[
\psi_S(\lambda^*) \leq \psi_S(\lambda^* - 1) \quad \text{and} \quad \psi_S(\lambda^*) \leq \psi_S(\lambda^* + 1)
\]
Substituting the relevant values in Eq. (3) for the condition \( \psi_S(\lambda^*) \leq \psi_S(\lambda^* - 1) \), and with further simplification and rearranging the terms, the following inequality is obtained as
\[
\lambda^* (\lambda^* - 1) \leq \frac{2(\alpha - \beta(P_R)^*) S_m}{q^2 C_m I}
\] (11)

Similarly, substituting the relevant values in Eq. (3) for the condition \( \psi_S(\lambda^*) \leq \psi_S(\lambda^* + 1) \), and with further simplification and rearranging the terms, the following inequality is obtained as
\[
\frac{2(\alpha - \beta(P_R)^*) S_m}{q^2 C_m I} \leq \lambda^* (\lambda^* + 1)
\] (12)

By combining the inequalities shown in equations (11) and (12), the following expression is obtained as
\[
\lambda^* (\lambda^* - 1) \leq \frac{2(\alpha - \beta(P_R)^*) S_m}{q^2 C_m I} \leq \lambda^* (\lambda^* + 1)
\]

**4. Numerical Investigation**

In the current section, the optimality of ordering policies and shipment frequencies has been tested for an integrated supply chain with the help of numerical
data. Based on the following data, the numerical example is devised here to illustrate the model.

The values of the model parameters: $S_R = \text{INR 100 per order}$, $S_m = \text{INR 500 per setup}$, $C_R = \text{INR 280 per unit}$, $C_m = \text{INR 200 per unit}$, $P_R = \text{INR 322 per unit}$, $\tau_R = \text{INR 120 per shipment}$, $\tau_m = \text{INR 480 per shipment}$, $\alpha = 2000$, $\beta = 5$, $k = \text{INR 0.18 per INR per year}$. Based on the step-by-step procedure developed above, the optimal values of decision variables and objective function are computed for joint two-echelon inventory model and the results are tabulated in Table 1.

**Table 1**: Variation of Demand, Inventory Levels, Shipment Frequencies and Total Relevant Costs w.r.t. $\gamma$ - Values

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Demand</th>
<th>Decision Variables and Objective Function</th>
<th>% Decrease in</th>
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<tr>
<td></td>
<td>$D$</td>
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Table 1 shows the analysis of variation of the optimality of replenishment quantity, shipment frequency, and total relevant cost of the coordinated supply chain as well as individual entities with respect to the variation in gamma. For a particular value of gamma equal to 0.5, the total relevant cost of the supply chain is optimal at the optimal values of replenishment quantity and shipment frequency. Further, Figure 1 shows the analysis of the variation of total relevant cost of the supply chain with respect to the simultaneous variation of shipment frequency and retailer’s replenishment quantity. From Figure 1, it is evident that the shape of the curve representing the total relevant cost of the supply chain is convex. It is attributed to the fact that trade-off occurs between the total ordering/setup cost and carrying cost at the optimal values of replenishment quantity and shipment frequency. At the non-optimal values of ordering quantity
and shipment frequency, trade-off do not occur between the ordering/setup cost and carrying cost.

**Fig. 1** Variation of Total Relevant Cost of the Coordinated Supply Chain \( (\psi_s) \) w.r.t Number of Shipments \( (\lambda) \) and Ordering Quantity \( (q) \)

Further, from table 1, it is also evident that the variation in optimal values of replenishment quantities, number of shipments and total relevant costs of the manufacturer, retailer and the supply chain is non-linear with respect to gamma. From, table 1 and figure 2 and 3, it is observed that the percentage decrease in decision variables and objective function is non-linear with respect to gamma. It is due to the fact that demand decreases non-linearly at the retailer point as the value of gamma increases. Consequently, retailer orders less number of items which in turn decreases the total relevant cost of the supply chain.

Further, it is interested to carry the sensitivity analysis to analyse the variation in optimality of decision variables and objective function with respect to model parameters. From table 2, it is observed that there is no variation in the optimality of shipment frequency with respect to model parameters. Similarly, with respect to the variation in the manufacturer’s unit cost, there is no change in the optimality of decision variables and objective function. It is due to the fact that the effect of the component associated with manufacturer’s unit cost is zero on the optimality of replenishment quantities and total relevant costs.
Fig. 2 Retailer’s Optimal Ordering Quantity w.r.t gamma ($\gamma$)

Fig. 3 Optimal Total Relevant Cost of the Supply Chain w.r.t gamma ($\gamma$)
Table 2: Sensitivity Analysis

<table>
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5. Conclusions and Future Scope

In this work, a mathematical model is developed for a joint two-echelon inventory system with a single manufacturer and a single retailer. In the model development, the demand is expressed as a non-linear function of retailer’s unit selling price. Inventory associated costs like ordering/setup costs, carrying costs and transportation costs are considered for model development. The optimality criterion is demonstrated for the decision variables and objective function in the form of corollary and proofs. Based on the optimality criterion, a computer programme is written in MATLAB and the model is solved. From the findings of the research, it is concluded that the curve representing the total relevant cost of the supply chain is convex with respect to replenishment quantity and shipment frequency. The optimality of replenishment quantities and total relevant costs decreases non-linearly with respect to gamma whereas the shipment frequency remains same. Also, from the sensitivity analysis, it is concluded that all the model parameters have the significant influence over the optimality of replenishment quantities and total relevant costs, except unit cost of the manufacturer. Research findings and the novelty of the current model can be useful in consumer goods industries. Managerial decisions like replenishment quantities and the number of shipments can be decided with the help of this model. The current work can be extended to a three-echelon supply chain with wide variations and assumptions in demand function.

References


Joint two-echelon inventory model


Received: October 25, 2014; Published: November 26, 2014