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Some Results of Studies on the Stability and Controlability Properties of the Switched Systems of Differential Equations

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Abstract

In this paper we provide an overview of the state of tjhe art related to the study of the switched systems of differential equations. Such a panoramic study follows a bidirectional path. The first deals with the study of the considered systems of differential equations with regards the pooling of the same in two clusters, namely linear switched systems dependent on a continuous or discrete variable (such an independent variable usually represents time); and nonlinear switched and stochastic switched systems. The second direction deals with the study of these systems taking into account the more applied techniques to characterize the stability and controllability thereof. We also have presented bi-dimensional switched systems.

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Keywords: Switched systems, H_∞ control, Lyapunov function, linear matrix inequalities

1 Introduction

The concepts of absolute stability, H_∞ control, super stability and others, usually derived for systems of ordinary and partial differential equations, are also applied to the switched systems of differential equations. Such systems are a particular case of hybrid systems, which are applied very broadly in Control Theory. The growing interest in the study of interconnected or switched is justified because many real-world processes (medical, biological, communication networks, etc.), are modeled through them. The next section in this paper is devoted to collect some of the most relevant results obtained in recent years mainly about continuous-time and discrete-time linear switched systems. Some important results on the study of nonlinear and stochastic switched systems are summarized in the third section. In section 4 we expose on the main techniques that have been applied to the study of switched systems. Finally, section 5 deals with a particular case of these systems, two-dimensional switched systems, which have also received special attention recently.

2 Linear continuous-time and discrete-time switched linear systems

In the case of this type of switched systems, we can refer to various publications. For example the work [2], this addresses the identification of linear switched systems from input-output data. The main challenge of this problem is that the data is only available as a mixture of observations generated by a finite set of different linear interacting subsystems, so that there is known, a priori which of the subsystems has generated the data. To overcome this difficulty a dispersed optimization approach is presented. The issue of the identification of each sub-model as a combinatorial optimization problem is formally raised. This is indeed an NP-complete problem that can be relaxed to a minimization (convex) problem with norm l_1. The author presents sufficient conditions for this relaxation to be accurate. The whole identification procedure allows extracting vector parameters (related to different subsystems)

one after another, without any prior pooling of data, according to their respective sub-models generators.
Article [4] addresses the problem of synthesis of the reconstruction of the state for linear switched systems. Based solely on the continuous output, an observer is proposed ensuring the reconstruction of all the state (continuous and discrete) in a finite time. For the design of the observer an exact sliding differentiator mode is used, which allows the convergence of the observation trajectories, in a finite time, to the actual trajectories. The design scheme includes both cases: zero control input and nonzero control input. While in [5] the authors address the problem of state observation for a class of switched linear systems with unknown inputs. The higher order observer proposed in sliding mode, provides a finite estimate of convergence for the continuous state vector of the system, despite the presence of unknown inputs. The design procedure, which assumes knowledge of the discrete state switched system is based on the principles of decoupling the disturbance and output hierarchical type observer. In order to confront the switching nature of the plant under observation, intentionall jumps (discontinuities) into the state space observer are forced. Implementing an additional observer allows reconstruction of unknown inputs, which can be important in the context of fault detection.
In [17] the problem of fault estimation and accommodation for a class of switched systems with time-varying delay is considered. An adaptive fault estimation algorithm is proposed, that can also be estimated as a constant or time-varying failure. Moreover, a delay dependent criterion is obtained in order to reduce the conservatism in the design of the algorithm for errors estimation. It is designed, on the basis of the errors estimation, a fault tolerant controller based on an observer, to ensure the stability of the closed loop system.
In the work [10] the global exponential stability to a class of neutral switched systems with delay of state varying in a time interval and two kinds of disturbances is investigated. Criteria based on linear matrix inequalities, dependent and non-dependent on delay to ensure exponential stability of the considered systems for any signal switching are proposed. One Razumikhin type approach and the Leibniz-Newton formula to find the stability conditions are applied. In this paper structured and unstructured uncertainties are considered. In the publication [12] the problem on optimal switching for linear switched systems is addressed. The uniqueness of the approach applied here, lies in describing the switching action by multiple control inputs. This allows one to include the switched system in a larger family of systems and apply the Pontryagin Minimum Principle for solving the optimal control problem. This approach does not impose any restriction on the switching sequence or on the number of such sequences. This is in contrast with the search of algorithms based on a fixed number of switching sequences that is set a priori. In this new approach,

the optimal solution can be determined by solving a two point boundary value problem.
In the article [14] the Small Gain Theorem for addressing linear switched systems is generalized. The authors develop a switching strategy that can guarantee the stability of the feedback interconnection of a linear switched subsystem and a linear perturbation with bounded norm. A new version of the Small Gain Theorem is especially important to resolve two different problems: the first is the possibility of improving the robustness of control feedback system properly handling the connections within a given set of linear regulators. The second problem is the design of a switching stabilization strategy for switched time-delay systems. These two cases are considered: state and switching output feedback. It is notable that, as sufficient conditions for the existence of a solution is of the delay-dependent type, they are less conservative compared to previous results reported in the literature.
In the article [20] the problem of optimal control for discrete-time switched systems is considered. This problem is formulated as an optimization problem involving both continuous variables and discrete variables, which can be transformed to a discrete optimization problem. A metric is introduced in the space of switching sequences, and a complete discrete appropriate function is constructed. Then an algorithm is developed that combines the method of complete discrete function and descent method to find the solution. In the work [24] the stability analysis and the design of switched system consistent of linear descriptor systems is developed, having the same descriptor matrix. When all descriptor systems are stable, it is shown that if the describer matrix and all the system matrices are commutative in pairs, then the switched system is stable under arbitrary switching . This is an extension of the well-known result of the authors K. S. Narendra and Balakrishnan J. (see [58]) for linear switched systems with phase space models for descriptor switched systems. Under the same condition of switching it is also demonstrated that in the case when all descriptor systems are unstable, if there exists a stable convex combination for unstable descriptor systems, then it is possible to establish a class of switching laws which stabilizes the system. Finally a brief discussion on how to obtain the stable convex combination by efficiently solving a matrix inequality, and the relaxation of switching condition for the stabilization of the connection system is made.
In the work [28] the singular value decomposition for a class of time-dependent linear systems is considered. The class considered in this paper describes linear switched systems controlled by time. On basis of an adequate description of input-output data, the method for the calculation of singular values and singular vectors is obtained. The singular value decomposition allows one to characterize the input and output dominant signals key, using singular vectors that form orthogonal systems in input-output space. This decomposition is

then applied to the switched linear systems to improve transient response. In [30] a design problem of robust stabilization for an uncertain class of switched linear systems through controllers of variable gain is discussed. Uncertainties are taken in to consideration and are assumed to satisfy the condition of accordingly, and switching between the subsystems is determined by using the nominal system. The proposed controller for the robust stabilization consists of a commutation rule, a state feedback law with a matrix of constant gain, and a state feedback law with a matrix of variable gain in tune with adjustable parameters. Here a method is presented in order to design the general switching rule, the feedback gain matrices and the adjusting law of the adjustable parameter. Also in [32] a weak characterization of distinguishable of two controlled linear systems is obtained. Furthermore, the observability of a linear system which is controlled by special switching is considered, and the necessary and sufficient conditions for observability are obtained too.

In the article [34] the author applies the concept of super-stability of linear switched systems, as a particular case of time-dependent linear systems. He introduces a generalized concept of super-stability, applied to complex matrices, and extended super-stability, in order to get a new result that guarantees asymptotic stability of a switched system under arbitrary switching. This work also discusses the relationship between sets of extended super-stable matrices, and sets of stable and simultaneously triangulable matrices. Ibeas shows that the stable triangulable matrices constitute a proper subset of the set of extended super-stable matrices, noting that the stability result presented is a generalization of previous stability theorems, well known for broader class of switched dynamic systems.

In the paper [66] a suitable type La Salle principle is applied to continuous-time linear switched systems to characterize invariant sets and their associated laws of change. An algorithm to determine algebraically these invariants is also proposed. The main novelty of this approach is that it does not require any average dwell time condition on switching laws. As it is not focused on restricted classes of control, it can describe the asymptotic properties of the switched systems considered.

In [38] the efforts are devoted to the derivation of stability delay-dependent criteria, under a switching law promoted by the state for switched delay-dependent systems. A linear state transformation is introduced to transfer to the switched system with time delay, and the Lyapunov-Krasovskii functional is applied to analyze the stability of these systems. This method can be applied to all cases in which the individual switched systems are unstable.

In the case of paper [39], a concept of L_2-gain vector for switched systems is presented. Each subsystem does not necessarily have L_2-gain in the classic sense, but is supposed to have individual L_2-gain during any interval of time when the subsystem is active. Then the stability is obtained from this vector

of L_2- gain under some constraints for inactive storage functions. Asymptotic stability is also achieved if the observability in the norm property in a small time is also imposed. A theorem of small gain for feedback switched systems with L_2-gain vector guaranteed for each subsystem also is proved. The small gain condition is given in terms of the L_2-gains of the coupled active subsystems and the changes of the inactive storage functions. A design method of the switching law is also obtained to achieve the L_2- gain vector.
The model reduction is also addressed in [47], in this case for a class of linear discrete-time switched systems with polytopic uncertainty. This paper initially explores the stability criterion for general switched discrete-time systems, and then the approach of dependence for the considered systems in the model of reduction of the solution is introduced. A reduced-order model is constructed and the corresponding existence conditions are derived through linear matrix inequalities of the formulation type. Admissible switching signals and the desired matrices of the reduced model are consistently obtained from such conditions so that the system for errors of the resulting model is robust, exponentially stable and has an exponential H_∞ performance.
A powerful approach for the analysis of the stability of the switched continuous-time systems is based on the use of Optimal Control Theory to characterize the "most unstable" switching law. This reduces the problem of the stability determination under arbitrary switching to analyzing the stability for specific the "most unstable" switching law. For discrete-time switched systems, the variational approach has received considerably less attention. This approach is based on the use of a necessary optimality condition of first order as a maximum principle, and usually this is not sufficient to fully characterize the "most unstable" switching law. In the paper [56] a simple and autonomous derivation of a necessary optimality condition of second order, for discrete-time bilinear control systems is proved. This provides new information that can be derived using a maximum principle of the first order. Likewise several applications of this principle to analyze the stability of discrete-time linear switched systems of the second order are demonstrated.
In [59] the stabilization of linear commutated systems with bounded inputs is considered. The authors design admissible evaluations of linear states and the switching law that has a minimum dwell time for the stabilized system. Initially, for each subsystem with restricted entries a linear state feedback stabilizer and an invariant set of the closed loop system are simultaneously constructed, so that input constraints are satisfied if and only if the states of the closed loop system are provided in this set. Then, by constructing a quadratic Lyapunov function for each closed loop subsystem, the minimum dwell time is deducted and switching strategy based on this time, to ensure the stability of the system are presented.
The authors Qing-Kui, Jun Zhao, Xiang-Jie and Dimirovski in [65], investi-

gate the problem of tracking control based on an observer for linear switched systems with variable time delay. They consider the possibility of designing switching control laws based on measurable outputs, when states are not available. For this they use the switching method based on hysteresis of a single Lyapunov-Krasovskii functional for stability analysis and controller design. When introducing the system descriptor approach, the consideration of mixed-mode is avoided, since $x(t)$ can be viewed as a state variable, and therefore it is not necessary substituting this function wherever it be, on the right-hand side of the system. The formula of variation of constants is used to overcome the difficulties caused for the estimation error and exotic disturbance. The scheme used, free of weighting matrix, facilitates the stability analysis and control of the synthesis.

Referring to switched linear models, using the notion of a switching interval, in [67] the posterior distributions of all parameters in the model are obtained. This includes the start and end points of the switching interval and the parameters that determine the nature of the switching. This is done by studying three cases of the problem: a permanent switching on a finite interval, a permanent switching in an infinite interval, and a temporary switching in a finite interval. The analysis is general in the sense that it can be applied to any problem that can be formulated as a linear model.

The work [69] is devoted to the study of controllability and observability for impulse and linear switched systems. First, some new results are derived on the variation of parameters for these systems. Thus, less conservative sufficient conditions, and conditions for controllability and observability of the state of such systems are established; and for systems without impulsive input control, necessary and sufficient conditions for controllability and observability are obtained. In the work [70] global stabilization method is proposed for a class of plane systems with input constraints of saturation type using state feedback and a switching strategy. The effectiveness of the method for analyzing the characteristics of the trajectories of second-order linear systems with input saturation was proved.

In real systems there are physical constraints such as voltage limitations imposed on the switches, or restriction state due to system structure. The violation of these restrictions may cause instability or, in the worst case, cause bad operation. In [71] an algorithm for multimodal controllers in switching, using the method of predictive control of the model is proposed. The problem of robust stability of uncertain systems with multiple delays of switching, resulting from failure of the switch, is discussed in [74]. Based on the approach of average dwell time a suitable set of switching signals in this paper is designed, using the relation between the total activation time for the stable and the unstable subsystem. First, it is shown that the resulting closed loop system is robust and exponentially stable for some allowable upper bound of delays, if the nominal system with zero delay is exponentially stable under these switching laws. It is proved particularly that a maximum upper bound to the delay can be obtained from the linear matrix inequality. Also in [76] the problem of control of switched linear systems with input saturation is considered. Here is presented a synthesis method based on the minimum dwell time switching together with saturated feedbacks. Sufficient conditions for stability in terms of linear matrix inequalities are also proposed. Moreover, in [79] the questions about stability in finite time analysis, and the stabilization of a linear continuous-time connection system are addressed. The authors propose several sufficient conditions for finite time boundedness and the stability of the system. By comparing the obtained results, it is shown that while most known is the information on the switching signal, fewer conservatives results can be derived. Then, based on the results of stability analysis of finite-time controllers, static state and dynamic output feedback controllers for linear switched finite-time systems, which stabilize linear connection systems are designed.

In [85] a recent study on the practical stabilization of discrete-time switched systems it is continued. In this paper, after briefly reviewing some notions of practical stabilization, a sufficient condition is proved for the asymptotic practical stabilization. Then attention is focused on a class of switched discrete-time systems, namely affine switched systems, and an approach to estimate the minimum bound for practical stabilization it is presented. Based on this approach, several new sufficient conditions are also presented for asymptotic global practical stabilization of one class of such systems. As such classes of systems of this type are often derived by polling continuous-time switched systems, some preliminary results on the relationship between affine switched discrete-time and continuous-time systems are finally presented.

The article [88] deals with the problem of the existence of a common positive linear Lyapunov function for positive switched linear systems with stable and switchable, two by two, subsystems. Here three families of such systems are considered: composed of continuous-time subsystems only; of discrete-time subsystems only; and mixed continuous-time and discrete-time subsystems, respectively. It is shown that a positive linear common Lyapunov function can always be constructed for the underlying system, regardless of whether subsystems are discrete or continuous-time or of the mixed type. First the case when the number of subsystems is two is considered, then the obtained result spans for the general case.

An interesting result for linear discrete-time switched systems is [91]; in this paper the characterization of the asymptotic stability for this class of systems was studied. First the system dynamics to a symbolic configuration under the framework of symbolic topology is translated. Then, using ergodic measure theory, an estimation of the lower bound of the Hausdorff dimension of the set of asymptotically stable sequences is obtained. It is demonstrated in the paper that the Hausdorff dimension of the set of asymptotically stable switching sequences is positive if and only if the corresponding linear switched system have at least one asymptotically stable switching sequence. This result reveals an underlying fundamental principle: under condition that the switching to be arbitrary, a linear switched system or possesses a countless number of asymptotically stable switching sequences or does not have any of them. The authors also obtained frequency indices and densities to identify sequences of asymptotically stable switched system.

In [94] the equivalence between several issues: switched convergence, asymptotic stability and exponential stabilization for linear switched systems on which no force acts is established; and implications for the problem of optimal switching in inifinite time are discussed. The author shows that for a general cost function under mild assumptions, the finiteness of the optimal cost is equivalent to the asymptotic stability of the switched linear system. Finally, the equality between the optimal costs for the switched system and the corresponding relaxed differential inclusion is proved. The same author, in the work [95] addresses some issues of robustness for a class of linear interconnected systems with nonlinear disturbance; he illustrates that the switching law based on state-space-partition- known can lead to chattering in the case that systems are subjected to disturbances. To overcome this, he proposes a change in state feedback law to a set of level different from zero. The author also proves that this new switching law can prevent chattering. A change law based in an observer with good robustness performance in order to direct the switched system towards stability is also proposed.

Sun Zhendong in [96] studied the phenomenon of stabilization of switched linear discrete-time systems. Here the author proposes, based on the abstraction-aggregation methodology, a feedback state law, for switching, of the "smart trajectory" type, which is a concatenation of feedback state for a finite set of switching trajectories each defined over a finite interval of time. The author proved that this set is universal in the sense that any stabilizable linear switched system admits a stabilizable switching law. Then a computer procedure is developed to calculate the stabilizable switching law in this set.

The traditional approach to design a controller for each subsystem of a switched system will increase the complexity of controller, therefore, at [97] sufficient conditions for design an uniform feedback controller for linear switched systems are obtained, and this common controller can be used for all subsystems. Thus, in this paper the problem of stabilizing the output for a particular class of linear switched systems under this uniform feedback controller has been studied.

Other papers devoted to studying linear switched systems are, for example: [73], [61], [89], [72], [53], [31], [45], [34], [19], [29], [40], [57], [68], [80], [83], [84], [90], [98]. These reports will be tackled in subsequent sections.

3 Nonlinear and stochastic switched systems

Nonlinear and stochastic switched systems have been less covered by the scientific community, as their characteristics, together with the switching itself, makes it much more complex and inaccessible. However, there have also been dedicated research efforts. In this sense we can highlight the paper [1], which focuses in the regularization technique of the proximal point for a class of optimal control processes, governed by affine switched systems. Here switching of the control systems described by nonlinear ordinary differential equations, which are affine respect to the input is considered. The affine structure of dynamic models in this study establishes some continuity/proximity properties, and permits to consider these models as a convex control systems. In this paper it is proved that, for some classes of cost functional, the associated optimal control problem corresponds to a conventional convex optimization problem on a suitable Hilbert space. The latter problem can be reliably solved using standard optimization algorithms and consistent first-order regularization schemes. A numerical conceptual approach based on the classic gradient type method and proximal point techniques, is particularly proposed.

In the article [9] the design of the switching signal (switching law) for robust global exponential stability of nonlinear switched systems is investigated. Here criteria based on linear matrix inequalities and dependent on delay, to design the switching signal and ensure global exponential stability are proposed. Unweighted matrices and nonnegative inequality approaches are used to arrive at less conservative stability results.

Filipovic, in [22] proposes a method for the stability analysis of deterministic switched systems. The author introduces two examples of motivation (the unholonomic system and the limited pendulum). The finite collection of models comprises nonlinear models, and the switching sequence is arbitrary. It is assumed that there is no jump in status at the instant of change, and there is no periodic behavior, ie there is a finite number of switches in each bounded interval. For the analysis of such systems multiple Lyapunov functions are used, and the global exponential stability is demonstrated. The author shows that the exponentially stable equilibrium of the systems is relevant for practice, because such systems are robust against disturbances. Methods based on the Lyapunov stability theorem, to study the stability and design of the switching law for nonlinear fuzzy switched systems of the Takagi-Sugeno type, with switching caused by the state, are presented in [37]. Furthermore, these methods can be applied to cases in which all the individual systems are unstable. Parallel distributed compensation is used to design fuzzy controllers based on the diffuse models of the Takagi-Sugeno type. The stability analysis is reduced to a problem of finding a common Lyapunov function for a set of linear matrix inequalities.

In the work [42] a new method for dealing with the problem of robust H_∞ control for a class of switched systems with nonlinear uncertainty, using a sliding integral mode of control is developed. A robust H_∞ integral sliding surface is constructed, so that the sliding mode is stable and robust, with a prescribed disturbance attenuation level, for a class of switching signals with average dwell time. Furthermore, variable structure controllers to keep the switched system (your state) on the sliding surface from the initial moment are designed.
The article [50] is dedicated to the development of a decentralized approach to the robust stability and the problem of stabilization of a class of continuous-time switched systems with nonlinearities and with bounded uncertainties within a cone. This class consists of nominally linear coupled subsystems with unknown state but bounded variable time delay. The authors show that multiple schemes of switching ontrollers provide an effective and powerful mechanism to deal with complex systems. A decentralized structure dependent on delay, which ensures global asymptotic stability with local attenuation of the disturbance at subsystem level is developed. A decentralized control switching scheme based on state feedback, to ensure global stabilization of the system with bounded L_2 performance is then thconstructed.
The stability properties of singularly perturbed switched systems with time delay and impulsive effects are studied in [54]. In this work it is assumed that such systems are constituted by both unstable and stable subsystems. Some stability criteria are established by using the technique of multiple Lyapunov functions and the average dwell time approach. The results show that impulses contribute to the goal of obtaining stability properties even when the system consists only of unstable subsystems.
A polynomial approach is presented in [55] to deal with the stability analysis of nonlinear switched systems under arbitrary switching, using dissipation inequalities. It is shown that a representation of the original problem as a polynomial continuous system allows one to use the dissipation inequalities for the stability analysis of polynomial systems. With this method, and from a theoretical point of view, an alternative way to find a common Lyapunov function for switched nonlinear systems is provided. The article [60] presents an approach to the reconstruction of the robust state to a class of non-linear switched systems affected by uncertainties in the model. Under the assumption that the continuous state is available for measurement, an approach is presented based on concepts and methodologies derived from the theory of sliding mode control. With measurements of the state free from noise, the time required to reconstruct the discrete state, after a transition, can be made arbitrarily small by sufficiently increasing a certain observed tuning parameter. The article [63] addresses the stabilization bounded input-bounded output type of switched piecewise linear systems in the presence of delays and nonlinear disturbances. On the basis of a quadratic piecewise Lyapunov function,

the main contribution of this work is to obtain sufficient conditions of stabilization bounded input-bounded output type as an algebraic matrix Riccati equation. Robust quadratic stability for such systems is also discussed.

A new analysis of the stability of impulsive switched systems with time delay, with subsystem is not necessarily stable, is presented in the work [75]. A sufficient condition for the uniform asymptotic stability for nonlinear impulsive switched systems is obtained. Using these results and the minimum (maximum) time of permanence, an easily verifiable condition on uniform asymptotic stability for linear impulsive switched systems with time delay is derived. In this paper the synthesis of control is also is discussed.

Finite time stability refers to, not limiting the system for a finite fixed time interval. For switched systems the stability property in finite time can be affected significantly by the switching behavior; however, this has not been considered in most of the investigations. In the work [78] stability analysis in finite time and stabilization problems of switched nonlinear discrete-time systems are addressed. First sufficient conditions to ensure the existence of a class of nonlinear discrete-time switched systems, subject to a disturbance in finite time, bounded in norm and under arbitrary switching are given. Below, these results are extended to switched nonlinear discrete systems, H_∞ unbounded in finite time. Finally, based on the results for unboundedness in finite time, a feedback state controller for the H_∞ stabilization of a switched, nonlinear, discrete-time system, in finite time is designed.

In the article [86] the problem of stabilization of a class of nonlinear time delay switched systems is investigated. Using the Principal Differential Value Theorem, nonlinear switched systems become linear systems dependent on a parameter. By using the multiple Lyapunov function approach and the principle of convexity, and by means of observer-based output feedback, a sufficient condition for stabilization is proposed for the original system, which is expressed in terms of linear matrix inequalities. In addition, the control method is extended to a class of nonlinear switched systems with uncertainties bounded in the norm. A new sufficient condition is proposed, which ensures that the class of undetermined switched system to be asymptotically stabilizable.

In [87] the matter of stabilization of switched nonlinear systems, with passive and not passive subsystems, is discussed. For any given average dwell time, if the rate of the total activation time of passive subsystems, in some interval time, is greater than any given constant, no matter how small this is, then the feedback controllers can be designated with respect to the mean dwell time and the rate of activation time of passive subsystems, to achieve exponential stabilization.

Other works in which nonlinear switched systems include are studied: [92], [62], [3], [33], [81], [99].

Researchers Feng Wei, Tian Jie and Zhao Ping study in the article [21] dis-

cuss a class of stochastic switched systems, instant stability and the stability of the test trajectory, respectively, and they make two main contributions. First, based on accurate estimates of the powers of a solution of special non-switched stochastic system using the concepts of a Lyapunov function and describing the switching laws with the average dwell time and the subsystems, they proved three sufficiency theorems for p-th instantaneous stability for stochastic switched systems. And secondly, for the stability of the test trajectory of such systems they obtain two sufficiency theorems based on the results of the p-th instant stability, for $p > 2$ and $p = 2$, respectively.
In [77] a quadratic optimal control problem for discrete-time linear stochastic switched systems, with no autonomous subsystems disturbed by gaussian random noise is studied. The objective is to design, jointly, a deterministic switching sequence and continuous feedback law in order to reduce the expectation of a quadratic cost function of finite horizon. Both the function and the optimal control strategy are characterized analytically. In the paper a numerical relaxation framework is developed to efficiently calculate a control strategy with a guaranteed performance of upper bound. It is also shown that by choosing the relaxation parameter sufficiently small, the performance of the resulting control strategy can be made arbitrarily close to the optimal one.
And finally, [82] has shown that it is not true that the conjecture that stability of periodic switching implies absolute asymptotic stability of infinite random products of a finite set of square matrices. This paper shows that this conjecture is valid in terms of Markov probabilities. In particular, let be $S_k \in \mathbb{C}^{n\times n}; 1 \leq k \leq K$, K matrices given arbitrarily, and $\sum_K = \{(k_j)_{j=1}^{+\infty} \setminus 1 \leq k_j \leq K$ for each $j \geq 1\}$, where $n, K \geq 2$. They studied the exponential stability of the following discrete dynamic switching time S: $x_j = S_{k_j} \cdots S_{k_1} x_0, j \geq 1$ and $x_0 \in \mathbb{C}^n$, where $(k_j)_{j=1}^{+\infty} \in \sum_K$ may be a sequence of arbitrary commutations. For a row probability vector $\mathbf{p} = (p_1 \cdots p_K) \in \mathbb{R}^K$ and an irreducible Markov transition matrix $P\ inR^{K\times K}$ with $\mathbf{p}^P = \mathbf{p}$, is denoted by $\mu_{\mathbf{p}}, P$ the Markov probability on $\sum_K$ corresponding to $(\mathbf{p}, P)$. Using symbolic dynamics and ergodic-theoretical approaches, we show that if S has periodically switched stability then, (i) S is surely almost-$\mu_{\mathbf{p}}, P$ exponentially stable; (ii) the set of stable switching sequences $(k_j)_{j=1}^{+\infty} \in \sum_K$ has the same Hausdorff dimension of $\sum_K$. Therefore, the periodic switching stability dynamic linear discrete-time switching means that the system is exponentially stable for almost all switching sequences.

4 On the main techniques that have been applied to the study of switched systems

To study switching systems from a qualitative point of view various techniques have been applied, such as: the singular value decomposition, the minimum principle of Pontryagin, the Ricatti equation, delay dependence, H_∞ control, etc. However, to treat interconnected linear systems the most used has been the combination of three techniques: building a proper Lyapunov function, in the variant that is best suited to each case; the averaging time of the system in a given state and conditions in the form of linear matrix inequalities.

Such tools have also been applied to study nonlinear switched systems. For example, in [81] the problem of designing a reduced-order robust fault detection filter for a class of switched nonlinear systems with time-varying delay is investigated. It is assumed that the nonlinearities satisfy the global Lipschitz conditions. By using a generalized form, based on an observer of the fault detection filter, as a generator of waste, the problem is formulated in the framework of H_∞ filtering. Based on the approach of average dwell time, and on the technique of the Lyapunov functional, a sufficient delay dependent criterion on the existence of a robust filter for the failure detection to ensure the exponential stability and prescribed L_2-gain bound for residual system error is obtained. In addition, the parameters of a permissible filter are obtained from the solution of a convex optimization problem.

Moreover, the authors Huang, Venkatesh, Xiang C and Lee in [33] studied the L_2-stability for systems with a single input and a single output (SISO systems) with periodical and non-periodical switching gains, described by integral equations that can be transformed to the standard form of differential equations. For the latter, the literature on stability mainly reflects the application of quadratic forms as Lyapunov function candidates, leading generally to conservative results. Exceptions are some recent results, especially for linear second-order differential equations, obtained by control of the trajectory or optimization to reach the worst case gain in the switching sequence. By contrast, in this paper a different Lyapunov framework is used to derive the L_2-stability condition for a class of SISO (linear and nonlinear) systems, in integral form, with monotonic, odd monotone and relaxed monotonous non linearity; and, in each case, with periodical and non-periodical switching gains. The results obtained, in the frequency domain, are reminiscent of, firstly, the Nyquist criterion for feedback non time-dependent systems; and secondly, the Popov criteria for feedback nonlinear time invariant systems with Lure type nonlinearity. Although the coincidence with some recent results reported in the literature, for periodic gains, the results of these researchers have been derived essentially independently of the Popov frame, and are different for certain classes of nonlinearities. Some questions kept open with respect to, for example, the synthesis of the multipliers and the numeric interpretation of the results. Apart from the novelty of the results as are applied to the problem of dwell time, they show an interesting phenomenon of switching systems: fast switching can lead to stability, which provides an alternative framework for the analysis of vibrational stability.

In work [99] the problem of reliable robust control for a class of nonlinear switched systems with time delay and switch failures under asynchronous switching is investigated. The authors propose a type of method for reliable controller design when the switching instants of the controller experiment delays with respect to the system; and use the dwell time approach for the stability analysis. Furthermore, they formulate sufficient conditions for the existence of a reliable controller, in terms of a set of linear matrix inequalities. Then they extend the proposed method to consider switched systems with delay and Lipschitz nonlinearities, and structured uncertainty.

Returning to the linear case in [6] sufficient conditions for stabilization of fuzzy Takagi-Sugeno type linear switching systems, of discrete-time is presented. These conditions are obtained when the control state feedback law is used. The results obtained in this paper are formulated in terms of linear matrix inequalities.

In the publication [7] the problem of design of failure detection filters for the switching discrete-time systems with delays in a variable interval time is investigated. The waste generator is built based on filter detection failure dependent of the mode, that is, the failure detection filter designed is also a switching system. Through building a new switched Lyapunov functional, a new criterion for the residual system is obtained. Based on this, a sufficient condition is established for the existence of the described filter in terms of linear matrix inequalities. This filter guarantees that the difference between the residual signal and the failure signal generated are to be as small as possible.

The researchers Cai and Mijanovic consider linear continuous-time interconnected systems, associated with linear state restoration during mode changes; the so called linear hybrid systems that can usually be found in switching control systems based on the absence of discontinuity in the transfers during the control of switching. They use an approach of multiple Lyapunov functions, to develop constructive tools for stability analysis of linear hybrid systems. In particular, they obtain a procedure based on linear matrix inequalities to calculate upper bounds of the dwell time for the uniform global exponential stability of the linear hybrid systems, and apply such techniques to a cooling process, which is regulated by multiple proportional integral switched controllers through undisturbed transfer. Their results are presented in [8].

Also, in [13] the problem of simultaneous failure detection and control for linear continuous-time switching systems is approached. Here a H_1 formulation of this type of problem, using a dynamic observer, is presented. In essence, a single unit dependent on modes of the system called detector/controller is designed, in which the detector is a dynamic observer, and the feedback controller is based on the dynamic state observer. It is shown that this observer can be used effectively to address the drawbacks of current methods of design for this type of problem. The authors use a characterization of extended linear matrix type inequality, to reduce the conservatism by introducing additional matrix variables, in order to remove the coupling of the Lyapunov matrices with the matrices of the system. In fact, the idea presented in this document is based on the time average dwell time and approaches of reduction of the conservatism and applying the advantages of the dynamic observer, which leads to some sufficient conditions to solubility of the problem of simultaneous failure detection and control, in terms of viable condition of linear matrix type inequalities.

The document [16] is focused on the problem of fault tolerant active control for switched systems with time delay. Here an adaptation algorithm for estimating failures using the diagnostic observer that can estimate the detection signal quickly and accurately is proposed. In addition, a delay-dependent criterion is obtained, in order to reduce the conservatism of the adaptive observer design. Relying on the fault estimation information an observer-based fault tolerance is designed; to ensure the stability of the closed loop system controller. Sufficient conditions are derived for the existence of an adaptive observer and a fault tolerant controller in terms of linear matrix inequalities.

In [18] criteria for the exponential stability of delay-dependent switching systems are presented, consisting of a family of stable subsystems and a family of unstable subsystems, with variable delay time intervals. Two cases are considered with respect to the delay: one in which the delay time-dependent function is differentiable and bounded; and the other that this function is continuous and bounded. It is very difficult to analyze the stability of such systems, due to the existence of delay and unstable subsystems. By introducing some free weighting matrices, a new Lyapunov-Krasovskii functional is built. Taking advantage of the benefits of average dwell time technique, not only overcomes this difficulty, but also sufficient conditions for such criteria are derived, which are formulated in terms of linear matrix inequalities. While in [25] the problem of stability analysis for linear descriptor interconnected systems is studied. Assuming that all subsystems are stable and there is no impulse in the switching instants, the authors establish a new switching condition by pairs, under which the system is stable. Also shown is that when given switching conditions hold, there is a common quadratic Lyapunov function for the subsystems. These results are natural and significant extensions of the existing results for switched systems in the representation of state space.

Also, [26] investigates the attenuation properties of the disturbance, in the sense of definitive uniform boundedness, for a class of nonlinear switched sys-

tems with parametric uncertainties and external disturbances. The goal is to characterize the conditions in the switching system that can achieve a finite level of disturbance attenuation. Firstly arbitrary switching signals are considered, and a necessary and sufficient condition is given. Secondly conditions on how to restrict the switching signals to achieve finite levels of disturbance attenuation are investigated. Two cases are considered here, depending on whether all the subsystems are uniformly bounded or not. Both switched systems of discrete-time and continuous-time are considered, and the techniques used are based on multiple polyhedral Lyapunov functions and extensions.
A new method of calculating the invariant set for switched linear continuous-time systems, with arbitrary switching and disturbance is presented in [27]. The proposed method is based on the existence of a transformation placing all the matrices of the linear switched system in a convenient way to satisfy certain properties. The method, which provides final bounds and an invariant set with polyhedral shape and/or ellipsoidal/mixed polyhedral sets, is fully systematic once the cited transformation is obtained, and provides a new sufficient condition for the practice stability. The authors show that the processing required by the method can be easily found in the known case, where the subsystem matrices generate solvable Lie algebra, and provides an algorithm to seek such transformation in the general case. An example comparing the bounds obtained by the proposed method with those obtained from a common quadratic Lyapunov function calculated through linear matrix inequalities, shows a clear advantage of the new method, in some cases.
In [36] the invariant principle of La Salle for switched linear systems is studied. Unlike most of the existing results, in which each switching mode on the system must be asymptotically stable, in this case the switching modes are only allowed to be Lyapunov stable. Under certain assumptions of ergodicity, an extension of the La Salle invariance principle is proposed for the global asymptotic stability of switched linear systems, under condition that the kernels of the derivatives of a certain common quadratic Lyapunov function, with respect to the switching modes, are disjoint (except at the origin). Moreover, in the article [43] the problem about disturbance tolerance/rejection type of a family of linear switched systems subject to saturation of switching and L_2 disturbances is considered. Here conditions in terms of linear and bilinear matrix inequalities are established, for a given set of linear feedback gains, a given connection scheme and a given bound for the norm L_2 of the disturbance; under which the resulting switched system has the property that the trajectories that begin in a bounded set in phase space remain within the set or within a bounded higher set. With these conditions satisfied the problems of evaluating the closed loop system capacity to tolerate/reject the disturbance; and the design of feedback gain and the connection scheme, can be formulated and solved as optimization problems with restrictions. Disturbance tolerance

is taken as the greater bound on the disturbances for which the trajectories for a given set remain bounded. The rejection of the disturbance is measured by the L_2 gain restricted to the set of tolerance of the perturbations. In the case when all the systems are identical, the switched system is reduced to a single system under a law of feedback switching. And it is shown that this simple system has stronger capability for tolerance/ rejection of disturbances, than can be attained by the simple linear feedback law.

The article [44] is dedicated to the sliding control mode of a switched continuous-time system with variable time delay in its state. Using the average dwell time approach and the Lyapunov piecewise function technique here for the first time a sufficient condition to ensure the exponential stability of the unforced system is proposed, with the estimation of the attenuation given explicitly. A sufficient condition for the existence of a dynamical sliding mode of the reduced order, and an explicit parameterization of the surface of the desired sliding is also obtained. These conditions are solved using the complementary linearization cone method; then an adaptive sliding mode controller is designed for the movement of arrival, so that the trajectories of the resulting closed-loop system can be driven on a prescribed sliding surface, and these trajectories remain there throughout the subsequent instants of time. All conditions obtained in this work are delay dependent.

The problem of exponential robust admissibility to a class of singular continuous-time indefinite switched systems, with variable delay intervals in time, is investigated in [46]. By defining a properly built Lyapunov function depending on decay rate; and using the average dwell time approach, a sufficient condition is derived, depending on delay range, in order for the nominal system to be regular, impulse-free and exponentially stable. The results obtained provide a solution to one of the basic problems for singular continuous-time switched systems with delay in time, namely, to identify a switching signal for which the singular continuous-time switched system with time delay is regular, impulse-free and exponentially stable. In [48] the problem of asynchronous switching control for a class of nonlinear switched systems with average dwell time for both continuous-time and discrete-time contexts is investigated. The so-called asynchronous switching means that the switching between candidate controllers and system modes are asynchronous. By allowing the Lyapunov type function to grow during the time of performance of active subsystems, the stability results are first obtained extended to switched systems with average dwell time, in a non-linear environment. Later the stabilization control problem is solved in an asynchronous way for linear switching cases. Given the scale of growth and the scale of decay of the Lyapunov type function, and the maximum delay of asynchronous switching, the minimum average dwell time for admissible switching signals and the corresponding controller gains is obtained.

In [49] the delay depending performance and the synthesis control for a class of hybrid discrete-time linear systems with delays variables over time is investigated. Here, under arbitrary switching, hybrid Lyapunov-Krasovskii functionals are employed to establish the desired results. A generalized H_2 approach is adopted, and a new parameterized characterization with linear matrix inequalities is developed to ensure delay-dependent asymptotic stability. The aim in [51] is to design a robust connection control for a class of continuous-time systems subject to linear fractional uncertainty and time delays of the interval type. The controller is based on state feedback, the stability conditions in the form of linear matrix inequalities are derived with the use of an improved Lyapunov-Krasovskii functional. On the other hand, from the minimizing of an ensured cost function, the commutation law, as well as the state feedback gains are determined. In [52] the H_2 generalized control problem is investigated for a class of linear switched discrete-time systems with unknown constant delays. There new linear matrix inequalities are established, based on the Lyapunov-Krasovskii switching functional, to ensure the asymptotic stability and the generalized H_2 performance. In the article the generalized H_2 state feedback and dynamic feedback output controllers are designed.
In the work [64] the problem of tracking control for switched and linear time-dependent delay systems, with stabilizable and no-stabilizable subsystems is investigated. Sufficient conditions are developed for solubility of the posed problem. This problem is soluble if stabilizable and no-stabilizable subsystems satisfy certain conditions and admissible switching law between them. In the solutions the average dwell time approach and methods based on the piecewise Lyapunov functional, for the stability analysis and controller design are applied. By introducing the integral controllers and a free-weighting matrix scheme, some restrictive assumptions for the switched systems are avoided.
Researchers Xiangze Lin, Haibo Du and Li Shihua; in the work [80], study the boundedness in finite time and the weighted L_2-gain in finite time, for a class of switched systems with delay and time-varying exogenous perturbations. They applied the average dwell time technique in order to provide sufficient conditions that ensure that linear switched system with time delay be bounded in finite time and have weighted L_2-gain in finite time. These conditions are delay dependent and are given in terms of linear matrix inequalities.
Sufficient conditions for the exponential stability of thr switched systems dependent on delay, and sufficient conditions for the exponential robust stability of the switched systems with uncertainty and two type of delay, using the average dwell time method and free weighting matrix, are presented in [93]. Here the interaction between different delay times is considered. Sufficient conditions are derived without the need for each subsystem to be stable, and these are expressed as linear matrix inequalities that can be resolved easily. Moreover, in the work [98] the problem of the L_2-L_∞ robust filtering for switched

continuous-time systems under asynchronous switching is investigated. When there is such kind of switching between the filter and the system, sufficient conditions are derived based on the average dwell time approach, for the existence of a linear filter to ensure that the filtering error of the system is to be exponentially stable, whith established weighted L_2- L_∞ yield for the switched systems; and filtering parameters can be obtained by solving a set of matrix inequalities.

Regarding the application of H_∞ control, we can cite papers [92], [42], [78], [81], which have already been considered in the previous section.

Moreover, in [15] the problem of H_∞ filtering for linear discrete-time switched systems under arbitrary switching laws is studied. The authors offer new sufficient conditions for the solubility of the problem by using quadratic switched Lyapunov functions. Based on the Finsler's lemma, two sets of slack variables with special structure to provide additional degrees of freedom in optimization of the guaranteed H_∞ performance are introduced. Comparing with existing methods, the one proposed here has better performance and less conservatism.

In the article [100] the problem of H_∞ exponential reduction of switched system models with delay, for continuous-time, under switching signals with average dwell time is studied. The delay in consideration varies within an interval. The attention of the authors focuses on the building of desired models of reduced order, which ensure that the resulting error systems, under switching signals average dwell time type, are exponentially stable with bounded H_∞ norm. Through the introduction of a matrix block and making use of the average dwell time approach sufficient conditions are derived, depending on delay, to ensure the existence of models of reduced order; and these conditions are formulated in terms of linear matrix strict inequalities. Due to the absence of non-convex restrictions, one can try to build admissible model of reduced order.

The research in [40] addresses the problem of H_∞ control of linear discrete-time systems with time-varying delay. To highlight the relationships between all combinations of delayed states, a general Lyapunov-Krasovskii functional, from which is established the relationship between the transformation of the switched system and the classical Lyapunov-Krasovskii method is introduced. The basic idea here is the formulation of the original delay systems as switched systems with arbitrary switching. Therefore, the stability and the H_∞ performance problems can be reduced to the corresponding problems for such switched systems. This fact unites two different research fields. In the report [41] a new method for the problem of H_∞ robust control for a class of undefined switched systems, by constructing a single robust H_∞ sliding surface is developed. The method consists of two stages: one is to build a single sliding surface, so that the equivalent sliding movement of reduced order, restricted to the sliding surface, is robustly stabilizable with H_∞ disturbance attenuation

level, by virtue of a hysteresis switching law; and the other phase is to design variable structure controllers for subsystems to drive the state of the system until reaching the only sliding surface in finite time and remain on it thereafter. And in [45] researchers study the H_∞ reduction for linear continuous-time switched systems, with variable delay in time. For a given stable system, attention is focused on the construction of a reduced-order model such that the approximated system to be exponentially stable with prescribed weighted H_∞ restricted behavior. Applying the average delay time approach and the piecewise Lyapunov function technique, sufficient conditions in terms of a linear matrix inequality, to ensure exponential stability and H_∞ weighting behavior for the approximate system are proposed. The model reduction problem is solved using the projection approach, which cloisters the model reduction problem within a sequential optimization problem subject to linear matrix inequalities restrictions, using the complementary linearization cone algorithm.
Another way has been to obtain Ricatti type equations, for example, in the article [31] the problem of the exponential stability and stabilization for switched linear systems with time delay is considered. The uncertainties in the parameters of the system are time-varying and unknown, but bounded in norm. Delay in the states of the system is also time dependent. By using an improved Lyapunov-Krasovskii functional, a rule for switching exponential stability and stabilization in terms of the solution of Ricatti type equations is designed. This approach allows the calculation of the dimensions that characterize the rhythm of exponential stability of the solution.
Margaliot and Hespanha, in [53], consider the problem of computing the mean square of the gain for switched linear systems. They develop a new approach based on an attempt to characterize the "worst case" switched law, i.e. switching law that produces the maximum possible profit. The main result provides a sufficient condition ensuring that the "worst case" switching law can be characterized explicitly using Ricatti differential equations, corresponding to linear subsystems. This condition is automatically satisfied for first-order single-input single-output systems. In this case a complete solution to the problem of mean square for gain is obtained. And in the work [73] some conditions that can provide stabilization for linear switched systems with polytopic uncertainties, through its linear quadratic closed loop state feedback regulator is investigated. Closed loop switched systems can stabilize open loop unstable systems, or stable open loop systems but for which there is no solution for a common Lyapunov matrix. It is shown, for linear continuous-time switched system, that if there is a solution to some Ricatti type equation, which is associated with the closed loop system and share with it a common Lyapunov matrix, then the switched linear system is stable. And for discrete-time switched systems a matrix inequality is obtained to calculate a common Lyapunov matrix and the solution for feedback closed loop systems. These linear quadratic

regulators of loop-closed state feedback, guarantee global asymptotic stability for any linear switched systems with any switching signal sequence. See also [63].

4.1 On the application of techniques typical of other areas

During the bibliographic review we have observed that, for the study of interconnection systems, sometimes techniques from other disciplines have been applied. We refer to this aspect as something that could be taken as a curiosity; however, we believe it also reflects the importance of the study of such systems today.
For example, the results of the article [29] have been motivated by the viral mutation problem in HIV infection. Under simplified assumptions, the dynamic of viral mutation can be viewed as a linear positive switched system. Using positive linear Lyapunov functions results for the synthesis of stabilization, guaranteed performance and optimal control laws to switched linear systems are presented. These results are then applied to one simplified model of mutation of human viral immunodeficiency. The optimum switching control law is compared with the law obtained through a function of guaranteed costs.
In the work [35] the authors develop a representation of controllers based on multiple models using techniques of Artificial Intelligence, such as the Theory of Graphs, Neural Networks, Genetic Algorithms and Fuzzy Logic. Thus, Graph Theory is used to describe in a formal and concise way, the connection mechanism between the various methods of parameterization of a plant represented by a switched system. Moreover, the interpretation of the controllers for multiple models, within the framework of Artificial Intelligence, allows the application of each specific technique to improve the design of controllers based on multiple models. Such obtained controllers are thus compared with the classical simple model based controllers. The authors show through simulation examples, that an improvement in the transient response by use of techniques based on multi estimate can be achieved. Moreover, they present a method to synthesize Neural Networks multiple controllers based on individual designs models, extending the applicability of this type of technique to a more general class of control. Further they propose some applications of Genetic Algorithms and Fuzzy Logic for the design of multi-model controllers. In particular, the mutation operation of Genetic Algorithms inspires a robustness test, consisting of a random modification of estimates used to select the one that leads to better performance of identification to online parameterization of the adaptive controller. Such a test is useful for plants operating in an environment of noise. The proposed robustness test improves the selection used to parameterize the adaptive controller in comparison with classical multi-model

schemes in which the choice of the parameterization of the controller is basically made based on the identification accuracy of each plant model. Moreover, the Fuzzy Logic approach suggests new ideas for design estimation of multiple structures, which can be applied to a wide variety of adaptive controller, such as controller design in robotic manipulation. It is important, no doubt, to have the ability to ensure the existence of a common quadratic Lyapunov function for a given switched system because this is proof of its asymptotic stability, but equally important is the ability to find this function in order to obtain more specific information on the behavior of the switched system being analyzed.

The article [61] describes the development of a methodology based on particle swarm optimization for calculating a common quadratic Lyapunov function; and once the existence of that function has been guaranteed several comparative analyzes are presented to show the strengths and advantages of the proposed methodology. Another example of application of particle swarm optimization in these problems is [19]. In [92] the monitoring adaptive control scheme is addressed for switched nonlinear systems with unknown control signal gain. The applied approach in this case relaxes the assumption that the upper limit of the control gain function is a known constant, and the limits of the external disturbances are known and the approximation errors of the Neural Networks are known too. Here are employed Neural Networks type "radial basis functions" to approximate unknown functions; and an H_∞ controller is introduced to enhance the robustness. The adaptive laws for updating and the admissible switching signals are derived from the method of multiple switching Lyapunov function. It was demonstrated in the work that the resulting closed loop system is asymptotically stable in Lyapunov sense, so the characterization of output tracking error and the attenuation of H_∞ disturbance, are well defined. Finally, a simulation example of Forced Duffing system is provided to illustrate the effectiveness of the proposed control system, which significantly improves transient performance.

5 Bi-dimensional switched systems. Geometric approach as a technique for study of interconnection systems

Plane switched systems also occupy an important place within the set of systems of differential equations with piecewise continuous right-hand side. We focus on the classification and study the behavior of the trajectories of certain types of families of two-dimensional interconnection systems.

Thus, in the work [57] the problems under decoupling disturbance, without stability and with quadratic stability, for linear connected systems in the so-called geometric approach are formulated. First are given necessary and sufficient conditions for the solubility of the problem without stability. Secondly, sufficient conditions are given in order for the problem of quadratic stability to be soluble. Furthermore, sufficient conditions are given to switched linear systems, composed of two subsystems, in order to the quadratic stability problem has a solution.

The authors Perez and Benitez in [62], study the problem of finding the initial state for which the solution of a class of connection systems, consisting of unstable, non-linear and second order subsystems, is converging. They describe and apply a method to set up the regions in which it is possible to define a switching law, so that the solution of the class of nonlinear switched systems converges to the origin of coordinates. They prove that, under certain conditions, these regions are bounded by closed curves (closed solutions) of the switched system. They also present a sufficient condition for the solution closed curve type to be a limit cycle.

In the article [68] on a pair of subsystems, the author is interested in characterizing the problem from two points of view: from the point of view of the state of activation, and from the point of view of the time domain and he reveals the inherent connection between them. To this end, first polar coordinates are used to represent the geometric property of switching, and the dynamic behavior caused by it. Then to put geometrized control of switching in correspondence to the implicit stabilization mechanism behind this control. This achievement lies on the intuitive classification of switching control in two types, namely, chattering switching and rotation switching; and then rigorously clarify their distinction and properties. Moreover, the forced uniform convergence of the trajectories is shown by the presentation of its estimated decay rate, and also to take into account the limit cycles and sliding movements generated by the switching.

In [72] the stability of a switched repetitive system described by a finite number of linear iterated periodically differential equations in the same order (fixed order) is considered. Here some new results that characterize stability dependence of the system on the switching frequency are presented. Some of these results are immediately applicable to the stability analysis of the systems sampled data. The approach to the problem, which is based on the Baker-Campbell-Hausdorff formula, and known theorems about the location of the eigenvalues of the system matrix in the complex plane, has a simple geometric interpretation and allows obtaining qualitative and quantitative conditions for the system.

The document [83] addresses the stability properties of a positive switched linear continuous-time and discrete-time system. For the discrete-time case, some necessary and sufficient conditions are derived for asymptotic stability of coupled second order systems. Similar conditions are also established for a finite number of systems of second order. Furthermore, some results on the stability for higher order systems are provided. In particular, for discrete-time, a common linear Lyapunov function, to ensure the stability of the switched positive systems via geometric properties it can be easily found. In the case of continuous-time, a finite number of second order systems is considered; and some equivalent conditions are developed for the stability of such systems.

In [89] a new technique is presented for numerical analysis of the stability of linear switched systems: radial mesh approach, based on uniform partitions of the state space in terms of ray directions, that allow one to consider families of refinable polytopes of adjustable complexity, with respect to the invariance. In this context, the existence of a polyhedral Lyapunov function, which is typical for a family of asymptotically stable subsystems, can be efficiently checked by applying simple iterative algorithms. It is stated in the work that the technique can be used to test the stability of linear switched switched systems, classes of time-dependent linear systems and differential inclusions. Also the preliminary results on another related problem, namely the construction multiple polyhedral Lyapunov function for specifying stabilization switching sequences for a switched system constructed from a family of stable linear subsystems are presented. Many real systems can be modeled as switched systems, the stability problem is a challenge even for linear subsystems.

In the article [90] the problem of the stability of linear second order switched systems with a finite number of subsystems under arbitrary switching law is investigated. The authors obtain necessary and sufficient conditions, in polar coordinates, for stability based on a worst case analysis method. The fundamental idea of this work is the partition of the entire state space into several regions, and to reduce stability analysis of all subsystems to the analysis of one or two worst subsystems in each region. The results are an extension of the results for analysis of the stability of a linear second order switched system with two subsystems, under arbitrary switching.

In [3] the problem of stabilization of nonlinear switched systems with a single affine entry is considered. The main idea is to transform the nonlinear system in to a controllable equivalent linear system. It first defines the concept of state modal feedback linearization. Then a set of conditions are developed for state modal feedback linearization for certain class of non-linear second-order systems. Considering two special structures, easily verifiable conditions are proposed, suitable for the existence of transformations for linearization of state modal feedback.

Duarte-Mermoud, Ordóñez-Hurtado and Zagalak in [19] show that the existence of a common quadratic Lyapunov function for a switched linear system guarantees the global asymptotic stability, even if the progress in the search for conditions of existence (non-existence) of a common quadratic Lyapunov function is significant, especially in the switched linear systems consisting of

N second order systems or two systems of order n. The general problem of N systems of order n is still open. This article demonstrates a sufficient condition for the non-existence of a common quadratic Lyapunov function for N systems of order n. Based on the obtained condition a new method for determine the non-existence of a common quadratic Lyapunov function by particle swarm optimization is described.
Xu and Antsaklis Xuping in [84], study and solve the problem of asymptotic stabilizing of switched systems which are formed by unstable second order linear time invariant subsystems. They derive necessary and sufficient conditions for asymptotic stability. If a switched system is asymptotically stabilizable then conic switching laws proposed in the document, are used for the construction of a switching law which asymptotically stabilizes the system. Switched systems consisting of two subsystems with unstable foci are discussed, and then the results are extended to switched systems with unstable nodes and saddle points. The results are also applicable to switched systems which consist of more than two subsystems.
Other results on second-order switched systems can be viewed at [11] and [23].

Conclusions

As seen from the bibliographic review, switched discrete-time and continuous-time linear systems occupy an important place in the qualitative theory of systems of differential equations, and no less important is the class of second order systems, which has also received special attention. Also among the techniques used to address the qualitative study of these types of systems, the determination of an appropriate Lyapunov function has been predominant. Although the geometric approach and the "worst case" approach to characterize asymptotic stability of families of linear switched systems have also been used.

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