Soft Semi Open Sets with Respect to Soft Ideals

Rodyna A. Hosny

Department of Mathematics, Faculty of Science
Taif University, Taif, Saudi Arabia
and
Department of Mathematics, Faculty of Science
Zagazig University, Zagazig, Egypt

Deena Al-Kadi

Department of Mathematics, Faculty of Science
Taif University, Taif, Saudi Arabia

Copyright © 2014 Rodyna A. Hosny and Deena Al-Kadi. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract
In the present paper, the concepts of soft semi open and soft semi closed sets modulo a soft ideal in the soft topological spaces are presented. These notions represent generalization of the concepts of soft semi open and soft semi closed sets. The notion of soft semi compactness based on soft ideal are proposed. Furthermore, some behaviors and features of such concepts are investigated with illustrative examples.

Mathematics Subject Classification: 54A05, 54A10, 54A20

Keywords: Soft sets, Soft topology, Soft ideal, Soft semi compact space.

1 Introduction
Many disciplines including engineering, medicine, economics, and sociology, are highly dependent on the task of modeling uncertain data. When the uncertainty is highly complicated and difficult to characterize, classical mathematical approaches are often insufficient to derive effective or useful models.
Testifying to the importance of uncertainties that cannot be defined by classical means, researchers are introducing alternative theories every day. In addition to classical probability theory, some of the most important results on this topic are fuzzy sets, intuitionistic fuzzy sets [1], vague sets [5], interval mathematics, and rough sets [13]. However, all of these new theories have inherent difficulties which are pointed out in [11]. A possible reason is that these theories possess inadequate parameterization tools [9].

Molodtsov [11] has established the concept of a soft set to deal with problems of incomplete information. The concept of soft sets enhanced the application potential of the different generalizations of crisp sets due to the additional advantage of parametrization tools. Xiao et al. [18] discussed the relationship between soft sets and information systems and showed that soft sets are a class of special information systems. The notion of topological space for soft sets and their several properties were formulated by Shabir and Naz [17] and Cagman et al. [3] separately in 2011. Hussain et al. [7] investigated the properties of soft open and soft closed sets, soft neighborhood and soft closure. Also, they defined and discussed the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces. The study on soft compactness for a soft topological space was initiated by Zorlutuna et al. in [19]. In [4, 8] the notations of soft semi open sets in soft topological spaces have been introduced. Mahanta and Das in [8] introduced and studied the notions of soft semi compactness.

In present work, some concepts in soft topological spaces such as soft $I$-semi open, soft $I$-semi closed sets will be introduced. Also, the notion of soft semi compactness with respect to soft ideals will be defined. Several characterizations of these concepts will be investigated with illustrative examples.

## 2 Preliminaries

In this section, we recall some basic well-known definitions and results of soft set theory that are useful for subsequent discussion. These notions and more detailed explanations related to them and their properties can be found in [2, 3, 4, 7, 8, 9, 11, 17, 19].

**Definition 2.1** Let $X$ be an initial universe and $A$ be a non-empty set of parameters. Let $\mathcal{P}(X)$ denote the power set of $X$. $F_A$ is called a soft set over $X$, where $F$ is a mapping given by $F: A \rightarrow \mathcal{P}(X)$. The family of all soft sets over $X$ is denoted by $\mathbb{SS}(X, A)$. For two soft sets $F_A, H_A$ over a common universe $X$ (i) $F_A$ is said to be a soft subset of $H_A$, if $F(e) \subseteq H(e)$ for all $e \in A$. 
Symbolically it is written as $F_A \subseteq H_A$. (ii) $F_A$ and $H_A$ are called soft equal, if $F_A \subseteq H_A$ and $H_A \subseteq F_A$. Symbolically it is written as $F_A = H_A$.

**Definition 2.2** A soft set $F_A$ over $X$ is called a null (resp., an absolute) soft set, denoted by $\emptyset_A$ (resp., $X_A$), if $e \in A$, $F(e) = \emptyset$ (resp., $F(e) = X$).

In particular $(X, A)$ will be denoted by $\tilde{X}_A$.

**Definition 2.3** Let $F_A, H_A \in SS(X, A)$. Then,

(i) The soft union of two soft sets $F_A$ and $H_A$ over $X$, denoted by $K_A = F_A \sqcup H_A$, is defined as $K(e) = F(e) \cup H(e)$ for all $e \in A$.

(ii) The soft intersection of two soft sets $F_A$ and $H_A$ over $X$, denoted by $K_A = F_A \cap H_A$, is defined as $K(e) = F(e) \cap H(e)$ for all $e \in A$.

(iii) The soft difference of two soft sets $F_A$ and $H_A$ over $X$, denoted by $K_A = F_A - H_A$ is defined as $K(e) = F(e) - H(e)$ for all $e \in A$.

(iv) The soft complement of $F_A$, denoted by $(F_A)^c = \tilde{X}_A - F_A$ is defined by $(F^c(e)) = X - F(e)$ for all $e \in A$. Clearly, $((F_A)^c)^c = F_A$.

(v) The soft symmetric difference of two soft sets $F_A$ and $H_A$, denoted by $K_A = F_A \triangle H_A$, is defined by $(F_A \setminus H_A) \sqcup (H_A \setminus F_A)$. Equivalently, $(F_A \sqcup H_A) \setminus (F_A \cap H_A)$. Clearly, $(\tilde{X}_A)^c = \emptyset_A$ and $(\emptyset_A)^c = \tilde{X}_A$.

**Definition 2.4** Let $F_A$ be a soft set over $X$ and $x \in X$. Then, $x \in F_A$, if $x = x(e) \in F(e)$ for all $e \in A$.

**Definition 2.5** Let $x \in X$ and $\alpha \in A$, then a soft point $x_\alpha$ denotes the soft set over $X$ for which $x(\alpha) = \{x\}$.

In the following, we give some basic results of soft topological spaces.

**Definition 2.6** Let $\tau$ be a collection of soft sets over $X$ with a fixed set of parameters $A$, then $\tau \subseteq SS(X, A)$ is called a soft topology on $X$, if it satisfies the following axioms:

(i) $\emptyset_A, X_A$ belong to $\tau$. 
(ii) The soft union of any number of soft sets in $\tau$ belongs to $\tau$.

(iii) The soft intersection of any two soft sets in $\tau$ belongs to $\tau$.

The triplet $(X, A, \tau)$ is called a soft topological space or soft space. Every member of $\tau$ is called a soft open set. A soft set $H_A$ is called soft closed in $(X, A, \tau)$, if $(H_A)^c$ is a soft open set. The family of soft closed sets is denoted by $\tau^c$.

**Definition 2.7** Let $(X, A, \tau)$ be a soft topological space and $F_A \in SS(X, A)$. Then,

(i) The soft interior of $F_A$ is the soft set \( \text{int} F_A = \bigcup \{ K_A \mid K_A \text{ is soft open set and } K_A \subseteq F_A \} \). i.e. It is the largest soft open subsets of $F_A$.

(ii) The soft closure of $F_A$ is the soft set \( \text{cl} F_A = \bigcap \{ H_A \mid H_A \text{ is soft closed set and } F_A \subseteq H_A \} \). i.e. It is the smallest soft closed supersets of $F_A$.

**Lemma 2.8** Let $(X, A, \tau)$ be a soft topological space and $F_A$ be a soft set. Then,

(i) If $H_A$ is soft $\tau$-open, then $H_A \cap \text{cl} (F_A) \subseteq \text{cl} (H_A \cap F_A)$.

(ii) If $H_A$ is soft $\tau$-closed, then $\text{int}(F_A \cup H_A) \subseteq \text{int}(F_A) \cup H_A$.

**Definition 2.9** A soft set $F_A$ of a soft topological space $(X, A, \tau)$ is called

(i) Soft semi open if, there exists a soft open set $H_A$ such that $H_A \subseteq F_A \subseteq \text{cl} H_A$ (equivalently, if $F_A \subseteq \text{cl} \text{int} F_A$).

(ii) Soft semi closed if, there exists a soft closed set $K_A$ such that $\text{int} K_A \subseteq F_A \subseteq K_A$ (equivalently, if $\text{int} \text{cl} F_A \subseteq F_A$).

**Definition 2.10** A family $\Psi$ of soft sets is said to be a soft cover of a soft set $F_A$, if $F_A \subseteq \bigcup \{ U_i \in \Psi \mid i \in I \}$. It is a soft open cover, if each member of $\Psi$ is a soft open set. A soft subcover of $\Psi$ is a subfamily of $\Psi$ which is also a soft cover of a soft set $F_A$.

**Definition 2.11** A soft cover of a soft set is called a soft semi open cover, if every member of the soft cover is a soft semi open set.

**Definition 2.12** A soft topological space $(X, A, \tau)$ is called
(i) Soft compact space if each soft open cover of $\tilde{X}_A$ has a finite soft subcover.

(ii) Soft semi compact space if each soft semi open cover of $\tilde{X}_A$ has a finite soft subcover.

Lemma 2.13 Every soft compact space is also soft semi compact space.

Definition 2.14 A family $\Psi$ of soft sets is said to have the finite intersection property (FIP for short), if the soft intersection of the members of each finite subfamily of $\Psi$ is not null soft set.

3 Soft Semi Open Sets Modulo a Soft Ideal

First, we introduce the concept of soft ideal over $X$ and give several related properties.

Definition 3.1 A soft ideal $I$ over $X$ is a collection of soft sets over $X$ which satisfies the following properties:

(i) If $F_A \in I$ and $H_A \subseteq F_A$, then $H_A \in I$.

(ii) If $F_A \in I$ and $H_A \in I$, then $F_A \sqcup H_A \in I$.

Lemma 3.2 The soft intersection $I \cap J$ of two soft ideals $I$ and $J$ is a soft ideal but the soft union is not a soft ideal in general.

Proof Let $I$ and $J$ be soft ideals over $X$.

(i) Let $F_A \in I \cap J$ and $H_A \subseteq F_A$, then $F_A \in I$ and $F_A \in J$. Therefore, $H_A \in I$ and $H_A \in J$. Consequently, $H_A \in I \cap J$.

(ii) Let $F_A \in I \cap J$ and $H_A \in I \cap J$, then $F_A \in I$, $H_A \in I$, $F_A \in J$ and $H_A \in J$. Consequently, $F_A \sqcup H_A \in I$ and $F_A \sqcup H_A \in J$ and so $F_A \sqcup H_A \in I \cap J$.

The next part of the Lemma is proved by the following example.

Example 3.3 Let $X=\{a, b, c\}$, $A=\{\alpha_1, \alpha_2\}$ and choose $I=\{\emptyset_A, C_A, D_A, E_A\}$, $J=\{\emptyset_A, M_A\}$ to be two soft ideals such that $C_A(\alpha_1) = \{b\}$, $C_A(\alpha_2) = \emptyset$. 
\( D_A(\alpha_1) = \{ c \}, \quad D_A(\alpha_2) = \emptyset. \)

\( E_A(\alpha_1) = \{ b, c \}, \quad E_A(\alpha_2) = \emptyset. \)

\( M_A(\alpha_1) = \{ a \}, \quad M_A(\alpha_2) = \emptyset. \)

Then, \( I \cup J = \{ \emptyset_A, C_A, D_A, E_A, M_A \} \) is not a soft ideal.

**Example 3.4**

(i) \( I = \{ \emptyset_A \} \) is a minimal soft ideal on \( X \).

(ii) \( I = \{ F_A \in \mathcal{SS}(X, A) \mid \cup F_A(e) \text{ is finite} \} \) is a soft ideal in \( X \), for each \( e \in A \).

(iii) \( I = \{ F_A \in \mathcal{SS}(X, A) \mid \cup F_A(e) \text{ is countable} \} \) is a soft ideal in \( X \), for each \( e \in A \).

(iv) For any soft set \( F_A \), \( I_{F_A} = \{ H_A \mid H_A \subseteq F_A \} \) is a soft ideal in \( X \).

**Lemma 3.5** [16] Let \( X \) be an universal set and \( I \) be a collection of soft sets on \( X \). Then \( I \) need not to be a soft ideal on \( X \), even if the collection \( I_e = \{ F_A(e) \mid F_A \in I \} \) defines an ideal in \( X \), for each \( e \in A \).

**Example 3.6** Let \( X = \{ a, b, c \} \); \( A = \{ \alpha_1, \alpha_2 \} \) and \( I = \{ \emptyset_A, C_A, D_A, E_A, L_A, M_A \} \) where \( C_A, D_A, E_A, M_A \) are a soft set over \( X \), defined as follows

\( C_A(\alpha_1) = \{ b \}, \quad C_A(\alpha_2) = \{ a \}. \)

\( D_A(\alpha_1) = \{ a, b \}, \quad D_A(\alpha_2) = \{ c \}. \)

\( E_A(\alpha_1) = \{ b \}, \quad E_A(\alpha_2) = \{ a, c \}. \)

\( L_A(\alpha_1) = \{ b \}, \quad M_A(\alpha_2) = \{ c \}. \)

\( M_A(\alpha_1) = \{ a \}, \quad M_A(\alpha_2) = \{ c \}. \)

Then \( I_{\alpha_1} = \{ \emptyset_A, \{ a \}, \{ b \}, \{ a, b \} \} \) and \( I_{\alpha_2} = \{ \emptyset_A, \{ a \}, \{ c \}, \{ a, c \} \} \) are ideals on \( X \). However, the \( I \) is not a soft ideal on \( X \) because \( C_A \cup D_A = \{ \{ a, b \}, \{ a, c \} \} \notin I \).

A soft ideal \( I \) on \( X \) does not guarantee that \( I_e \) is an ideal on \( X \) for each \( e \in A \), as shown below.
Example 3.7  Let $X=\{a, b\}; A=\{\alpha_1, \alpha_2\}$ and $\mathcal{I}=\{\tilde{\emptyset}_A, C_A, D_A, E_A\}$ where $C_A, D_A, E_A$ are a soft set over $X$, defined as follows

$C_A(\alpha_1)=X, C_A(\alpha_2)=\emptyset.$

$D_A(\alpha_1)=\{a\}, D_A(\alpha_2)=\emptyset.$

$E_A(\alpha_1)=\{b\}, E_A(\alpha_2)=\emptyset.$

Then $\mathcal{I}$ is a soft ideal on $X$. However, the $\mathcal{I}_{\alpha_1}$ is not an ideal on $X$ because $X\in\mathcal{I}_{\alpha_1}$ for $\alpha_1\in A$, even if $\mathcal{I}_{\alpha_2}$ defines an ideal in $X$ for $\alpha_2\in A$.

Now we move one step forward to introduce soft semi open and soft semi closed sets modulo a soft ideal in a soft topological space $(X, A, \tau)$ as a generalization of the concepts soft semi open and soft semi closed sets [4, 8]. Various properties related to these concepts will be studied.

Definition 3.8 A soft set $F_A$ in a soft topological space $(X, A, \tau)$ is called a soft semi open modulo a soft ideal $\mathcal{I}$ (written as, soft $\mathcal{I}$-semi open), if there exists a soft open set $H_A$ such that $(H_A - F_A)\in \mathcal{I}$ and $(F_A - \text{cl } H_A)\in \mathcal{I}$. A soft set $F_A$ is called soft $\mathcal{I}$-semi closed if and only if $(F_A)^c$ is a soft $\mathcal{I}$-semi open. In other words, a soft set $F_A$ is called soft $\mathcal{I}$-semi closed, if there exists a soft closed set $K_A$ such that $(\text{int } K_A - F_A)\in \mathcal{I}$ and $(F_A - K_A)\in \mathcal{I}$.

It is easy to see that $\tilde{\emptyset}_A$ and $\tilde{X}_A$ are always soft $\mathcal{I}$-semi open and soft $\mathcal{I}$-semi closed sets.

Corollary 3.9 $F_A$ is always soft $\mathcal{I}$-semi open, for $F_A\in \mathcal{I}$.

Proof Obvious, since $\tilde{\emptyset}_A$ is a soft open set in a soft topological space $(X, A, \tau)$.

Lemma 3.10 Let $F_A$ be soft $\mathcal{I}$-semi open set, then

(i) If $H_A=\tilde{\emptyset}_A$, then $F_A\in \mathcal{I}$.

(ii) If $H_A=\tilde{X}_A$, then $\tilde{X}_A - F_A\in \mathcal{I}$.

Proof Obvious.

Lemma 3.11 In a soft topological space $(X, A, \tau)$ with a soft ideal $\mathcal{I}$, the following statements hold.
(i) [2] Every soft open (resp., closed) set is a soft semi open (resp., closed) set.

(ii) Every soft semi open (resp., semi closed) is a soft $\mathcal{I}$-semi open (resp., $\mathcal{I}$-semi closed) set.

**Proof** Obvious.

The next example will be shown that the converse of Lemma 3.11 does not hold.

**Example 3.12** Let $X=\{a, b, c\}$, $A=\{\alpha_1, \alpha_2\}$, $\tau=\{\emptyset_A, X_A, F_1A, F_2A, F_3A, F_4A, F_5A, F_6A, F_7A\}$ and $\mathcal{I}=\{\emptyset_A, C_A\}$ where $F_1A, F_2A, F_3A, F_4A, F_5A, F_6A, F_7A$ and $C_A$ are soft sets over $X$, defined as follow

\[
F_1A(\alpha_1)=\{a, b\}, F_1A(\alpha_2)=\{a, b\} \\
F_2A(\alpha_1)=\{b\}, F_2A(\alpha_2)=\{a, c\} \\
F_3A(\alpha_1)=\{b, c\}, F_3A(\alpha_2)=\{a\} \\
F_4A(\alpha_1)=\{b\}, F_4A(\alpha_2)=\{a\} \\
F_5A(\alpha_1)=\{a, b\}, F_5A(\alpha_2)=X \\
F_6A(\alpha_1)=X, F_6A(\alpha_2)=\{a, b\} \\
F_7A(\alpha_1)=\{b, c\}, F_7A(\alpha_2)=\{a, c\}. \\
C_A(\alpha_1)=\{b\}, C_A(\alpha_2)=\{b\}.
\]

(i) A soft set $G_A$, which is defined as follow $G_A(\alpha_1)=\{b, c\}$, $G_A(\alpha_2)=\{a, b\}$, is soft semi open, but it is not soft open.

(ii) A soft set $K_A$, which is defined as follow $K_A(\alpha_1)=\{a\}$, $K_A(\alpha_2)=\{a, c\}$, is a soft $\mathcal{I}$-semi open and it is not soft semi open set.

**Theorem 3.13** Let $\mathcal{I}$ be a soft ideal on a soft topological space $(X, A, \tau)$. Then, the concepts of soft $\mathcal{I}$-semi open set and soft semi open set are the same, if $\mathcal{I}=\{\emptyset_A\}$.

**Proof** Obvious.
Next, we discuss some of the properties of soft $\mathcal{I}$-semi open set.

**Theorem 3.14** Let $\mathcal{I}$ and $\mathcal{J}$ be two soft ideals on a soft topological space $(X, A, \tau)$, then the following statements hold:

(i) If $\mathcal{J}$ is soft finer than $\mathcal{I}$ (i.e $\mathcal{I} \sqsubseteq \mathcal{J}$), then every soft $\mathcal{I}$-semi open set is a soft $\mathcal{J}$-semi open set.

(ii) If $F_A$ is a soft $\mathcal{I} \cap \mathcal{J}$-semi open set set, then it is simultaneously soft $\mathcal{I}$-semi open and soft $\mathcal{J}$-semi open set.

**Proof** Obvious.

**Theorem 3.15** Let $\mathcal{I}$ be a soft ideal on a soft topological space $(X, A, \tau)$. If both $F_A$ and $G_A$ are soft $\mathcal{I}$-semi open sets, then so is their soft union $F_A \sqcup G_A$.

**Proof** let $F_A$ and $G_A$ be soft $\mathcal{I}$-semi open sets of soft topological space $(X, A, \tau)$, then there exist respectively soft open sets $H_A$ and $K_A$ such that $(H_A - F_A) \in \mathcal{I}$, $(F_A - \text{cl } H_A) \in \mathcal{I}$, $(K_A - G_A) \in \mathcal{I}$ and $(G_A - \text{cl } K_A) \in \mathcal{I}$. Choose $M_A = H_A \sqcup K_A$ and observe that $(M_A - (F_A \sqcup G_A)) = ((H_A - F_A) - G_A) \cup ((K_A - G_A) - F_A) \in \mathcal{I}$. Also, $(F_A \sqcup G_A) - \text{cl } (M_A) = ((F_A - \text{Cl } (H_A)) - \text{cl } (K_A)) \cup ((G_A - \text{cl } (K_A)) - \text{cl } (H_A)) \in \mathcal{I}$. Since $M_A$ is a soft open set therefore, $F_A \sqcup G_A$ is soft $\mathcal{I}$-semi open set.

Next example shows that the soft intersection of two soft $\mathcal{I}$-semi open sets is not necessary soft $\mathcal{I}$-semi open set.

**Example 3.16** Let $X=\{a, b, c\}$, $A=\{\alpha_1, \alpha_2\}$, $\tau=\{\emptyset_A, \bar{X}_A, F_{1A}, F_{2A}, F_{3A}\}$ and soft ideal $\mathcal{I}=\{\emptyset_A, C_A, D_A, E_A\}$ where $F_{1A}, F_{2A}, F_{3A}, C_A, D_A, E_A$ are soft set over $X$, defined as follow

$F_{1A}(\alpha_1)=\{a\}$, $F_{1A}(\alpha_2)=\{c\}$.

$F_{2A}(\alpha_1)=\{c\}$, $F_{2A}(\alpha_2)=\{a\}$.

$F_{3A}(\alpha_1)=\{a, c\}$, $F_{3A}(\alpha_2)=\{a, c\}$.

$C_A(\alpha_1)=\{\emptyset_A\}$, $C_A(\alpha_2)=\{c\}$.

$D_A(\alpha_1)=\{\emptyset_A\}$, $D_A(\alpha_2)=\{a\}$.

$E_A(\alpha_1)=\{\emptyset_A\}$, $E_A(\alpha_2)=\{a, c\}$. 
Let $G_A$ and $H_A$ be two soft sets such that $G_A(\alpha_1) = \{a, b\}$, $G_A(\alpha_2) = \{\emptyset_A\}$ and $H_A(\alpha_1) = \{b, c\}$, $H_A(\alpha_2) = \{\emptyset_A\}$. Choose $M_A = G_A \cap H_A$ where $M_A(\alpha_1) = \{b\}$, $M_A(\alpha_2) = \{\emptyset_A\}$. It is clear that $G_A$ and $H_A$ are soft $\mathcal{I}$-semi open sets but their soft intersection $M_A$ is not soft $\mathcal{I}$-semi open set.

**Theorem 3.17** Let $G_A$ be a soft open set and $F_A$ be a soft $\mathcal{I}$-semi open set in a soft topological space $(X, A, \tau)$, then the soft intersection $F_A \cap G_A$ is a soft $\mathcal{I}$-semi open set.

**Proof** Suppose that $F_A$ is a soft $\mathcal{I}$-semi open set, then there is a soft open set $H_A$ such that $(H_A - F_A) \in \mathcal{I}$ and $(F_A - \text{cl } H_A) \in \mathcal{I}$. Since $G_A$ is a soft open set, then $(H_A \cap G_A)$ is soft open set such that $(H_A \cap G_A) - (F_A \cap G_A) = ((H_A - F_A) \cap G_A) \in \mathcal{I}$. From Lemma 2.10, we have $(F_A \cap G_A) - \text{cl } (H_A \cap G_A) - (\text{cl } H_A \cap G_A) \subseteq ((F_A - \text{cl } H_A) \cap G_A) \in \mathcal{I}$. Consequently, $F_A \cap G_A$ is soft $\mathcal{I}$-semi open set.

**Theorem 3.18** Let $\mathcal{I}$ be a soft ideal on a soft topological space $(X, A, \tau)$. If $F_A$ and $G_A$ are both soft $\mathcal{I}$-semi closed sets, then $F_A \cap G_A$ is a soft $\mathcal{I}$-semi closed set.

**Proof** Obvious.

**Definition 3.19** A soft set $F_A$ is called

(i) Soft dense in a soft set $H_A$, if $H_A \subseteq \text{cl } F_A$.

(ii) Soft dense, if $\text{cl } F_A = \bar{X}_A$.

**Lemma 3.20** Let $\mathcal{I}$ be a soft ideal on a soft topological space $(X, A, \tau)$. If $F_A$ is a soft dense in $H_A$ and soft open set with $F_A \subseteq H_A$, then $H_A$ is soft $\mathcal{I}$-semi open.

**Proof** Obvious.

**Corollary 3.21** Let $\mathcal{I}$ be a soft ideal on a soft topological space $(X, A, \tau)$ and $F_A$ be soft open set, then $\text{cl } F_A$ is a soft $\mathcal{I}$-semi open set.

**Proof** These is immediate consequences of Lemma 3.20.
**Definition 3.22** A soft topological space \((X, A, \tau)\) is called

(i) *Soft hyper connected*, if every non-empty soft open set over set \(X\) is soft dense.

(ii) *Soft locally indiscrete*, if every soft open set over set \(X\) is soft closed.

**Theorem 3.23** Let \(\mathcal{I}\) be a soft ideal on a soft hyper connected topological space \((X, A, \tau)\) and the collection of soft open subsets of \(X\) satisfies the (FIP). Then, the following statements hold.

(i) If \(F_A\) is a soft \(\mathcal{I}\)-semi open set and \(F_A \subseteq G_A\), then \(G_A\) is also soft \(\mathcal{I}\)-semi open set.

(ii) If \(F_A\) is a soft \(\mathcal{I}\)-semi open set and \(G_A\) is any soft set, then \(F_A \sqcup G_A\) is soft \(\mathcal{I}\)-semi open set.

(iii) If both \(F_A\) and \(G_A\) are soft \(\mathcal{I}\)-semi open set, then so is their soft intersection \(F_A \cap G_A\).

**Proof** (i) Suppose that \(F_A\) is a soft \(\mathcal{I}\)-semi open set and \(F_A \subseteq G_A\). There is a soft open set \(H_A\) such that \((H_A - F_A) \in \mathcal{I}\) and \((F_A - \text{cl } H_A) \in \mathcal{I}\). Note that soft open set \(H_A \neq \emptyset\) and \((X, A, \tau)\) is a soft hyper connected topological space, thus \((H_A - G_A) \subseteq (H_A - F_A) \in \mathcal{I}\), moreover, \((G_A - \text{cl } H_A) = (G_A - \overline{X}_A) = \emptyset \in \mathcal{I}\). Thus, \(G_A\) is a soft \(\mathcal{I}\)-semi open set.

(ii) Follows immediately from (i).

(iii) Let \(F_A\) and \(G_A\) be two soft \(\mathcal{I}\)-semi open sets, suppose that \(F_A \cap G_A \neq \emptyset\), otherwise \(F_A \cap G_A\) will be trivially soft \(\mathcal{I}\)-semi open set. By assumption, there are soft open sets \(H_A\) and \(K_A\) such that \((H_A - F_A) \in \mathcal{I}\), \((F_A - \text{cl } H_A) \in \mathcal{I}\), \((K_A - G_A) \in \mathcal{I}\) and \((G_A - \text{cl } K_A) \in \mathcal{I}\). Consider the soft open set \(H_A \cap K_A\), which is non-empty. Since the collection of soft open sets satisfies the (FIP), then \((H_A \cap K_A) - (F_A \cap G_A) = ((H_A - F_A) \cap K_A) \cup (H_A \cap (K_A - G_A)) \in \mathcal{I}\) and \((F_A \cap G_A) - \text{cl}(H_A \cap K_A) = (F_A \cap G_A) - \overline{X}_A = \emptyset \in \mathcal{I}\), it follows that \(F_A \cap G_A\) is soft \(\mathcal{I}\)-semi open set.

**Lemma 3.24** Let \(\mathcal{I}\) be a soft ideal on a soft hyper connected topological space \((X, A, \tau)\) and the collection of soft open sets satisfies the (FIP). If \(\text{cl } F_A\) is soft \(\mathcal{I}\)-semi open set, then \(F_A\) is soft \(\mathcal{I}\)-semi open set.

**Proof** Let \(\text{cl } F_A\) is a soft \(\mathcal{I}\)-semi open set, then there is a soft open set \(H_A\) such that \((H_A - \text{cl } F_A) \in \mathcal{I}\), \((\text{cl } F_A - \text{cl } H_A) \in \mathcal{I}\). Consider the soft open set \(G_A = (H_A - \text{cl } F_A) \in \mathcal{I}\), \((\text{cl } F_A - \text{cl } H_A) \in \mathcal{I}\). Thus, \(G_A\) is a soft \(\mathcal{I}\)-semi open set.
- cl $F_A$) $\in \mathcal{I}$, by assumption. We have $(G_A - F_A) = (H_A - \text{cl } F_A) - F_A = G_A \in \mathcal{I}$ and $(F_A - \text{cl } G_A) = (F_A - \text{cl}(H_A - \text{cl } F_A)) = F_A - X_A \in \mathcal{I}$. This shows that $F_A$ is soft $\mathcal{I}$-semi open set.

**Corollary 3.25** Let $\mathcal{I}$ be a soft ideal on a soft hyper connected topological space $(X, A, \tau)$, and suppose that the collection of soft open sets satisfies the (FIP). Then $F_A$ is a soft $\mathcal{I}$-semi open set, if and only if $\text{cl } F_A$ is a soft $\mathcal{I}$ semi open set.

**Proof** It is obvious from Corollary 3.21 and Lemma 3.24.

**Theorem 3.26** Let $\mathcal{I}$ be a soft ideal on a soft locally indiscrete topological space $(X, A, \tau)$ and $F_A$ is soft $\mathcal{I}$-semi open set, then there is soft open set $H_A$ such that $F_A \triangle H_A \in \mathcal{I}$.

**Proof** Obvious.

The next concept of soft $\mathcal{I}$ pre-open will be presented in a soft topological space $(X, A, \tau)$ as a generalization of the concept pre-open modulo an ideal [15].

**Definition 3.27** A soft set $F_A$ in a soft topological space $(X, A, \tau)$ is called a soft pre-open modulo a soft ideal $\mathcal{I}$ (written as, soft $\mathcal{I}$ pre-open), if there exists a soft open set $H_A$ such that $(F_A - H_A) \in \mathcal{I}$ and $(H_A - \text{cl } F_A) \in \mathcal{I}$.

**Theorem 3.28** Let $\mathcal{I}$ be a soft ideal on a soft locally indiscrete topological space $(X, A, \tau)$. Then, $F_A$ is soft $\mathcal{I}$-pre open set, if it is soft $\mathcal{I}$-semi open set.

**Proof** Let $F_A$ be soft $\mathcal{I}$-semi open set, then there exists a soft open set $H_A$ such that $(H_A - F_A) \in \mathcal{I}$ and $(F_A - \text{cl } H_A) \in \mathcal{I}$. Since $(X, A, \tau)$ is soft locally indiscrete space, then $\text{cl } H_A = H_A$. Hence $(F_A - H_A) \in \mathcal{I}$ and $(H_A - \text{cl } F_A) \subseteq (H_A - F_A) \in \mathcal{I}$ for some a soft open set $H_A$. Thus $F_A$ is soft $\mathcal{I}$-pre open set.

## 4 Soft Semi Compact Space Modulo Soft Ideal

Generalization of soft open sets to soft semi open sets in soft topological spaces also demands generalization of compactness. This section is devoted to introduce soft semi compactness with respect to soft ideal in soft topological spaces.

**Definition 4.1** A soft topological space $(X, A, \tau)$ together with a soft ideal is called soft compact modulo a soft ideal $\mathcal{I}$ or just soft $\mathcal{I}$-compact space, if for every soft open cover $\{F_iA | i \in \Gamma\}$ of $\check{X}_A$, there is a finite subfamily $\{F_1A, F_2A, \ldots, F_nA\}$ such that $\check{X}_A - \{\bigcup_{i=1}^{n} F_iA\} \in \mathcal{I}$. 

Definition 4.2 A soft topological space \((X, A, \tau)\) together with a soft ideal is called soft semi compact modulo a soft ideal \(I\) or just soft \(I\)-semi compact space, if for every soft semi open cover \(\{F_iA \mid i \in \Gamma\}\) of \(\tilde{X}_A\), there is a finite subfamily \(\{F_1A, F_2A, \ldots, F_nA\}\) such that \(\tilde{X}_A - \{\bigcup_{i=1}^n F_iA\} \in I\).

Next theorems with immediate proof and then omitted.

Theorem 4.3 Let \(I\) be a soft ideal on a soft topological space \((X, A, \tau)\). Then, the concepts of soft \(I\)-semi compact space and soft semi compact space coincide, if \(I = \{\emptyset_A\}\).

Theorem 4.4 Let \(I\) be a soft ideal on a soft topological space \((X, A, \tau)\). If \(\tilde{X}_A\) is a soft \(I\)-semi compact space, then it is soft \(I\)-compact space.

Theorem 4.5 Let \((X, A, \tau)\) be a locally indiscrete soft topological space together with a soft ideal, then soft semi compact modulo a soft ideal \(I\) and soft compact modulo a soft ideal \(I\) are equivalent.

Theorem 4.6 Let \(I\) and \(J\) be two soft ideals on a soft topological space \((X, A, \tau)\). If \(I \subseteq J\), then every soft \(I\)-semi compact space is a soft \(J\)-semi compact space.

Theorem 4.7 Let \(I\) be soft ideal on a soft topological space \((X, A, \tau)\), then \(\tilde{X}_A\) is soft \(I\)-semi compact space if and only if for every family \(\{U_iA \mid i \in \Gamma\}\) of soft semi closed sets of \(\tilde{X}_A\) for which \(\bigcap_{i \in \Gamma} U_iA = \emptyset_A\), there exists finite \(\Gamma_0\) of \(\Gamma\) such that \(\bigcap_{i \in \Gamma_0} U_{iA} \in I\).

Proof Let \(\{U_{iA} \mid i \in \Gamma\}\) be a family of soft semi closed sets of \(\tilde{X}_A\) for which \(\bigcap_{i \in \Gamma} U_{iA} = \emptyset_A\). Then, \(\{X_{iA} - U_{iA} \mid i \in \Gamma\}\) is a family of soft semi open sets of \(\tilde{X}_A\) such that \(\bigcup_{i \in \Gamma}(X_{iA} - U_{iA}) = \tilde{X}_A\) i.e., the family \(\{(X_{iA} - U_{iA}) \mid i \in \Gamma\}\) is a soft semi open cover of \(\tilde{X}_A\). Since \((X, A, \tau)\) is soft \(I\)-semi compact, then there exists finite \(\Gamma_0\) of \(\Gamma\) such that \(\tilde{X}_A - \bigcup_{i \in \Gamma_0}(X_{iA} - U_{iA}) \in I\). Consequently, \(\bigcap_{i \in \Gamma_0} U_{iA} \in I\). Conversely, let \(\{H_{iA} \mid i \in \Gamma\}\) be a family of soft semi open cover of \(X_{iA}\), then \(\bigcup_{i \in \Gamma} H_{iA} = \tilde{X}_A\). Hence, \(\{(X_{iA} - H_{iA}) \mid i \in \Gamma\}\) be a family of soft semi closed sets of \(\tilde{X}_A\) and so \(\tilde{X}_A - \bigcup_{i \in \Gamma} H_{iA} = \emptyset_A\). Therefore, \(\bigcap_{i \in \Gamma}(\tilde{X}_A - H_{iA}) = \emptyset_A\). By assumption exists finite \(\Gamma_0\) of \(\Gamma\) such that \(\bigcap_{i \in \Gamma_0}(\tilde{X}_A - H_{iA}) \in I\). Thus \(\tilde{X}_A - \bigcup_{i \in \Gamma_0} H_{iA} \in I\). Consequently, \((X, A, \tau)\) is soft \(I\)-semi compact.

Theorem 4.8 Let \(I\) be soft ideal on a soft topological space \((X, A, \tau)\). Then, semi closed subset of an \(I\)-semi compact soft topological space is a soft \(I\)-semi compact.
Proof Let $F_A$ be a semi closed and $\{G_{i_A} \mid i \in \Gamma\}$ be a semi open cover of $F_A$. Since $\{G_{i_A} \mid i \in \Gamma\}\sqcup(\bar{X}_A - F_A)$ is a soft semi open cover of $F_A$ and $\bar{X}_A$ is $\mathcal{I}$-semi compact soft topological space, hence there exists a finite subcover $\{G_{i_A} \mid i \in \Gamma_0\}\sqcup(\bar{X}_A - F_A)$ of $\bar{X}_A$ such that $\bar{X}_A - [\{G_{i_A} \mid i \in \Gamma_0\}\sqcup(\bar{X}_A - F_A)] \in \mathcal{I}$. Thus $\{G_{i_A} \mid i \in \Gamma_0\}$ is a finite sub cover $F_A$. Therefore $F_A$ is soft $\mathcal{I}$-semi compact.

Acknowledgements. The authors are grateful to the anonymous referee for their careful checking of the details and for their helpful comments that improved this paper.

References


Received: September 3, 2014; Published: October 26, 2014