A Vague Soft Set Theoretic Approach to
Multiattribute Decision Making Problems

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Abstract

In this paper, we introduce an algorithm for decision making and object recognition problems involving vague soft sets and demonstrate the application of this algorithm in a decision making problem set in an imprecise environment.

Mathematics Subject Classifications: 68T10, 68T20

Keywords: Vague soft sets, Soft sets; Fuzzy soft sets; Decision making; Application of soft sets

1. INTRODUCTION

Soft set theory was first proposed by Molodtsov (see [1]) as a general mathematical tool for dealing with uncertain, fuzzy, vague or ill-defined objects as it is free from the inherent difficulties and limitations affecting mathematical tools that are traditionally used to deal with uncertainties, imprecision and vagueness such as the theory of probability, fuzzy set theory (see [2]) and rough set theory (see [3]). Soft set theory has been successfully applied by Molodtsov

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Vague set (see [9]) is a set of objects, each of which has a grade of membership whose value is a continuous subinterval of [0, 1]. Such a set is characterized by a truth-membership function and a false-membership function. Thus, a vague set is indeed a generalized form of fuzzy set, albeit a more accurate version. Soft set theory has been regarded as an effective mathematical tool to deal with uncertainties. However it is difficult to be used to represent the vagueness of problem parameters in problem-solving and decision-making contexts as one of the main problems faced in the real world is that objects in the universal set often does not precisely satisfy the parameters associated to each of the elements in a set. The notion of fuzzy soft sets (see [4]) only partially resolves this problem. This led to the introduction of the notion of vague soft sets by Xu et al. (see [5]). This concept which is the combination of soft sets and vague sets is an extension to the notion of soft sets. It is an improvement to the notion of fuzzy soft sets and can better represent the vagueness of parameters associated to each of the elements in a set and is an effective method to overcome the problem of assigning a suitable value for the grade of membership of an element in a set since the exact grade of membership may be unknown.

Work on the application of soft set theory and fuzzy soft set theory in decision making problems was initiated by Maji et al. (see [6]) who used the concept of knowledge reduction in rough set theory and applied it to solve problems involving soft sets. Yang et al. (see [7]) introduced the concept of interval-valued fuzzy soft sets and analyzed its application in a decision-making problem. Finally, Feng et al. (see [8]) introduced an adjustable approach to fuzzy soft set based decision making problems using the concept of level fuzzy soft sets which was also introduced by them in the same paper. They also introduced the notion of weighted fuzzy soft sets and demonstrated its application in a decision making problem.

Since then, a number of real life problems in engineering, economics, physical sciences, biological sciences and social sciences which involve imprecise data have been actively researched and solved by researchers in this area. The solution to most of these problems involves the use of mathematical principles that are based on uncertainty and imprecision. Some of these problems are essentially humanistic in nature and thus subjective (e.g. the level of human understanding and vision systems) while others are objective. Yet the problems are firmly set in an imprecise environment. Presently work on the application of soft sets and various forms of fuzzy soft sets in decision-making problems are progressing rapidly.

Problems involving decision making and object recognition set in an imprecise environment has been of paramount importance in recent years. In this paper, we present an algorithm to solve decision making problems and object recognition problems involving vague soft sets. We then demonstrate the usage of this
algorithm by solving a decision making problem involving vague soft sets.

2. **PRELIMINARIES**

In this section, we present some of the basic definitions and results pertaining to the theory of vague soft sets. These information would be useful for subsequent discussions as it will be used throughout this paper.

Throughout this section, let \( I \) be the closed unit interval \( I = [0, 1] \) and let \( A \) be a set of parameters for the universe \( U \).

**Definition 2.1** ([1]). Let \( U \) be an initial universe set and let \( A \) be the set of parameters. Let \( P(U) \) denote the power set of \( U \). A pair \((F, A)\) is called a soft set over \( U \), where \( F \) is a mapping given by \( F : A \rightarrow P(U) \).

**Definition 2.2** ([9]). Let \( X \) be a space of points (objects) with a generic element of \( X \) denoted by \( x \). A vague set \( V \) in \( X \) is characterized by a truth-membership function \( t_V : X \rightarrow [0, 1] \) and a false-membership function \( f_V : X \rightarrow [0, 1] \). The value \( t_V(x) \) is a lower bound on the grade of membership of \( x \) derived from the evidence for \( x \) and \( f_V(x) \) is a lower bound on the negation of \( x \) derived from the evidence against \( x \). The values \( t_V(x) \) and \( f_V(x) \) both associate a real number in the interval \([0, 1]\) with each point in \( X \), where \( t_V(x) + f_V(x) \leq 1 \).

This approach bounds the grade of membership of \( x \) to a subinterval \([t_V(x), 1 - f_V(x)]\) of \([0, 1]\).

When \( X \) is continuous, a vague set \( V \) can be written as \( = \int [t_V(x), 1 - f_V(x)]/x \), where \( x \in X \).

When \( X \) is discrete, a vague set \( V \) can be written as \( = \sum^n_{i=1} [t_V(x_i), 1 - f_V(x_i)]/x_i \), where \( x_i \in X \).

**Definition 2.3** ([5]). Let \( U \) be a universe, \( E \) be a set of parameters, \( V(U) \) be the power set of vague sets on \( U \) and \( A \subseteq E \). A pair \((\hat{F}, A)\) is called a vague soft set over \( U \) where \( \hat{F} \) is a mapping given by \( \hat{F} : A \rightarrow V(U) \) and \( V(U) \) is the power set of \( U \).

In other words, a vague soft set over \( U \) is a parameterized family of vague sets of the universe \( U \). Every set \( \hat{F}(e) \) for all \( e \in A \), from this family may be considered as the set of \( e \)–approximate elements of the vague soft set \((\hat{F}, A)\). Hence the vague soft set \((\hat{F}, A)\) can be viewed as consisting of a collection of approximations of the following form:

\[
(\hat{F}, A) = \{\hat{F}(x_i) : i = 1, 2, 3, \ldots \} = \left\{ \frac{[t_{\hat{F}(e_i)}(x_i), 1 - f_{\hat{F}(e_i)}(x_i)]}{x_i} : i = 1, 2, 3, \ldots \right\}
\]
for all $e \in A$ and for all $x \in U$.

**Example 2.4** ([10]). Consider a vague soft set $\left(\hat{F}, E\right)$, where $U$ is a set of six houses under consideration of a decision maker to purchase, which is denoted by $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $E$ is a parameter set, where $E = \{e_1, e_2, e_3, e_4, e_5\}$

$=$ (expensive, beautiful, wooden, cheap, in the green surroundings).

The vague soft set $\left(\hat{F}, E\right)$ describes the “attractiveness of the houses” to this decision maker.

Suppose that

$\hat{F}(e_1) = \left(\begin{array}{c}0.1, 0.2\end{array} \begin{array}{c}0.9, 1\end{array} \begin{array}{c}0.3, 0.5\end{array} \begin{array}{c}0.8, 0.9\end{array} \begin{array}{c}0.2, 0.4\end{array} \begin{array}{c}0.4, 0.6\end{array} \begin{array}{c}h_1\end{array} \begin{array}{c}h_2\end{array} \begin{array}{c}h_3\end{array} \begin{array}{c}h_4\end{array} \begin{array}{c}h_5\end{array} \begin{array}{c}h_6\end{array}\right)$,

$\hat{F}(e_2) = \left(\begin{array}{c}0.9, 1\end{array} \begin{array}{c}0.2, 0.7\end{array} \begin{array}{c}0.6, 0.9\end{array} \begin{array}{c}0.2, 0.4\end{array} \begin{array}{c}0.3, 0.4\end{array} \begin{array}{c}0.1, 0.6\end{array} \begin{array}{c}h_1\end{array} \begin{array}{c}h_2\end{array} \begin{array}{c}h_3\end{array} \begin{array}{c}h_4\end{array} \begin{array}{c}h_5\end{array} \begin{array}{c}h_6\end{array}\right)$,

$\hat{F}(e_3) = \left(\begin{array}{c}0.0\end{array} \begin{array}{c}0.0\end{array} \begin{array}{c}1, 1\end{array} \begin{array}{c}1, 1\end{array} \begin{array}{c}0.0\end{array} \begin{array}{c}0.0\end{array} \begin{array}{c}h_1\end{array} \begin{array}{c}h_2\end{array} \begin{array}{c}h_3\end{array} \begin{array}{c}h_4\end{array} \begin{array}{c}h_5\end{array} \begin{array}{c}h_6\end{array}\right)$,

$\hat{F}(e_4) = \left(\begin{array}{c}0.8, 0.9\end{array} \begin{array}{c}0.0, 1\end{array} \begin{array}{c}0.5, 0.7\end{array} \begin{array}{c}0.1, 0.2\end{array} \begin{array}{c}0.6, 0.8\end{array} \begin{array}{c}0.4, 0.6\end{array} \begin{array}{c}h_1\end{array} \begin{array}{c}h_2\end{array} \begin{array}{c}h_3\end{array} \begin{array}{c}h_4\end{array} \begin{array}{c}h_5\end{array} \begin{array}{c}h_6\end{array}\right)$,

$\hat{F}(e_5) = \left(\begin{array}{c}0.9, 1\end{array} \begin{array}{c}0.2, 0.3\end{array} \begin{array}{c}0.1, 0.4\end{array} \begin{array}{c}0.1, 0.2\end{array} \begin{array}{c}0.2, 0.4\end{array} \begin{array}{c}0.7, 0.9\end{array} \begin{array}{c}h_1\end{array} \begin{array}{c}h_2\end{array} \begin{array}{c}h_3\end{array} \begin{array}{c}h_4\end{array} \begin{array}{c}h_5\end{array} \begin{array}{c}h_6\end{array}\right)$.

The vague soft set $\left(\hat{F}, E\right)$ is a parameterized family $\{\hat{F}(e_i), i = 1, 2, 3, 4, 5\}$ of vague sets on $U$, and $\left(\hat{F}, E\right)$

$=$ \{Expensive Houses

$= \left(\begin{array}{c}0.1, 0.2\end{array} \begin{array}{c}0.9, 1\end{array} \begin{array}{c}0.3, 0.5\end{array} \begin{array}{c}0.8, 0.9\end{array} \begin{array}{c}0.2, 0.4\end{array} \begin{array}{c}0.4, 0.6\end{array} \begin{array}{c}h_1\end{array} \begin{array}{c}h_2\end{array} \begin{array}{c}h_3\end{array} \begin{array}{c}h_4\end{array} \begin{array}{c}h_5\end{array} \begin{array}{c}h_6\end{array}\right)$,

Beautiful Houses

$= \left(\begin{array}{c}0.9, 1\end{array} \begin{array}{c}0.2, 0.7\end{array} \begin{array}{c}0.6, 0.9\end{array} \begin{array}{c}0.2, 0.4\end{array} \begin{array}{c}0.3, 0.4\end{array} \begin{array}{c}0.1, 0.6\end{array} \begin{array}{c}h_1\end{array} \begin{array}{c}h_2\end{array} \begin{array}{c}h_3\end{array} \begin{array}{c}h_4\end{array} \begin{array}{c}h_5\end{array} \begin{array}{c}h_6\end{array}\right), ... \}$.

**Definition 2.5** ([5]). For two vague soft sets $\left(\hat{F}, A\right)$ and $\left(\hat{G}, B\right)$ over a universe $U$, $\left(\hat{F}, A\right)$ is said to be a vague soft subset of $\left(\hat{G}, B\right)$, if $A \subseteq B$ and for all $a \in A$, $\hat{F}(a)$ and $\hat{G}(a)$ are identical approximations. In this case $\left(\hat{G}, B\right)$ is said to be a vague
soft superset of \((\hat{F}, A)\) and this relationship is denoted by \((\hat{F}, A) \subseteq (\hat{G}, B)\).

**Definition 2.6** ([5]). Two vague soft sets \((\hat{F}, A)\) and \((\hat{G}, B)\) over a universe of discourse \(U\) are said to be vague soft equal if \((\hat{F}, A)\) is a vague soft subset of \((\hat{G}, B)\) and \((\hat{G}, B)\) is a vague soft subset of \((\hat{F}, A)\).

**Definition 2.7** ([5]). Let \((\hat{F}, A)\) and \((\hat{G}, B)\) be vague soft sets over \(U\). Then “\((\hat{F}, A)\) and \((\hat{G}, B)\)”, denoted by \((\hat{F}, A) \tilde{\land} (\hat{G}, B)\) is a vague soft set defined by

\[
(\hat{F}, A) \tilde{\land} (\hat{G}, B) = (\hat{F}, A \times B)
\]

where

\[
t_{\hat{F}, (\alpha, \beta)}(x) = \min\{t_{\hat{F}, A}(x), t_{\hat{G}, B}(x)\}
\]

and

\[
1 - f_{\hat{F}, (\alpha, \beta)}(x) = \min\{1 - f_{\hat{F}, A}(x), 1 - f_{\hat{G}, B}(x)\},
\]

for all \((\alpha, \beta) \in A \times B\) and for all \(x \in U\).

### 3. METHODOLOGY AND ALGORITHM

In this section, we present a modified algorithm that can be used to solve problems involving decision making and object recognition among others. This algorithm is a generalized version of the algorithm introduced by Yang et al. (see [7]) which is based on the comparison of choice values of the different objects, that is the higher the choice value, the better the object is. However the algorithm introduced in this paper is based on the concept of vague soft set which is a much more accurate version of fuzzy soft sets, hence making this algorithm more advantageous to be used to solve real-world problems in the areas of decision making, object recognition and engineering. The algorithm introduced here is as given below:

**Step 1:**
Input the set of vague soft sets.

**Step 2:**
Input the parameter set \(P\) as observed by the observer.

**Step 3:**
Compute the corresponding resultant vague soft set obtained from the vague soft sets in Step 1.
Step 4:
For all \( h_i \in U \), compute the choice value \( c_i \) for each object \( h_i \) such that:
\[
c_i = [c_i^-, c_i^+] = \left[ \sum_{p \in P} t_{\bar{h}_p}(h_i), \sum_{p \in P} 1 - f_{\bar{h}_p}(h_i) \right].
\]

Step 5:
For all \( h_i \in U \), compute the score \( r_i \) of \( h_i \) such that:
\[
r_i = \sum_{h_j \in U} \left( (c_i^--c_j^-) + (c_i^+-c_j^+) \right).
\]

Step 6:
The decision is any one of the elements in \( S \) where \( S = \max_{h_i \in U} \{ r_i \} \). If there are more than one element which has the highest \( r_i \) score, then any one of those elements can be chosen.
Then we can conclude that the optimal choice for the house buyer is to buy house \( h_i \).

The algorithm introduced here is based on the concept of vague soft sets which is a generalized but more accurate version of classical soft sets and fuzzy soft sets. The main advantage of vague soft sets compared to the other generalizations of soft sets lies in the fact that vague soft sets gives us an interval of values for the membership function of an object with respect to a certain criteria. As such, we can be sure that the degree of compatibility of an object with respect to a certain criteria lies within a range which is given by the closed interval provided by vague soft sets. This additional feature of vague soft sets makes it a more practical mathematical tool to be used to solve real-world problems which often involve inaccurate or imprecise data as well as helps to better define an object and its degree of compatibility to a particular set. On the other hand, fuzzy sets are only characterized by a membership function which means when modelling use fuzzy sets, we are only able to tell the degree of compatibility of an object with respect to a set. However when dealing with real-life problems, it is often next to impossible to know everything there is to know about an object with respect to a particular set and subsequently say with absolute certainty that the degree of membership or degree of compatibility of an object with respect to a particular set is given by a specific value. Furthermore, since most of the problems that we often have to deal with in the real-world, are subjective and humanistic in nature and often cannot be quantified, the concept of soft sets and fuzzy soft sets are rather ill-defined and inadequate to be used to deal
with such problems. For example, in a decision making problem involving choosing a house to buy, the words “expensive, beautiful, cheap, modern” and so on are considered as part of the set of parameters. These words are fuzzy concepts in the real world as it is unreasonable to say whether a home is absolutely beautiful or not. All we can say is the degree of compatibility of the house to the criteria “beautiful”. In addition, this degree of compatibility can differ greatly from one person to another as it is influenced by many criteria among others, by the perception of an individual, the environment in which a person was brought up in and the part of the world a person is from. As such, it is better and more accurate to state or define the membership function of an object with respect to a certain criteria in terms of an interval instead of assigning a specific or exact value as it is often unknown and cannot be easily determined. Hence it is clear that soft sets and fuzzy soft sets need to be expanded or improved to increase its potential to be used in real-world applications as they are still ill-defined concepts. As such, a better concept such as the notion of vague soft set which is defined in a more accurate manner would be better defined to overcome the problems that are faced when using soft sets and fuzzy soft sets. Hence the algorithm introduced in this paper is more advantageous to be used in solving decision making problems or object recognition problems set in imprecise environments compared to existing algorithms in this area of research.

4. HYPOTHETICAL EXAMPLE

In this section, we demonstrate the technique presented in Section 3 by applying the algorithm introduced in this paper to a hypothetical example of a decision making problem involving vague soft sets and set in an imprecise environment.

Step 1:

Consider a house selection problem where a potential house buyer is considering a set of houses to be purchased. Suppose that there exist six houses $h_1, h_2, h_3, h_4, h_5$ and $h_6$ to be considered for purchasing. This set of houses are denoted by $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$.

Suppose that eight attributes $a_1$ (beautiful), $a_2$ (wooden), $a_3$ (cheap), $a_4$ (expensive), $a_5$ (in the green surroundings), $b_1$ (strategic location), $b_2$ (modern style), $b_3$ (in good condition) are taken into consideration in this decision making problem. The set of all these attributes are denoted as given below:
\( E = \{ \text{beautiful, wooden, cheap, expensive, in the green surroundings, strategic location, modern style, in good condition} \} \)

\( A = \{ a_1, a_2, a_3, a_4, a_5 \} \)

\( B = \{ b_1, b_2, b_3 \} \)

**Step 2:**

Suppose that the vague soft sets \((\hat{F}, A)\) and \((\hat{G}, B)\) describe the ‘houses having the characteristics’. These vague soft sets are defined as follows:

\((\hat{F}, A)\)
\[= \{\hat{F}(a_1), \hat{F}(a_2), \hat{F}(a_3), \hat{F}(a_4), \hat{F}(a_5)\} \]
\[= \{ \text{beautiful houses} = ([0.8, 0.9]/h_1, [0.4, 0.5]/h_2, [0.6, 0.8]/h_3, [0.1, 0.2]/h_4, [0.7, 0.9]/h_5, [0.5, 0.6]/h_6), \text{wooden houses} = ([0, 0]/h_1, [0.3, 0.4]/h_2, [0.2, 0.3]/h_3, [0.3, 0.5]/h_4, [0.8, 0.9]/h_5, [0.6, 0.7]/h_6), \text{cheap houses} = ([0.3, 0.5]/h_1, [0.8, 0.9]/h_2, [0.5, 0.7]/h_3, [0.7, 1.0]/h_4, [0.9, 1.0]/h_5, [0.2, 0.5]/h_6), \text{expensive houses} = ([0.5, 0.7]/h_1, [0.1, 0.2]/h_2, [0.3, 0.5]/h_3, [0.0, 0.3]/h_4, [0.0, 1.0]/h_5, [0.5, 0.8]/h_6), \text{houses in the green surroundings} = ([0.5, 0.8]/h_1, [0.9, 1.0]/h_2, [0.7, 0.9]/h_3, [0.6, 0.8]/h_4, [0.2, 0.5]/h_5, [0.7, 1.0]/h_6) \}. \]

\((\hat{G}, B)\)
\[= \{\hat{G}(b_1), \hat{G}(b_2), \hat{G}(b_3)\} \]
\[= \{ \text{houses with strategic location} = ([0.3, 0.5]/h_1, [0.7, 0.9]/h_2, [0.6, 0.7]/h_3, [0.2, 0.3]/h_4, [0.8, 0.9]/h_5, [0.9, 1.0]/h_6), \text{houses with modern style} = ([0.9, 1.0]/h_1, [0.8, 1.0]/h_2, [0.2, 0.5]/h_3, [0.1, 0.3]/h_4, [0.9, 1.0]/h_5, [0.5, 0.6]/h_6), \text{houses in good condition} = ([0.6, 0.7]/h_1, [0.1, 0.3]/h_2, [0.9, 1.0]/h_3, [0.2, 0.4]/h_4, [0.5, 0.7]/h_5, [0.8, 1.0]/h_6) \}. \]

The tabular representation of the vague soft sets \((\hat{F}, A)\) and \((\hat{G}, B)\) are given in Table 1 and 2.

**Step 3:**

Consider the vague soft sets \((\hat{F}, A)\) and \((\hat{G}, B)\) as defined above. If we define the
“AND” operation between \((\hat{F}, A)\) and \((\hat{G}, B)\), we have \(5 \times 3 = 15\) parameters of the form \(e_{ij}\), where \(e_{ij} = a_i \times b_j\) for all \(i = 1, 2, 3, 4, 5\) and \(j = 1, 2, 3\). Let the set of parameters, \(P\) be defined as follows:

\[
P = \{e_{11}, e_{12}, e_{13}, e_{21}, e_{22}, e_{23}, e_{31}, e_{32}, e_{33}, e_{41}, e_{42}, e_{43}, e_{51}, e_{52}, e_{53}\}.
\]

Let the resultant vague soft set for \(\hat{F}, A\) and \(\hat{G}, B\) be \((\hat{H}, P)\). By definition 2.7, we can write \((\hat{F}, A) \cap (\hat{G}, B) = (\hat{H}, P)\), where \(P\) is as defined above. Then for all \(p \in P\) and for all \(x \in U\), we have

\[
t_{H_p}(x) = \min\{t_{F_a}(x), t_{G_b}(x)\}
\]

and

\[
1 - f_{H_p}(x) = \min\{1 - f_{F_a}(x), 1 - f_{G_b}(x)\}.
\]

The tabular representation of the resultant vague soft set \((\hat{H}, P)\) is as given in Table 3.

**Step 4:**

For all \(h_i \in U\), where \(i = 1, 2, 3, 4, 5, 6\), compute the choice value \(c_i\) for each house \(h_i\) such that:

\[
c_i = [c_i^- , c_i^+] = \left[\sum_{p \in P} t_{H_p}(h_i), \sum_{p \in P} 1 - f_{H_p}(h_i)\right].
\]

The truth membership functions and false membership functions of \((\hat{H}, P)\) are obtained from Table 3. The choice values \(c_i\) for all \(h_i \in U\) are as shown in Table 4.

**Step 5:**

For all \(h_i \in U\), where \(i = 1, 2, 3, 4, 5, 6\), compute the score \(r_i\) for each house \(h_i\) such that:

\[
r_i = \sum_{h_j \in U} \left( (c_i^- - c_j^-) + (c_i^+ - c_j^+) \right).
\]

The scores \(r_i\) are obtained from the choice values \(c_i\) given in Table 4. The scores \(r_i\) for all \(h_i \in U\), where \(i = 1, 2, 3, 4, 5, 6\) are as shown in Table 5.
Table 1: A vague soft set $(\mathcal{F}, A)$.

<table>
<thead>
<tr>
<th>$U$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1$</td>
<td>$[0.8, 0.9]$</td>
<td>$[0.0]$</td>
<td>$[0.3, 0.5]$</td>
<td>$[0.5, 0.7]$</td>
<td>$[0.5, 0.8]$</td>
</tr>
<tr>
<td>$h_2$</td>
<td>$[0.4, 0.5]$</td>
<td>$[0.3, 0.4]$</td>
<td>$[0.8, 0.9]$</td>
<td>$[0.1, 0.2]$</td>
<td>$[0.9, 1.0]$</td>
</tr>
<tr>
<td>$h_3$</td>
<td>$[0.6, 0.8]$</td>
<td>$[0.2, 0.3]$</td>
<td>$[0.5, 0.7]$</td>
<td>$[0.3, 0.5]$</td>
<td>$[0.7, 0.9]$</td>
</tr>
<tr>
<td>$h_4$</td>
<td>$[0.1, 0.2]$</td>
<td>$[0.3, 0.5]$</td>
<td>$[0.7, 1.0]$</td>
<td>$[0.0, 0.3]$</td>
<td>$[0.6, 0.8]$</td>
</tr>
<tr>
<td>$h_5$</td>
<td>$[0.7, 0.9]$</td>
<td>$[0.8, 0.9]$</td>
<td>$[0.9, 1.0]$</td>
<td>$[0.0, 0.1]$</td>
<td>$[0.2, 0.5]$</td>
</tr>
<tr>
<td>$h_6$</td>
<td>$[0.5, 0.6]$</td>
<td>$[0.6, 0.7]$</td>
<td>$[0.2, 0.5]$</td>
<td>$[0.5, 0.8]$</td>
<td>$[0.7, 1.0]$</td>
</tr>
</tbody>
</table>

Table 2: A vague soft set $(\mathcal{G}, B)$.

<table>
<thead>
<tr>
<th>$U$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1$</td>
<td>$[0.3, 0.5]$</td>
<td>$[0.9, 1.0]$</td>
<td>$[0.6, 0.7]$</td>
</tr>
<tr>
<td>$h_2$</td>
<td>$[0.7, 0.9]$</td>
<td>$[0.8, 1.0]$</td>
<td>$[0.1, 0.3]$</td>
</tr>
<tr>
<td>$h_3$</td>
<td>$[0.6, 0.7]$</td>
<td>$[0.2, 0.5]$</td>
<td>$[0.9, 1.0]$</td>
</tr>
<tr>
<td>$h_4$</td>
<td>$[0.2, 0.3]$</td>
<td>$[0.1, 0.3]$</td>
<td>$[0.2, 0.4]$</td>
</tr>
<tr>
<td>$h_5$</td>
<td>$[0.8, 0.9]$</td>
<td>$[0.9, 1.0]$</td>
<td>$[0.5, 0.7]$</td>
</tr>
<tr>
<td>$h_6$</td>
<td>$[0.9, 1.0]$</td>
<td>$[0.5, 0.6]$</td>
<td>$[0.8, 1.0]$</td>
</tr>
</tbody>
</table>

Step 6:
The decision is any one of the elements in $S$ where $S = \max_{h_i \in U} \{r_i\}$. In the context of this example, $S = \max_{h_i \in U} \{r_i\} = \{h_6\}$ with a score of 25. Hence house $h_6$ is the best choice for a buyer. This decision/result is reasonable because it can be clearly seen that $c_6 > c_i$ for all $i = 1, 2, 3, 4, 5$, which means that $h_6$ has the highest choice value.

As such, it can be concluded that the optimal alternative/option is for the house buyer to purchase house $h_6$. The optimal ranking order of the choices for the house buyer are house $h_6$ (the best and most optimal choice), followed by house $h_5, h_3, h_1$ and house $h_4$ (the worst and least optimal choice).
Table 3: The resultant vague soft set \((\tilde{H}, P)\).

<table>
<thead>
<tr>
<th>(U)</th>
<th>(e_{11})</th>
<th>(e_{12})</th>
<th>(e_{13})</th>
<th>(e_{21})</th>
<th>(e_{22})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h_1)</td>
<td>[0.3, 0.5]</td>
<td>[0.8, 0.9]</td>
<td>[0.6, 0.7]</td>
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<td>[0, 0]</td>
</tr>
<tr>
<td>(h_2)</td>
<td>[0.4, 0.5]</td>
<td>[0.4, 0.5]</td>
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<td>[0.3, 0.4]</td>
<td>[0.3, 0.4]</td>
</tr>
<tr>
<td>(h_3)</td>
<td>[0.6, 0.7]</td>
<td>[0.2, 0.5]</td>
<td>[0.6, 0.8]</td>
<td>[0.2, 0.3]</td>
<td>[0.2, 0.3]</td>
</tr>
<tr>
<td>(h_4)</td>
<td>[0.1, 0.2]</td>
<td>[0.1, 0.2]</td>
<td>[0.1, 0.2]</td>
<td>[0.2, 0.3]</td>
<td>[0.1, 0.3]</td>
</tr>
<tr>
<td>(h_5)</td>
<td>[0.7, 0.9]</td>
<td>[0.7, 0.9]</td>
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<td>[0.8, 0.9]</td>
<td>[0.8, 0.9]</td>
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<tr>
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<td>[0.6, 0.7]</td>
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<table>
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<tr>
<th>(U)</th>
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<th>(e_{31})</th>
<th>(e_{32})</th>
<th>(e_{33})</th>
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<td>[0.3, 0.5]</td>
<td>[0.3, 0.5]</td>
<td>[0.3, 0.5]</td>
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<tr>
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<td>[0.7, 0.9]</td>
<td>[0.8, 0.9]</td>
<td>[0.1, 0.3]</td>
<td>[0.1, 0.2]</td>
</tr>
<tr>
<td>(h_3)</td>
<td>[0.2, 0.3]</td>
<td>[0.5, 0.7]</td>
<td>[0.2, 0.5]</td>
<td>[0.5, 0.7]</td>
<td>[0.3, 0.5]</td>
</tr>
<tr>
<td>(h_4)</td>
<td>[0.2, 0.4]</td>
<td>[0.2, 0.3]</td>
<td>[0.1, 0.3]</td>
<td>[0.2, 0.4]</td>
<td>[0.3, 0.5]</td>
</tr>
<tr>
<td>(h_5)</td>
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<td>[0.8, 0.9]</td>
<td>[0.9, 1.0]</td>
<td>[0.5, 0.7]</td>
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<tr>
<td>(h_6)</td>
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<td>[0.2, 0.5]</td>
<td>[0.2, 0.5]</td>
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<table>
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<tr>
<th>(U)</th>
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<th>(e_{51})</th>
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<td>[0.8, 1.0]</td>
<td>[0.1, 0.3]</td>
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<td>[0.3, 0.5]</td>
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<td>[0.2, 0.5]</td>
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<tr>
<td>(h_5)</td>
<td>[0, 0.1]</td>
<td>[0, 0.1]</td>
<td>[0.2, 0.5]</td>
<td>[0.2, 0.5]</td>
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</table>
In this paper, we introduced a vague soft set based modified algorithm to solve decision making problems and object recognition problems. We also demonstrated the usage of this algorithm using a hypothetical example involving a decision making problem set in an imprecise environment. In conclusion, vague soft set based multi-attribute decision making algorithms such as the one introduced in this paper can represent a wide range of possibilities and has great potential to be used to handle uncertainties and vagueness in problem parameters.
as well as to accurately represent knowledge and information which are subjective and humanistic in nature. This enables the explicit consideration of the most optimal results and least optimal results one can expect to obtain in decision making or object recognition problems.

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References


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