Abstract

This article is based on a research project that intents to answer the question: How to perfect students mathematics abilities in Ordinary Differential Equations with the use if models and simulations in engineering professional situations? It is considered pertinent to design a didactic proposal that allows overcoming the limits imposed by traditional teaching methods and that uses some of the technological tools applied to mathematical education.

To solve the research problem, a set of actions was designed and evaluated in which problematic situations were posited from real pre-established mathematical models, represented by first and second order ordinary differential equations both linear and non linear with given starting conditions that are solved in a numerical or analytical manner with the help of the Mathematica software.
Keywords: Ordinary differential equations, didactics and mathematical models.

1 Introduction

Ordinary differential equations courses offered to engineering students are characterized by their content high density and by a traditional teaching approach in the classroom. The experience of the authors in years of working with this topic has produced situations in which the academic management has questioned the high academic mortality and the methods used by professors to reach learning objectives in these courses. Besides, it has been seen a lack of motivation from students towards this topic.

This research project is important because it has an effect in the teaching-learning process of ordinary differential equations in a methodic, structured and measurable way that includes the use of models and simulations as part of the mathematical work. It will consolidate a didactic model that serves as a referent for teachers of these courses and related subjects. Besides, it will allow to establish indicators as to the way in which computer technologies facilitate learning by improving the comprehension of thematic content and the development of mathematical competences of future engineers.

2 Methodology

The methodology used for the completion of this research project was descriptive. The response of 2012 third term students of the Differential Equations course from District University Francisco José de Caldas, Engineering school, Electric Engineering program, was examined and valued.

Once the different activities and pedagogic actions proposed were applied through the academic term, results were obtained and, after processing and analyzing them, it was possible to establish their incidence in the teaching and learning process.

To design each of the activities presented to the students, representative problematic situations modeled mathematically with ordinary differential equations were selected and they were simulated with the help of the Mathematica software.

Design of a didactic model and a methodology for its practical introduction

1. Didactic strategy and heuristic procedure
In the figure 1, diverse elements, concepts, theories and proposals that were taken into account for the formulation of this project are shown:

Figure 1: Elements, concepts and theories used in the present project

From the identification of didactic, pedagogic, epistemic, and conceptual elements the heuristic procedure that indicates the way in which the proposed activities are taken into practice is derived:
<table>
<thead>
<tr>
<th><strong>Differential equations learning phases in engineering programs</strong></th>
<th><strong>Actions</strong></th>
<th><strong>Orienting questions</strong></th>
<th><strong>Heuristic resources</strong></th>
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</thead>
<tbody>
<tr>
<td>I. Characterization and diagnostic of the previous concepts required to understand and interiorize the different techniques and procedures to solve ordinary differential equations</td>
<td>Diagnostic</td>
<td>Which are the previous concepts required? What is the management level of these concepts in the classroom?</td>
<td>Uses engrams to solve the diagnostic test</td>
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<tr>
<td>II. Theoretic foundations required for the development of each activity guide</td>
<td>Theoretic class on the diverse methods to solve ODEs (Ordinary Differential Equations)</td>
<td>How can a student differentiate an ODE from a PDE (Partial Differential Equation)?</td>
<td>Relates ODEs with integral and differential calculus contents</td>
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<td>III. Strengthening and Understanding of different techniques to solve differential equations. Building of differential equations and their solutions</td>
<td>Manual solution of different ODEs</td>
<td>How can different ODEs solving methods be distinguished?</td>
<td>Identifies the different types of first and second order ODEs, and recognizes the solving techniques</td>
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<td>IV. Analysis of models that are solved with DE</td>
<td>Translate real situations to mathematical language</td>
<td>How can an ODE be built from a real situation?</td>
<td>Builds ODEs to model real situations</td>
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<tr>
<td>V. Solutions verifications, use of software, making of simple simulations and inferences</td>
<td>Use of the Mathematica software</td>
<td>How to verify solutions and compare them with real situations?</td>
<td>Uses the software to verify solutions and make simulations</td>
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</table>


2. Activities to carry out

Five activities were designed in which various real pre-established mathematical models are presented to students; these models are solved with the use of first and second order ODEs. A good theoretical foundation was given to students so that they could solve six differential equations that model real problems tied to some situations in an analytic and/or numerical manner with the Mathematica software; some real experiences were carried out. The purpose of these activities is that they help students better their understanding level in mathematics, acquire abilities in the solution of differential equations, and use them to solve problems in their professional lives. For this research project the following systems were used:

- Tank draining
- R-C (resistor capacitor) circuit in series
- R-L-C (resistor inductor capacitor) circuit
- Prey-predator model (LOTKA VOLTERRA)

2.1 Activity No. 1
2.1.1 Diagnostic test

With the results of this first test, it is intended to examine the knowledge level students have in mathematics, specifically in ODEs so that a classification according to the results can be made (strata).

2.1.2 Structure

This first diagnostic activity consists of measuring the theoretical conceptualization that students have in first and second order differential equations; besides, they must apply first order ODEs to the draining of cylindrical geometric tanks, using the separable variables method. The objective here is that students use the Torricelli principle in the modeling of the draining of a cylindrical tank, solve the differential equation by variable separation and, with the use of an initial condition of the problem, determine the time that takes the tank to drain.

2.1.3 Previous concepts

Algebraic concepts, derivation and integration techniques, solving of first order ODEs by separable variables.

2.1.4 Previous concepts
To identify the knowledge (engrams) that the student brings from algebra and differential and integral calculus through the solving of the diagnostic test and, thus, to be able to classify students in groups or strata of accomplishment, according to the grades obtained.

2.2 Activity No. 2

2.2.1 Theme: First order ODEs solved by separable variables (Continued)

2.2.2 Structure: “Draining of tanks model”

The second activity consists on the application of first order ODEs in the draining of tanks, by the separable variables method. The objective is that students use the Torricelli principle in the modeling of draining tanks of different geometric forms (cylindrical, conic, hemispheric) and compare discharge velocities, times of draining, capacities, etc.

2.2.3 Previous knowledge

Algebraic concepts, solving of rectangle triangles, similar triangles, functions derivation, integration techniques, and first order ODEs solving by separable variables.

2.2.4 Objectives

That the student posit a first order ODE by variable separation according to the shape of the tank, solve it manually, and test it with the help of the Mathematica software. Besides, that they relate such differential equations with real life situations and obtain solutions.

2.3 Activity No. 3

2.3.1 Theme: First order linear ODEs

2.3.2 Structure: “R-C Circuit. Cardiac defibrillator model.”

In this activity the objective is that the student relates a medical instrument called “cardiac defibrillator” used to homogenize heart beats of patients with a cardiac arrest, through electric shocks, with a first order linear ODE. The idea is to solve the equation and predict, from its solution, whether a specific patient has possibilities to live or not.

2.3.3 Previous concepts

Algebraic concepts, function derivation, integration techniques, first order linear ODEs solution by separable variables, electric circuits and capacitors concepts.

2.3.4 Objective
The objective is that the student identifies a first order linear ODE, solve it manually and make the verification of the solution with the Mathematica software. Besides, they must relate the differential equation with the cardiac defibrillator model and obtain solutions.

2.4 Activity No. 4

2.4.1 Theme: Second order linear homogeneous and non-homogeneous ODEs with constant coefficients

2.4.2 Structure: “R-L-C. Circuit. Mechanic-electric analogy”

2.4.3 Previous concepts

Second-degree algebraic equations solutions, some concepts of linear algebra, functions derivation, integration techniques and second order linear homogeneous and non-homogeneous ODEs with constant coefficients solution.

2.4.4 Objective

That the student learns to identify a second order linear homogeneous and non-homogeneous ODE with constant coefficients, solve it manually, verify the solution with the Mathematica software and compare the results. Besides, they must interpret the numerical-graphic simulation that makes the Mathematica software, change the differential equation parameters and find solutions.

2.5 Activity No. 5

2.5.1 Theme: Systems of non-linear ODEs

2.5.2 Structure: “Prey-predator model (Lotka-Volterra)”

In this activity the student must identify that the prey-predator model, created by Lotka-Volterra, is modelled through a non-linear ODEs system. The solution of this differential equations system can be made manually and with the help of the Mathematica software. This solution must be used to find maximal and minimal values of preys and predators and populations in balance.

2.5.3 Previous concepts

2.5.4 Objective

That the student learns to identify a system of non-linear differential equations, solve it manually, and then obtain a numeric solution with the Mathematica software and compare results. Besides, they must interpret data and dynamic graphics that offer the Mathematica software through a simulation process.

3 Results analysis

The grading range for each activity was 0 to 12, where 0 corresponds to lowest grade and 12 to the highest.
In the table 1, the results of grades obtained by each student in each of the test are presented. A student that did not attend a session is marked with N.A.

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Table 1: Grades obtained by student in each of the activities

In this table, it can be seen that only thirteen students took all tests and activities designed by the course.

In the figure 2, the distribution of students by category in each of the activities is shown. The categories correspond to each of the activities in the research project.

4 Conclusions

The students that participated in the set of activities evidenced some degree of mathematical abilities development for solving problems of ODEs through the
use of mathematical models and simulation. As it can be established from the statistic analysis of the sample, as activities developed throughout the class, students showed better grades and, therefore, the indicators of the construction of statements and comprehension of problems also improved.

From the 13 students who developed all activities, a contrast is established between the grades of the diagnostic test and the final test, it is observed that 84.61% improved their grades; the remaining 15.39 obtained the same grades in both tests.

In the designed activities with problematic situations, students showed development of mathematical abilities concerning the identification of the type of differential equations, the construction of statements, the solution of ordinary differential equations, and the use of software as a tool to perform simulations and inferences.

In the final survey, 42.11% of students identified themselves with the statement “Solving accurately the proposed statements was nice and motivating for my differential equations learning”. 36.84% of students identified themselves with the statement “NOT solving accurately the proposed statements caused to make a greater effort and, therefore, improved my learning of differential equations”. This situation indicates that the proposed activities impacted positively students and promoted to some degree autonomous learning and highlighted the importance of applications and mathematical software.

Each activity was developed around a mathematical model related with ODEs, the incorporation of the Mathematica software contributed to some degree to the understanding, analysis and making of inferences of the problematic situations presented to Electrical Engineering students.
Students showed their desire for “more exposure by the part of the professor” since, within the students culture there is a certain attachment to the traditional teaching / learning model, in which the professor assumes a paternalistic role that does not promote autonomous learning.

References


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