Fuzzy Multi Objective Transportation Model

Based on New Ranking Index on

Generalized LR Fuzzy Numbers

Y. L. P. Thorani and N. Ravi Shankar

Dept. of Applied Mathematics, GIS
GITAM University, Visakhapatnam, India

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Abstract

In this paper, we present a new method for analyzing a fuzzy multi objective transportation problem using a linear programming model based on a new method for ranking generalized LR fuzzy numbers. First, we present a new method for ranking generalized LR fuzzy numbers from its \( \lambda \)-cut. To the best of our knowledge till now there is no method in the literature to find the ranking order of the generalized LR fuzzy numbers to various linear and non-linear functions of LR fuzzy numbers from its \( \lambda \)-cut based on area, mode and spread. Our method can efficiently rank various LR fuzzy numbers, their images and crisp numbers which are consider to be a special case of fuzzy numbers (normal/nonnormal, triangular/trapezoidal, and general), which could not be ranked by the existing ranking methods. Our ranking method also satisfies the linearity property condition. For the corroboration, we used some comparative examples of different existing methods to illustrate the advantages of the proposed method. The reference functions of LR fuzzy numbers of fuzzy multi objective transportation problem are considered to be linear and non-linear functions. This paper develops a procedure to derive the fuzzy objective value of the fuzzy multi objective transportation problem, in that the fuzzy cost coefficients and the fuzzy time are LR fuzzy numbers. The method is illustrated with an example by various cases.
Keywords: Multi objective transportation; ranking index; LR fuzzy numbers; linear programming; $\lambda$-cut

1. Introduction

Transportation models play a significant role in logistics and supply chain management for reducing cost and time, for better service. In today’s dynamic market the demands on organizations to find improved ways to create and deliver value to customers becomes stronger. How and when to send the products to the customers in the quantities which they want in a cost-effective manner, become more challenging. To meet this challenge transportation models endow with a powerful framework. They ensure the competency movement and timely accessibility of raw materials and finished goods. Efficient algorithms had been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. In real world applications, these parameters may not be presented in a precise manner due to uncontrollable factors. Bellman and Zadeh [1] and Zadeh [2] introduced the notion of fuzziness to deal with imprecise information in making decisions. The solutions obtained by fuzzy linear programming were always efficient is proved by Zimmermann [3] and its fuzzy linear programming had developed into several fuzzy optimization methods for solving transportation problems. Several researchers had been making rigorous investigations on multiobjective transportation problem. Optimization of multiobjectives transportation problems was presented by Lee and Moore [4]. A bicriteria transportation problem was presented by Aneja and Nair [5]. Multiobjective linear transportation problem procedure which generates all non-dominated solutions were presented by Diaz [6,7] and Isermann [8]. Bhatia et al.[9,10], Gupta[11], Prasad et al.[12], Li et al. [13], Liu and Zhang [14] studied the time-cost minimizing transportation problem. The fractional function is one of the objectives in the multiobjective transportation problem. Swarup [15], Sharama and Swarup [16], Chandra and Saxena [17] studied that the multiobjective transportation problem which also relates to the fractional transportation problem. Bit et al. [18] consider a k-objective transportation problem fuzzified by fuzzy numbers and used $\alpha$-cut to obtain a transportation problem in the fuzzy sense expressed in linear programming form. Chanas and Kuchta[19] used fuzzy numbers of the type L-L to fuzzify cost coefficients in the objective function and $\alpha$-cut to express the objective function in the form of an interval. Hussien [20] studied the complete set of $\alpha$-possibility efficient solutions of multi objective transportation problem with possibilistic co-efficients of the objective functions. Li and Lai [21] proposed a fuzzy compromise programming approach to a multi objective linear transportation problem. For rating and ranking multiple aspect alternatives using fuzzy sets was
presented by Bass and Kwakeernaak [37]. For the ranking of fuzzy numbers a number of ranking approaches had been proposed. A ranking procedure for ordering fuzzy sets in which a ranking index was calculated for the LR fuzzy number from its \( \lambda \)-cut was proposed by Yager [38], it cannot rank generalized LR fuzzy numbers, but by the proposed method we can overcome the limitations and shortcomings of the existing method and the proposed method is relatively simple in computation and is easily logical. Liou and Wang [26] presented a method for ranking fuzzy numbers with integral values. Abbasbandy and Asady [23], Asady and Zendehnam [28] proposed that a fuzzy number is mapped to a real number based on the area measurement. A new approach for ranking fuzzy numbers by the distance method and the coefficient variance (CV) index method proposed by Cheng [27] had some drawbacks (for account see Examples 1 and 2 as mentioned in section 4 of this paper). For ranking fuzzy numbers with an area between the centroid points of fuzzy numbers and the original point was proposed by Chu and Tsao [22] to overcome the drawbacks that are existing in Cheng [27], but their approach still had some drawbacks (for account see Example 3 as mentioned in section 4 of this paper). Wang and Lee [29] made a revision on ranking fuzzy numbers with an area between the centroid and original points to improve Chu and Tsao’s approach [22]. Though some improvements are made, Wang and Lee’s approach cannot still differentiate two fuzzy numbers with the same centroid point (see Example 3 as mentioned in section 4). Wang et al. [30] proposed an approach to ranking fuzzy numbers based on lexicographic screening procedure and summarized some limitations of the existing methods (see Table 2 in Ref. [30]). However, Wang et al’s approach can not differentiate these kinds of fuzzy numbers as shown in Example 3 in Section 4 of this paper, as most of the existing approaches do. To overcome the limitations of the existing studies and simplify the computational procedures, we define a new ranking approach to various linear and non-linear functions of LR fuzzy numbers from its \( \lambda \)-cut and also uses mode and spreads in those cases where the discrimination is not possible is proposed. Based on the proposed fuzzy ranking method, we present a method for dealing with fuzzy multi-objective transportation model uses linear programming. In this paper, we consider fuzzy multi-objective transportation problem with fuzzy parameters in the case of the transportation process. Here, we let \( \tilde{c}_{ij} \) to be a fuzzy cost for shipping one unit product from \( i^{th} \) source to \( j^{th} \) destination, \( \tilde{t}_{ij} \) to be fuzzy time from \( i^{th} \) source to \( j^{th} \) destination. Here, the fuzzy cost coefficients, the fuzzy times are LR fuzzy numbers. By assigning a weight to the objectives according to their priorities the single objective function is obtained. Then, by using the proposed ranking method, transform a newly formed single objective fuzzy transportation problem to a crisp transportation problem in the linear programming problem form and it can be solved by any conventional method.
The rest of the paper is organized as follows: In section 2 preliminaries of LR fuzzy numbers, \( \lambda \)-cut of LR fuzzy number, reference functions and their inverses are presented. In section 3 new ranking approach to various linear and non-linear functions of LR fuzzy numbers from its \( \lambda \)-cut and also uses mode and spreads in those cases where the discrimination is not possible is given and an important result like linearity of ranking function which is the basis for defining the ranking procedure in section 3.1 is discussed and proved. In section 4, we compare the ranking results of the proposed method for various cases with the existing methods. In section 5, linear programming model for fuzzy multi objective transportation model to various linear and non-linear functions with generalized LR fuzzy numbers is discussed and, the proposed method to solve the total optimal fuzzy cost, and fuzzy time for fuzzy multi-objective transportation problem using linear programming is discussed with a numerical example and total optimal fuzzy cost, and fuzzy time for various cases are presented. Finally the conclusion is given in section 6.

### 2. Preliminaries

In this section, LR fuzzy numbers, \( \lambda \)-cut of LR fuzzy number, reference functions and their inverses are presented.

#### 2.1. LR fuzzy numbers and reference functions

In this section, LR fuzzy number, \( \lambda \)-cut of LR fuzzy number, reference functions and their inverses are reviewed [38].

**Definition 1** A fuzzy number \( \tilde{A} = (m,n,\alpha,\beta)_{LR} \) is said to be an LR fuzzy number if

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
L \left( \frac{m-x}{\alpha} \right), & x \leq m, \ \alpha > 0, \\
R \left( \frac{x-n}{\beta} \right), & x \geq n, \ \beta > 0, \\
1, & \text{otherwise}
\end{cases}
\]

where L and R are continuous, non-increasing functions that define the left and right shapes of \( \mu_{\tilde{A}}(x) \) respectively and \( L(0) = R(0) = 1 \).

Linear reference functions and nonlinear reference functions with their inverses are presented in Table I.
Table I: Reference functions and their inverses

<table>
<thead>
<tr>
<th>Function Name</th>
<th>Reference Function (RF)</th>
<th>Inverse of Reference function $\alpha \in (0,1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$RF_1(x) = \max {0, 1 -</td>
<td>x</td>
</tr>
<tr>
<td>Exponential</td>
<td>$RF_2(x) = e^{-x}, \ p \geq 1$</td>
<td>$RF_2^-(x) = -\ln(\alpha)/p$</td>
</tr>
<tr>
<td>Power</td>
<td>$RF_3(x) = \max {0, 1 - x^p}, \ p \geq 1$</td>
<td>$RF_3^-(x) = \sqrt[1-\alpha]{1}$</td>
</tr>
<tr>
<td>Exponential power</td>
<td>$RF_4(x) = e^{-x^p}, \ p \geq 1$</td>
<td>$RF_4^-(x) = \sqrt{-\ln(\alpha)}$</td>
</tr>
<tr>
<td>Rational</td>
<td>$RF_5(x) = 1/(1 + x^p), \ p \geq 1$</td>
<td>$RF_5^-(x) = \sqrt{(1 - \alpha)/\alpha}$</td>
</tr>
</tbody>
</table>

**Definition 2** Let $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ be an LR fuzzy number and $\lambda$ be a real number in the interval $[0, 1]$. Then the crisp set $A_\lambda = \{x \in X: \mu_\lambda(x) \geq \lambda\} = [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$ is said to be $\lambda$-cut of $\tilde{A}$.

3. New ranking approach to various linear and non-linear functions of LR fuzzy Numbers from its $\lambda$-cut based on area, mode and spread.

In this section, new ranking approach is presented for the ranking of LR fuzzy numbers. This method involves a procedure for ordering fuzzy sets in which a ranking approach $R(\tilde{A})$ is calculated for the fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ from its $\lambda$-cut.

3.1. Proposed ranking Method

The Centroid of a trapezoid is considered as the balancing point of the trapezoid (Fig.1). Divide the trapezoid into three triangles as APC, QCD and PQC respectively. Let $G_1$ be the Centroid of the triangle APC, $G_2$ be the Centroid of the triangle QCD and $G_3$ be the Centroid of the triangle PQC. The Centroid of the Centroids of these three triangles is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each Centroid point is a balancing point of each individual triangle, and the Centroid of these Centroid points is a much more balancing point of a generalized LR fuzzy number. Therefore, this point would be a better reference point than the Centroid point of the trapezoid.
Consider a generalized LR fuzzy number \( \tilde{A} = (m, n, \alpha, \beta; w)_{LR} \). (Fig.1). The Centroids of the three triangles are
\[
G_1 = \left( \frac{2m + n - \alpha}{3}, \frac{w}{3} \right), \quad G_2 = \left( \frac{3n + \beta}{3}, \frac{w}{3} \right) \quad \text{and} \quad G_3 = \left( \frac{m + 2n}{3}, \frac{2w}{3} \right)
\]
respectively.

Equation of the line \( \overline{G_1G_2} \) is \( y = \frac{w}{3} \) and \( G_3 \) does not lie on the line \( \overline{G_1G_2} \). Therefore \( G_1, G_2 \) and \( G_3 \) are non-collinear and they form a triangle.

We define the Centroid \( G_A(x_0, y_0) \) of the triangle with vertices \( G_1, G_2 \) and \( G_3 \) of the generalized LR fuzzy number \( \tilde{A} = (m, n, \alpha, \beta; w)_{LR} \) as
\[
G_A(x_0, y_0) = \left( \frac{3m + 6n - \alpha + \beta}{9}, \frac{4w}{9} \right)
\]
As a special case, for triangular L-R fuzzy number \( \tilde{A} = (m, n, \alpha, \beta; w)_{LR} \) i.e., \( m = n \) the Centroid of Centroids is given by
\[
G_{\tilde{A}}(x_0, y_0) = \left( \frac{9m - \alpha + \beta}{9}, \frac{4w}{9} \right)
\]
The ranking function of the generalized LR fuzzy number \( \tilde{A} = (m, n, \alpha, \beta; w)_{LR} \) which maps the set of all fuzzy numbers to a set of real numbers is defined as:
\[
R(\tilde{A}) = (x_0 \times y_0) = \left( \frac{3m + 6n - \alpha + \beta}{9} \times \frac{4w}{9} \right)
\]
This the Area between the Centroid of the Centroids \( G_{\tilde{A}}(x_0, y_0) \) as defined in Eq.(1) and the original point.
The ranking function of the generalized LR fuzzy number \( \tilde{A} = (m, n, \alpha, \beta; w)_{LR} \) of Eq.(1) from its \( \lambda \)-cut is defined as

\[
R(\tilde{A}) = 2 \left( \int_{0}^{\infty} \left( 3m - \alpha L^{-1}(\lambda) \right) d\lambda + \int_{0}^{1} \left( 6n + \beta R^{-1}(\lambda) \right) d\lambda \right)
\]  

(4)

Since \( R(\tilde{A}) \) is calculated from the extreme values of \( \lambda \)-cut of \( \tilde{A} \), rather than its membership function, it is not required knowing the explicit form of the membership functions of the fuzzy numbers to be ranked. That is unlike most of the ranking methods that require the knowledge the membership functions of all fuzzy numbers to be ranked. This centroid of centroid index is still applicable even if the explicit form the membership function of the fuzzy numbers is unknown.

The ranking indexes for Eq.(4) with various linear and non-linear functions are:

**Case (i)** \( L(x) = R(x) = \max \{0, 1 - |x|\} \)

\[
R(\tilde{A}) = \frac{8w}{81} \left[ 3m + 6n - \frac{\alpha + \beta}{2} \right]
\]

**Case (ii)** \( L(x) = R(x) = e^{-x} \)

\[
R(\tilde{A}) = \frac{8w}{81} \left[ 3m + 6n - \alpha + \beta \right]
\]

**Case (iii)** \( L(x) = \max \{0, 1 - |x|\} \) and \( R(x) = e^{-x} \)

\[
R(\tilde{A}) = \frac{8w}{81} \left[ 3m + 6n - \frac{\alpha}{2} + \beta \right]
\]

**Case (iv)** \( L(x) = e^{-x} \) and \( R(x) = \max \{0, 1 - |x|\} \)

\[
R(\tilde{A}) = \frac{8w}{81} \left[ 3m + 6n + \frac{\beta}{2} \right]
\]

**Case (v)** \( L(x) = e^{-px} \) and \( R(x) = \max \{0, 1 - x^p\} \)

\[
R(\tilde{A}) = \frac{8w}{81} \left[ 3m + 6n - \frac{\alpha}{p} + \frac{\beta p}{p + 1} \right]
\]

The Mode (M) of the generalized LR fuzzy number \( \tilde{A} = (m, n, \alpha, \beta; w)_{LR} \) is defined as:

\[
M(\tilde{A}) = \frac{1}{2} \int w (m + n) dx = \frac{w}{2} (m + n)
\]  

(5)
The Spread ($S$) of the generalized LR fuzzy number $\tilde{A} = (m, n, \alpha, \beta; w)_l$ is defined as:

$$S(\tilde{A}) = \int_0^w (\alpha + \beta + n - m) \, dx = w(\alpha + \beta + n - m)$$  \hspace{1cm} (6)

The Left spread ($LS$) of the generalized LR fuzzy number $\tilde{A} = (m, n, \alpha, \beta; w)_l$ is defined as:

$$LS(\tilde{A}) = \int_0^w \alpha \, dx = w\alpha$$  \hspace{1cm} (7)

The Right spread ($RS$) of the generalized LR fuzzy number $\tilde{A} = (m, n, \alpha, \beta; w)_l$ is defined as:

$$RS(\tilde{A}) = \int_0^w \beta \, dx = w\beta$$  \hspace{1cm} (8)

An algorithm for finding the ranking of two generalized LR fuzzy numbers $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1; w_1)_l$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2; w_2)_l$, by using the ranking indexes for Eq.(4) with various linear and non-linear functions, and using the equations from (5-8), is defined as follows

**Step 1: Find $R(\tilde{A}_1)$ and $R(\tilde{A}_2)$**

(i) If $R(\tilde{A}_1) > R(\tilde{A}_2)$ then $\tilde{A}_1 > \tilde{A}_2$

(ii) If $R(\tilde{A}_1) < R(\tilde{A}_2)$ then $\tilde{A}_1 < \tilde{A}_2$

(iii) If $R(\tilde{A}_1) = R(\tilde{A}_2)$ The comparison is not possible, then go to step 2.

**Step 2: Find $M(\tilde{A}_1)$ and $M(\tilde{A}_2)$**

(i) If $M(\tilde{A}_1) > M(\tilde{A}_2)$ then $\tilde{A}_1 > \tilde{A}_2$

(ii) If $M(\tilde{A}_1) < M(\tilde{A}_2)$ then $\tilde{A}_1 < \tilde{A}_2$

(iii) If $M(\tilde{A}_1) = M(\tilde{A}_2)$ The comparison is not possible, then go to step 3.

**Step 3: Find $S(\tilde{A}_1)$ and $S(\tilde{A}_2)$**

(i) If $S(\tilde{A}_1) > S(\tilde{A}_2)$ then $\tilde{A}_1 < \tilde{A}_2$

(ii) If $S(\tilde{A}_1) < S(\tilde{A}_2)$ then $\tilde{A}_1 > \tilde{A}_2$

(iii) If $S(\tilde{A}_1) = S(\tilde{A}_2)$ The comparison is not possible, then go to step 4.
Step 4: Find $LS(\tilde{A}_1)$ and $LS(\tilde{A}_2)$
(i) If $LS(\tilde{A}_1) > LS(\tilde{A}_2)$ then $\tilde{A}_1 > \tilde{A}_2$
(ii) If $LS(\tilde{A}_1) < LS(\tilde{A}_2)$ then $\tilde{A}_1 < \tilde{A}_2$
(iii) If $LS(\tilde{A}_1) = LS(\tilde{A}_2)$ The comparison is not possible, then go to step 5.

Step 5: Examine $w_1$ and $w_2$
(i) If $w_1 > w_2$ then $\tilde{A}_1 > \tilde{A}_2$
(ii) If $w_1 < w_2$ then $\tilde{A}_1 < \tilde{A}_2$
(iii) If $w_1 = w_2$ then $\tilde{A}_1 \approx \tilde{A}_2$

3.2. In this section an important result which is the basis for defining the ranking procedure in section 3.1 is discussed and proved.

**Proposition 3.2.1.**
The ranking function of centroid of centroids (Eq.4) is a linear function of the normal LR fuzzy number $\tilde{A} = (m, n, \alpha, \beta; w)_{LR}$
If $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1; w_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2; w_2)_{LR}$ are two normalized LR fuzzy numbers, then
(i) $R(k_1\tilde{A}_1 \oplus k_2\tilde{A}_2) = k_1R(\tilde{A}_1) \oplus k_2R(\tilde{A}_2)$
(ii) $R(-\tilde{A}) = -R(\tilde{A})$
(iii) $R((\tilde{A}) \oplus (-\tilde{A})) = 0$

**Proof (i)**

**Case (i)** Let $k_1, k_2 > 0$

$(k_1\tilde{A}_1 \oplus k_2\tilde{A}_2) = (k_1m_1 + k_2m_2, k_1n_1 + k_2n_2, k_1\alpha_1 + k_2\alpha_2, k_1\beta_1 + k_2\beta_2)$
Using Eq. (4), we get

$R(k_1\tilde{A}_1 \oplus k_2\tilde{A}_2) = \frac{8}{81_0} \left\{ 3(k_1m_1 + k_2m_2) - (k_1\alpha_1 + k_2\alpha_2)L^{-1}(\lambda) + 6(k_1n_1 + k_2n_2) + (k_1\beta_1 + k_2\beta_2)R^{-1}(\lambda) \right\}$

$= k_1 \left( \frac{8}{81_0} 3m_1 + 6n_1 - \alpha_1L^{-1}(\lambda) \right) + k_2 \left( \frac{8}{81_0} 3m_2 + 6n_2 - \alpha_2R^{-1}(\lambda) \right)$

$= k_1R(\tilde{A}_1) + k_2R(\tilde{A}_2)$

$\therefore R(k_1\tilde{A}_1 \oplus k_2\tilde{A}_2) = k_1R(\tilde{A}_1) + k_2R(\tilde{A}_2)$
Similarly, the result can be proved for the case (ii) \( k_1 > 0, k_2 < 0 \) and case (iii) \( k_1 < 0, k_2 > 0 \).

**Proof (ii)**

Let \( \tilde{A} = (m,n,\alpha,\beta; w)_{LR} \) and
\[-\tilde{A} = (-\beta, -\alpha, -m, -n; l)\]
\[R(-\tilde{A}) = \frac{8}{81} \int_0^1 (-\beta R^{-1}(x) + \alpha L^{-1}(\lambda) - 6n - 3m) d\lambda\]
\[= -\frac{8}{81} \int_0^1 (3m + 6n - \alpha L^{-1}(x) + \beta R^{-1}(x))\]
\[.: R(-\tilde{A}) = -R(\tilde{A})\]

**Proof (iii)**

\[R((\tilde{A} \oplus (-\tilde{A})) = R(\tilde{A}) \oplus R(-\tilde{A}) \quad \text{by (i)}\]
\[= R(\tilde{A}) \oplus R(\tilde{A}) \quad \text{by (ii)}\]
\[= 0\]

**4. A comparison of the ranking results of the proposed method for various cases with the existing methods**

In this section, some numerical examples from the existing literature and two proposed examples and the comparative study with figures are used to illustrate the proposed method for ranking generalized LR fuzzy numbers using \( \lambda \)-cut, for various cases is given.

**Example1.**
Consider set 1 with two fuzzy numbers [22], \( \tilde{A} = (2,2,0.1,0.1; 1)_{LR} \) and \( \tilde{B} = (3,3,0.9,1; 1)_{LR} \), as shown in Fig. 2.
By using various ranking indexes with various linear and nonlinear functions as defined in section 3.1 the ranking order of \( \tilde{A} \) and \( \tilde{B} \) in all the cases is \( \tilde{A} < \tilde{B} \), the ranking results by the proposed method is given in Table II. From Fig. 2 of set 1, it can be seen that the result obtained by our approach is reliable with human instinct. However, by the CV index proposed by Cheng [27], the ranking order is \( \tilde{A} > \tilde{B} \), which is unreasonable.

**Example 2.**
Consider set 2 with three fuzzy numbers [23], \( \tilde{A} = (6,6,1;1)_{LR} \), \( \tilde{B} = (6,6,0,1;1)_{LR} \), and
\( \tilde{C} = (6,6,0,1;1)_{LR} \) as shown in Fig. 2.

By using various ranking indexes with various linear and nonlinear functions as defined in section 3.1 the ranking order of \( \tilde{A} \), \( \tilde{B} \) and \( \tilde{C} \) in case(i, ii, iii and v) is \( \tilde{C} > \tilde{B} > \tilde{A} \) and in case(iv) the ranking order is \( \tilde{B} > \tilde{C} > \tilde{A} \). The ranking results by the proposed method is given in Table II. However, by Chu and Tsao’s approach [22], the ranking order is \( \tilde{B} > \tilde{C} > \tilde{A} \). Cheng [27], proposed CV index, through is approach the ranking order is \( \tilde{A} > \tilde{B} > \tilde{C} \). From Fig. 2, it is easy to see that the ranking results obtained by the existing approaches [22, 27] are unreasonable and are not consistent with human intuition. On the other hand, by Abbasbandy and Asady’s approach [23], the ranking results is \( \tilde{C} > \tilde{B} > \tilde{A} \), which is the same as the one obtained by our approach. However, our approach is simpler in the computation procedure.

**Example 3.**
Consider set 3 with two fuzzy numbers [24], \( \tilde{A} = (6,6,3,3;1)_{LR} \), \( \tilde{B} = (6,6,1,1;1)_{LR} \) as shown in Fig. 2.

By using various ranking indexes with various linear and nonlinear functions as defined in section 3.1 the ranking order of \( \tilde{A} \) and \( \tilde{B} \) in case(i-ii) is \( R(\tilde{A})=R(\tilde{B}) \) therefore by using Eq. 6 and Eq. 7 we get the result as \( \tilde{A} > \tilde{B} \) and in case (iii-v) the ranking order is \( \tilde{A} > \tilde{B} \), the ranking results by the proposed method is given in Table II. Because fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) have the same mode and symmetric spread, most of existing approaches fail. Asady and Zendehnam [28], Chu and Tsao [22], Wang and Lee [29] and Yao and Wu [25], get the same ranking order as \( \tilde{A} \approx \tilde{B} \). Abbasbandy and Asady’s [23] approach the ranking order of fuzzy numbers
is \( \tilde{A} \approx \tilde{B} \) and \( \tilde{A} > \tilde{B} \) for different index values i.e., when \( p=1 \) and \( p=2 \) respectively. By Wang et al. [30], approach the ranking order is \( \tilde{A} > \tilde{B} \). Tran-Duckein [31] get the variance results when \( D_{\text{max}} \) and \( D_{\text{min}} \) are used respectively. Liu [32], Matarazzo and Munda [33], Yao and Lin [34], approaches gave different ranking order when different indices of optimism are taken. But by our ranking approach the ranking results in all the cases is same i.e., \( \tilde{A} > \tilde{B} \).

**Example 4.**
Consider two sets set 4 and set 5 with three fuzzy numbers from Yao and Wu [25], as shown in Fig. 2.

Set 4: \( \tilde{A} = (0.5, 0.5, 0.1, 0.5; 1)_{LR}, \tilde{B} = (0.7, 0.7, 0.3, 0.3; 1)_{LR}, \text{ and } \tilde{C} = (0.9, 0.9, 0.5, 0.1; 1)_{LR} \);

By using various ranking indexes with various linear and nonlinear functions as defined in section 3.1 the ranking order of \( \tilde{A} \), \( \tilde{B} \) and \( \tilde{C} \) in all the cases is \( \tilde{C} > \tilde{B} > \tilde{A} \). The ranking results by the proposed method is given in Table II. The ranking results by the proposed method and the ranking results of the existing methods [22, 25, 27, 31, 35] the ranking order is \( \tilde{C} > \tilde{B} > \tilde{A} \) which is the same as the one obtained by our approach. However, our approach is simpler in the computation procedure. The ranking results of other approaches are given in Table III. By the CV index approach [27], the ranking order is \( \tilde{A} > \tilde{C} > \tilde{B} \), which is counter-intuition (see set 4 of Fig 2).

Set 5: \( \tilde{A} = (0.4, 0.7, 0.1, 0.2; 1)_{LR}, \tilde{B} = (0.7, 0.7, 0.4, 0.2; 1)_{LR}, \text{ and } \tilde{C} = (0.7, 0.7, 0.2, 0.2; 1)_{LR} \);

By using various ranking indexes with various linear and nonlinear functions as defined in section 3.1 the ranking order of \( \tilde{A} \), \( \tilde{B} \) and \( \tilde{C} \) in case(i, ii, iii and v) is \( \tilde{C} > \tilde{B} > \tilde{A} \) and in case(iv) the ranking order is \( \tilde{B} > \tilde{C} > \tilde{A} \). The ranking results by the proposed method is given in Table II. The ranking results of other approaches are given in Table III. By Bortolan-Degani [35] the ranking order is \( \tilde{C} > \tilde{B} = \tilde{A} \) and by the CV index [27] the ranking order is \( \tilde{A} > \tilde{B} > \tilde{C} \), both the ranking orders is counter-intuition (see set 5 of Fig 2).

**Example 5.**
Consider set 6 with two fuzzy numbers [26], \( \tilde{A} = (2, 2, 1, 3; 1)_{LR}, \tilde{B} = (2, 2, 1, 2; 1)_{LR} \) as shown in Fig. 2.
By using various ranking indexes with various linear and nonlinear functions as defined in section 3.1 the ranking order of \( \tilde{A} \) and \( \tilde{B} \) in all the cases is \( \tilde{A} > \tilde{B} \), the ranking results by the proposed method is given in Table II. Liou and Wang [26] approach obtained the different ranking order when different optimistic indices are adopted. Chu and Tsao [22] approach obtain the ranking order as \( \tilde{A} > \tilde{B} \) which is same as the proposed approach. But our approach is simpler in the computation procedure. By Deng et al. [36] approach the ranking order is \( \tilde{B} > \tilde{A} \). From Fig. 2 of set 6, we can conclude that \( \tilde{A} > \tilde{B} \) is more consistent with human intuition.

### Table III: Comparative results of Example 4

<table>
<thead>
<tr>
<th>Methods</th>
<th>Set 4</th>
<th>Set 5</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \tilde{A} )</td>
<td>( \tilde{B} )</td>
<td>( \tilde{C} )</td>
<td>( \tilde{A} )</td>
</tr>
<tr>
<td>Bortolan-Degani [35] Results</td>
<td>0.30</td>
<td>0.33</td>
<td>0.44</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chu-Tsao [22] Results</td>
<td>0.2999</td>
<td>0.350</td>
<td>0.3993</td>
<td>0.2847</td>
</tr>
<tr>
<td></td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yao-Wu [25] Results</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.575</td>
</tr>
<tr>
<td></td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sign distance (p = 1) [1]</td>
<td>1.2</td>
<td>1.4</td>
<td>1.6</td>
<td>1.15</td>
</tr>
<tr>
<td>Results</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sign distance (p = 2) [1]</td>
<td>0.8869</td>
<td>1.0194</td>
<td>1.1605</td>
<td>0.8756</td>
</tr>
<tr>
<td>Results</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cheng distance [27] Results</td>
<td>0.79</td>
<td>0.8602</td>
<td>0.9268</td>
<td>0.7577</td>
</tr>
<tr>
<td></td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cheng CV uniform distribution [27] Results</td>
<td>0.0272</td>
<td>0.0214</td>
<td>0.0225</td>
<td>0.0328</td>
</tr>
<tr>
<td></td>
<td>( \tilde{A} &gt; \tilde{C} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} &gt; \tilde{C} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cheng CV proportional distribution [27] Results</td>
<td>0.0183</td>
<td>0.0128</td>
<td>0.0137</td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td>( \tilde{A} &gt; \tilde{C} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} &gt; \tilde{C} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asady-Zendehnam distance [28] Results</td>
<td>0.6</td>
<td>0.7</td>
<td>0.9</td>
<td>0.575</td>
</tr>
<tr>
<td></td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: “\( \square \)” indicates incorrect ranking results.
Example 6.
Consider set 7, the images of set 6, \(-\tilde{A} = (-3, -1, -2, -2; 1)_{LR}\), \(-\tilde{B} = (-2, -1, -2, -2; 1)_{LR}\) as shown in Fig. 2. By using various ranking indexes with various linear and nonlinear functions as defined in section 3.1 the ranking order of \(-\tilde{A}\) and \(-\tilde{B}\) in all the cases is \(-\tilde{A} < -\tilde{B}\), the ranking results by the proposed method is given in Table II.
From set 6 and set 7, \(\tilde{A} > B \Rightarrow -\tilde{A} < -\tilde{B}\)
Therefore the proposed ranking method can rank images also.

Example 7.
Consider set 8 with two fuzzy numbers \(\tilde{A} = (2, 2, 1, 3; 0.8)_{LR}\), \(\tilde{B} = (1, 1, 1, 1; 1)_{LR}\) as shown in Fig. 2.
By using various ranking indexes with various linear and nonlinear functions as defined in section 3.1 the ranking order of \(\tilde{A}\) and \(\tilde{B}\) in all the cases is \(\tilde{A} > \tilde{B}\), the ranking results by the proposed method is given in Table II. Therefore the proposed ranking method can rank fuzzy numbers with different heights and it can rank crisp numbers also. Yager [38], cannot rank generalized LR fuzzy numbers, but by the proposed method we can overcome the limitations and shortcomings of the existing method and the proposed method is relatively simple in computation and is easily logical. Till now in the existing literature there are no ranking methods to rank L-R fuzzy numbers with different heights.
Fig: 2 Eight sets of fuzzy numbers.
Table II: Ranking values of the proposed ranking method

<table>
<thead>
<tr>
<th>Fuzzy number</th>
<th>Case(i)</th>
<th>Case(ii)</th>
<th>Case(iii)</th>
<th>Case(iv)</th>
<th>Case(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{A} = (2,2,0,0.1,0.1; 1)_{u} )</td>
<td>1.7777</td>
<td>1.7777</td>
<td>1.7827</td>
<td>1.7925</td>
<td>1.7802</td>
</tr>
<tr>
<td>( \tilde{B} = (3,3,0,9,1; 1)_{u} )</td>
<td>2.6716</td>
<td>2.6765</td>
<td>2.7209</td>
<td>2.8049</td>
<td>2.6962</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>( \tilde{A} &lt; \tilde{B} )</td>
<td>( \tilde{A} &lt; \tilde{B} )</td>
<td>( \tilde{A} &lt; \tilde{B} )</td>
<td>( \tilde{A} &lt; \tilde{B} )</td>
<td>( \tilde{A} &lt; \tilde{B} )</td>
</tr>
<tr>
<td>( \tilde{A} = (6,6,1,1; 1)_{u} )</td>
<td>5.3333</td>
<td>5.3333</td>
<td>5.3827</td>
<td>5.4814</td>
<td>5.3580</td>
</tr>
<tr>
<td>( \tilde{B} = (6,6,0,1,1; 1)_{u} )</td>
<td>5.3777</td>
<td>5.4222</td>
<td>5.4271</td>
<td>5.3925</td>
<td>5.4024</td>
</tr>
<tr>
<td>( \tilde{C} = (6,6,0,1; 1)_{u} )</td>
<td>5.3827</td>
<td>5.4320</td>
<td>5.4320</td>
<td>5.3827</td>
<td>5.4074</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{B} &gt; \tilde{C} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
</tr>
<tr>
<td>( \tilde{A} = (6,6,3,3; 1)_{u} )</td>
<td>6</td>
<td>6</td>
<td>5.4814</td>
<td>5.7777</td>
<td>5.4074</td>
</tr>
<tr>
<td>( \tilde{B} = (6,6,1,1; 1)_{u} )</td>
<td>2</td>
<td>2</td>
<td>5.3827</td>
<td>5.4814</td>
<td>5.3580</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
</tr>
<tr>
<td>( \tilde{A} = (0.5,0.5,0.1,0.5; 1)_{u} )</td>
<td>0.4641</td>
<td>0.4345</td>
<td>0.4395</td>
<td>0.4543</td>
<td>0.4395</td>
</tr>
<tr>
<td>( \tilde{B} = (0.7,0.7,0.3,0.3; 1)_{u} )</td>
<td>0.6222</td>
<td>0.6222</td>
<td>0.6370</td>
<td>0.6666</td>
<td>0.6296</td>
</tr>
<tr>
<td>( \tilde{C} = (0.9,0.9,0.5,0.1; 1)_{u} )</td>
<td>0.7802</td>
<td>0.7604</td>
<td>0.7851</td>
<td>0.8543</td>
<td>0.7827</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
</tr>
<tr>
<td>( \tilde{A} = (0.4,0.7,0.1,0.2; 1)_{u} )</td>
<td>0.5382</td>
<td>0.5432</td>
<td>0.5481</td>
<td>0.5530</td>
<td>0.5432</td>
</tr>
<tr>
<td>( \tilde{B} = (0.7,0.7,0.4,0.2; 1)_{u} )</td>
<td>0.6123</td>
<td>0.6024</td>
<td>0.6222</td>
<td>0.6716</td>
<td>0.6172</td>
</tr>
<tr>
<td>( \tilde{C} = (0.7,0.7,0.2,0.2; 1)_{u} )</td>
<td>0.6222</td>
<td>0.6222</td>
<td>0.6320</td>
<td>0.6518</td>
<td>0.6271</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
<td>( \tilde{C} &gt; \tilde{B} &gt; \tilde{A} )</td>
</tr>
<tr>
<td>( \tilde{A} = (2,2,1,3; 1)_{u} )</td>
<td>1.8765</td>
<td>1.9753</td>
<td>2.0246</td>
<td>2.0246</td>
<td>1.9506</td>
</tr>
<tr>
<td>( \tilde{B} = (2,2,1,2; 1)_{u} )</td>
<td>1.8271</td>
<td>1.8765</td>
<td>1.9259</td>
<td>1.9753</td>
<td>1.8765</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
</tr>
<tr>
<td>( \tilde{A} = (-3,-1,-2,-2; 1)_{u} )</td>
<td>-1.4814</td>
<td>-1.4814</td>
<td>-1.5802</td>
<td>-1.7777</td>
<td>-1.5308</td>
</tr>
<tr>
<td>( \tilde{B} = (-2,-1,-2,-2; 1)_{u} )</td>
<td>-1.1851</td>
<td>-1.1851</td>
<td>-1.2839</td>
<td>-1.4814</td>
<td>-1.2345</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>( \tilde{A} &lt; \tilde{B} )</td>
<td>( \tilde{A} &lt; \tilde{B} )</td>
<td>( \tilde{A} &lt; \tilde{B} )</td>
<td>( \tilde{A} &lt; \tilde{B} )</td>
<td>( \tilde{A} &lt; \tilde{B} )</td>
</tr>
<tr>
<td>( \tilde{A} = (2,2,1,3; 0.8)_{u} )</td>
<td>1.5012</td>
<td>1.5802</td>
<td>1.6197</td>
<td>1.6197</td>
<td>1.5604</td>
</tr>
<tr>
<td>( \tilde{B} = (1,1,1,1; 1)_{u} )</td>
<td>0.8888</td>
<td>0.8888</td>
<td>0.9382</td>
<td>1.0370</td>
<td>0.9135</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
<td>( \tilde{A} &gt; \tilde{B} )</td>
</tr>
</tbody>
</table>
5. Linear Programming Model for Fuzzy Multi Objective Transportation Model

In this section, Fuzzy multiobjective transportation model with fuzzy cost and fuzzy time, mathematical formulation of fuzzy multiobjective transportation model is presented and we apply the proposed ranking method to solve the total optimal fuzzy cost, and fuzzy time for fuzzy multiobjective transportation problem using linear programming to various linear and non-linear functions with LR fuzzy numbers, so as to minimize the fuzzy cost, and fuzzy time.

Generally the fuzzy transportation problem is to transport various amounts of a single homogeneous commodity that are initially stored at various sources, to different destinations in such a way that the total fuzzy transportation cost is a minimum. Let there be m sources, i\textsuperscript{th} sources possessing $\tilde{a}_i$ fuzzy supply units of a certain product, n destinations (n may or may not be equal to m) with destination j requiring $\tilde{b}_j$ fuzzy demand units. Cost of shipping of an item from each of m sources to each of the n destinations are known either directly or indirectly in terms of mileage, shipping hours, etc. If the objective of a transportation problem is to minimize fuzzy cost, and fuzzy time, then this type of fuzzy problem is treated as a fuzzy multiobjective transportation problem. Here, we consider fuzzy transportation problem with two objectives in the following form of m \times n fuzzy matrix (Table IV) where each cell having a fuzzy cost $\tilde{c}_{ij}$, and fuzzy time $\tilde{t}_{ij}$.

Table IV: Fuzzy multi objective transportation model with fuzzy cost and fuzzy time

<table>
<thead>
<tr>
<th>Source/ Destination</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>j</th>
<th>...</th>
<th>n</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\tilde{c}<em>{11}$; $\tilde{t}</em>{11}$</td>
<td>$\tilde{c}<em>{1i}$; $\tilde{t}</em>{1i}$</td>
<td>$\cdots$</td>
<td>$\tilde{c}<em>{1j}$; $\tilde{t}</em>{1j}$</td>
<td>$\cdots$</td>
<td>$\tilde{c}<em>{1n}$; $\tilde{t}</em>{1n}$</td>
<td>$\tilde{a}_i$</td>
</tr>
<tr>
<td>B</td>
<td>$\tilde{c}<em>{21}$; $\tilde{t}</em>{21}$</td>
<td>$\tilde{c}<em>{2i}$; $\tilde{t}</em>{2i}$</td>
<td>$\cdots$</td>
<td>$\tilde{c}<em>{2j}$; $\tilde{t}</em>{2j}$</td>
<td>$\cdots$</td>
<td>$\tilde{c}<em>{2n}$; $\tilde{t}</em>{2n}$</td>
<td>$\tilde{a}_i$</td>
</tr>
<tr>
<td></td>
<td>$\cdots$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>I</td>
<td>$\tilde{c}<em>{m1}$; $\tilde{t}</em>{m1}$</td>
<td>$\tilde{c}<em>{mi}$; $\tilde{t}</em>{mi}$</td>
<td>$\cdots$</td>
<td>$\tilde{c}<em>{mj}$; $\tilde{t}</em>{mj}$</td>
<td>$\cdots$</td>
<td>$\tilde{c}<em>{mn}$; $\tilde{t}</em>{mn}$</td>
<td>$\tilde{a}_i$</td>
</tr>
<tr>
<td></td>
<td>$\cdots$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>M</td>
<td>$\tilde{c}<em>{i1}$; $\tilde{t}</em>{i1}$</td>
<td>$\tilde{c}<em>{in}$; $\tilde{t}</em>{in}$</td>
<td>$\cdots$</td>
<td>$\tilde{c}<em>{in}$; $\tilde{t}</em>{in}$</td>
<td>$\cdots$</td>
<td>$\tilde{c}<em>{in}$; $\tilde{t}</em>{in}$</td>
<td>$\tilde{a}_m$</td>
</tr>
<tr>
<td>Demand</td>
<td>$\tilde{b}_1$</td>
<td>$\tilde{b}_2$</td>
<td>$\cdots$</td>
<td>$\tilde{b}_j$</td>
<td>$\cdots$</td>
<td>$\tilde{b}_n$</td>
<td>$\tilde{b}_s$</td>
</tr>
</tbody>
</table>
5.1. Mathematical formulation of fuzzy multiobjective transportation model

Mathematically, the fuzzy multiobjective transportation problem in Table IV can be stated as:

Minimize $\bar{z}_k = \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \bar{p}_{ij} \right) x_{ij}$,

subject to $\sum_{j=1}^{n} x_{ij} = \bar{a}_i \quad i = 1, 2, \ldots, m$

$\sum_{i=1}^{m} x_{ij} = \bar{b}_j \quad j = 1, 2, \ldots, n$

where $\bar{z}_k = \{ \bar{z}_1, \bar{z}_2, \ldots, \bar{z}_k \}$ is a vector of k-objective functions.

If the objective function $\bar{z}_i$ denotes the fuzzy cost function,

Minimize $\bar{z}_i = \sum_{i=1}^{m} \sum_{j=1}^{n} \bar{c}_{ij} x_{ij}$,

If the objective function $\bar{z}_2$ denotes the fuzzy time function,

Minimize $\bar{z}_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \bar{t}_{ij} x_{ij}$,

Then it is a two objective fuzzy transportation problem. Use weights to consider the priorities of the objective.

Minimize $\bar{z} = w_1 \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \bar{c}_{ij} \right) x_{ij} + w_2 \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \bar{t}_{ij} \right) x_{ij}$

subject to $\sum_{j=1}^{n} x_{ij} = \bar{a}_i \quad i = 1, 2, \ldots, m$

$\sum_{i=1}^{m} x_{ij} = \bar{b}_j \quad j = 1, 2, \ldots, n$

and $w_1 + w_2 = 1$.

$x_{ij} \geq 0 \quad i = 1, 2, \ldots, m \quad j = 1, 2, \ldots, n$.

where $\bar{c}_{ij} = \left( m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij} \right)_{LR}$ : Fuzzy cost from $i^{th}$ source to $j^{th}$ destination

$\bar{t}_{ij} = \left( m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij} \right)_{LR}$ : Fuzzy time from $i^{th}$ source to $j^{th}$ destination

$\bar{a}_i = \left( m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij} \right)_{LR}$ : Fuzzy supply from $i^{th}$ source to $j^{th}$ destination

$\bar{b}_j = \left( m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij} \right)_{LR}$ : Fuzzy demand from $i^{th}$ source to $j^{th}$ destination
(i) \( L(x) = R(x) = \max \{0, 1 - |x|\} \),
(ii) \( L(x) = R(x) = e^{-x} \),
(iii) \( L(x) = \max \{0, 1 - |x|\} \) and \( R(x) = e^{-x} \),
(iv) \( L(x) = e^{-x} \) and \( R(x) = \max \{0, 1 - |x|\} \),
(v) \( L(x) = e^{-x} \) and \( R(x) = \max 0, 1 - |x| \)

\( L(x) \) = left shape functions; \( R(x) \) = right shape functions.

All \( \tilde{c}_{ij}, \tilde{t}_{ij}, \tilde{a}_i, \tilde{b}_j \) denotes a non-negative LR fuzzy numbers.

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} (\tilde{c}_{ij})x_{ij} : \text{Total fuzzy cost for shipping from } i^{th} \text{ source to } j^{th} \text{ destination.} \]
\[ \sum_{i=1}^{m} \sum_{j=1}^{n} (\tilde{t}_{ij})x_{ij} : \text{Total fuzzy time for shipping from } i^{th} \text{ source to } j^{th} \text{ destination.} \]

5.2. The total optimal fuzzy solution for fuzzy multiobjective transportation problem

In this section, the proposed method to solve the total optimal fuzzy cost, and fuzzy time for fuzzy multiobjective transportation problem using linear programming as follows:

Step 1: First test whether the given fuzzy multiobjective transportation problem is a balanced one or not. If it is a balanced one (i.e., sum of supply units equal to the sum of demand units) then go to step 3. If it is an unbalanced one (i.e sum of supply units is not equal to the sum of demand units) then go to step 2.

Step 2: Introduce dummy rows and/or columns with zero fuzzy costs, and time so as to form a balanced one.

Step 3: Consider the fuzzy linear programming model as proposed in section 5.1.

Step 4: Convert the fuzzy multiobjective transportation problem into the following crisp linear programming problem

Minimize \( \tilde{z} = w_1 \sum_{i=1}^{m} \sum_{j=1}^{n} R(\tilde{c}_{ij})x_{ij} + w_2 \sum_{i=1}^{m} \sum_{j=1}^{n} R(\tilde{t}_{ij})x_{ij} \)

subject to \( \sum_{j=1}^{n} x_{ij} \geq R(\tilde{a}_i) \) \( i=1,2,\ldots,m \)
\( \sum_{i=1}^{m} x_{ij} \geq R(\tilde{b}_j) \) \( j=1,2,\ldots,n \)

and \( w_1 + w_2 = 1. \)
x_{ij} \geq 0 \quad i = 1,2,\ldots,m; \quad j=1,2,\ldots,n

**Step 5:** Based on the case chosen in section 3.1, calculate the values of \( R(\tilde{c}_i), R(\tilde{t}_j), R(\tilde{a}_i), \) and \( R(\tilde{b}_j) \) \( \forall i, j \) by using the ranking procedure as mentioned in section 3.1 for the chosen fuzzy multi-objective transportation problem.

**Step 6:** For the values obtained in step 5, and using the assigned values of \( w_1 \) and \( w_2 \), the multi-objective transportation problem is converted into a single objective crisp transportation problem as follows:

Minimize \[ z = \sum_{i=1}^{m} \sum_{j=1}^{n} Q_{ij} x_{ij} \]

subject to \[ \sum_{j=1}^{n} x_{ij} \geq R(\tilde{a}_i) \quad i = 1,2,\ldots,m \]
\[ \sum_{i=1}^{m} x_{ij} \geq R(\tilde{b}_j) \quad j = 1,2,\ldots,n \]
\[ x_{ij} \geq 0 \quad i = 1,2,\ldots,m; \quad j=1,2,\ldots,n. \]

where \( Q_{ij} \) is a constant.

**Step 7:** To find the optimal solution \( \{x_{ij}\} \), solve the crisp linear programming problem obtained in step 6 by using software like TORA.

**Step 8:** Find the optimal total fuzzy transportation cost, and total fuzzy transportation time by substituting the optimal solution obtained in step 7 in the objective function of step 3.

In the following, we use the proposed method to deal with the fuzzy multi-objective transportation problem.

To illustrate the proposed model, consider a case of transportation process with three sources and three destinations, as a fuzzy multi-objective transportation problem so as to minimize the fuzzy cost, and fuzzy time. The cost coefficients, the time, the supply and demand in fuzzy multi-objective transportation problem are considered as LR fuzzy numbers. The fuzzy multi-objective transportation problem with fuzzy cost, fuzzy time, and fuzzy supply and fuzzy demand is shown in Table V and it is solved by using various cases.

**TABLE V:** Fuzzy Multi Objective Transportation Problem with Fuzzy Cost and Fuzzy Time

<table>
<thead>
<tr>
<th>Source/ Destination</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>supply (( \tilde{a}_i ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(6,7,2,2)</td>
<td>(5,7,2,3)</td>
<td>(8,10,3,2)</td>
<td>(5,7,4,2)</td>
</tr>
<tr>
<td></td>
<td>(9,11,2,2)</td>
<td>(9,10,3,2)</td>
<td>(4,5,2,2)</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>(3,5,1,2)</td>
<td>(7,8,2,3)</td>
<td>(8,7,2,4)</td>
<td>(7,8,3,2)</td>
</tr>
<tr>
<td></td>
<td>(7,10,1,2)</td>
<td>(12,14,3,3)</td>
<td>(5,8,2,1)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>(8,10,2,2)</td>
<td>(7,12,2,2)</td>
<td>(8,9,1,2)</td>
<td>(5,8,1,3)</td>
</tr>
<tr>
<td></td>
<td>(4,5,1,2)</td>
<td>(5,7,1,2)</td>
<td>(7,9,1,2)</td>
<td></td>
</tr>
<tr>
<td>Demand (( \tilde{b}_j ))</td>
<td>(5,8,2,4)</td>
<td>(8,9,4,1)</td>
<td>(4,6,2,2)</td>
<td></td>
</tr>
</tbody>
</table>

Note: “" indicates fuzzy time.
Fuzzy multi objective transportation model

The total optimal fuzzy cost and fuzzy time for fuzzy multiobjective transportation problem using fuzzy linear programming for various cases as follows:

Case (i) \( L(x) = R(x) = \max \{0, 1 - |x|\} \)

**Step 1:** The given fuzzy multi-objective transportation problem is a balanced one.

**Step 2:** Using step3 of the proposed model, the given fuzzy multi-objective transportation problem is converted into a single fuzzy objective problem as follows

Minimize \( \tilde{z} = 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} (\tilde{c}_{ij})x_{ij} + 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} (\tilde{t}_{ij})x_{ij} \)

subject to \( \sum_{j=1}^{3} x_{ij} = \tilde{a}_i \quad i = 1, 2, 3 \)

\( \sum_{i=1}^{3} x_{ij} = \tilde{b}_j \quad j = 1, 2, 3 \)

Here, \( w_1 = 0.5, w_2 = 0.5 \).

**Step 3:** The fuzzy multi-objective transportation problem is converted into the following crisp linear programming problem

Minimize \( z = 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} R(\tilde{c}_{ij})x_{ij} + 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} R(\tilde{t}_{ij})x_{ij} \)

subject to \( \sum_{j=1}^{3} x_{ij} = R(\tilde{a}_i) \quad i = 1, 2, 3 \).

\( \sum_{i=1}^{3} x_{ij} = R(\tilde{b}_j) \quad j = 1, 2, 3 \).

**Step 4:** Using section 3.1., the values of \( R(\tilde{c}_{ij}), R(\tilde{t}_{ij}), R(\tilde{a}_i) \) and \( R(\tilde{b}_j) \) \( \forall i, j \) are calculated and given in Table VI.

**TABLE VI: Ranks of Fuzzy Cost and Fuzzy Time of case (i)**

<table>
<thead>
<tr>
<th>Source/ Destination</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.92</td>
<td>5.67</td>
<td>8.24</td>
<td>5.52</td>
</tr>
<tr>
<td></td>
<td>9.18</td>
<td>8.54</td>
<td>4.14</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>3.90</td>
<td>6.86</td>
<td>6.61</td>
<td>6.76</td>
</tr>
<tr>
<td></td>
<td>8.04</td>
<td>11.85</td>
<td>6.17</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>8.29</td>
<td>9.18</td>
<td>7.75</td>
<td>6.32</td>
</tr>
<tr>
<td></td>
<td>4.19</td>
<td>5.67</td>
<td>7.45</td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>6.32</td>
<td>7.55</td>
<td>4.74</td>
<td></td>
</tr>
</tbody>
</table>

Note: “ ” indicates rank of fuzzy time
Step 5: Using step 6 of the proposed method convert the chosen fuzzy multiobjective transportation problem into the following crisp linear programming

Minimize: $(7.55)x_{11} + (7.07)x_{12} + (6.19)x_{13} +
(5.97)x_{21} + (9.35)x_{22} + (6.39)x_{23} +
(6.24)x_{31} + (7.42)x_{32} + (7.6)x_{33}.

subject to: $x_{11} + x_{12} + x_{13} = 5.52$ ;
$x_{12} + x_{22} + x_{32} = 6.76$ ;
$x_{13} + x_{23} + x_{33} = 6.32$ ;
$x_{11} + x_{12} + x_{13} = 6.32$ ;
$x_{21} + x_{22} + x_{23} = 7.55$ ;
$x_{31} + x_{32} + x_{33} = 4.74$ .

$x_{ij} \geq 0$, for all $i=1,2,3$ and $j=1,2,3$.

Step 6: Solve the crisp linear programming problem, obtained in step 5, by using TORA software the optimal solution obtained is:

$x_{12} = 2.02$, $x_{13} = 4.30$, $x_{21} = 5.53$, $x_{23} = 2.02$, $x_{32} = 4.74$.

Step 7: Using step 8 of the proposed model, the minimum fuzzy transportation cost and time respectively are $(110.43, 155.81, 35.99, 43.28)$ and $(107.89, 146.34, 28.97, 35.2)$.

Case (ii) $L(x) = R(x) = e^{-x}$

Step 1: The given fuzzy multiobjective transportation problem is a balanced one.

Step 2: Using step 3 of the proposed model, the given fuzzy multiobjective transportation problem is converted into a single fuzzy objective problem as follows

Minimize $\tilde{z} = 0.5\sum_{i=1}^{3}\sum_{j=1}^{3}(\tilde{c}_{ij})x_{ij} + 0.5\sum_{i=1}^{3}\sum_{j=1}^{3}(\tilde{r}_{ij})x_{ij}$

subject to

$\sum_{j=1}^{3} x_{ij} \approx \tilde{a}_{i}$ \hspace{0.5cm} $i=1,2,3$

$\sum_{i=1}^{3} x_{ij} \approx \tilde{b}_{j}$ \hspace{0.5cm} $j=1,2,3$

Here, $w_1 = 0.5$, $w_2 = 0.5$.

Step 3: The fuzzy multiobjective transportation problem is converted into the following crisp linear programming problem

Minimize $z = 0.5\sum_{i=1}^{3}\sum_{j=1}^{3}R(\tilde{c}_{ij})x_{ij} + 0.5\sum_{i=1}^{3}\sum_{j=1}^{3}R(\tilde{r}_{ij})x_{ij}$

subject to $\sum_{j=1}^{3} x_{ij} \approx R(\tilde{a}_{i})$ \hspace{0.5cm} $i=1,2,3$

$\sum_{i=1}^{3} x_{ij} \approx R(\tilde{b}_{j})$ \hspace{0.5cm} $j=1,2,3$
Step 4: Using section 3.1, the values of \( \tilde{c}_{ij}, \tilde{t}_{ij}, \tilde{a}_i \) and \( \tilde{b}_j \) for all \( i, j \) are calculated and given in Table VII.

**TABLE VII: Ranks of Fuzzy Cost and Fuzzy Time of case (ii)**

<table>
<thead>
<tr>
<th>Source/ Destination</th>
<th>1</th>
<th>2</th>
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<th>supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.92</td>
<td>5.72</td>
<td>8.19</td>
<td>5.43</td>
</tr>
<tr>
<td></td>
<td>9.18</td>
<td>8.49</td>
<td>4.14</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>3.95</td>
<td>6.91</td>
<td>6.71</td>
<td>6.71</td>
</tr>
<tr>
<td></td>
<td>8.09</td>
<td>11.85</td>
<td>6.12</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>8.29</td>
<td>9.18</td>
<td>7.80</td>
<td>6.41</td>
</tr>
<tr>
<td></td>
<td>4.24</td>
<td>5.72</td>
<td>7.50</td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>6.41</td>
<td>7.40</td>
<td>4.74</td>
<td></td>
</tr>
</tbody>
</table>

Note: “ ” indicates rank of fuzzy time

Step 5: Using step 6 of the proposed method convert the chosen fuzzy multiobjective transportation problem into the following crisp linear programming

Minimize: \((7.55)x_{11} + (7.10)x_{12} + (6.16)x_{13} + (6.02)x_{21} + (9.38)x_{22} + (6.41)x_{23} + (6.26)x_{31} + (7.45)x_{32} + (7.65)x_{33} \)

subject to: \(x_{11} + x_{21} + x_{31} = 5.43\) ; \(x_{12} + x_{22} + x_{32} = 6.71\) ; 
\(x_{13} + x_{23} + x_{33} = 6.41\) ; \(x_{11} + x_{12} + x_{13} = 6.41\) ;
\(x_{21} + x_{22} + x_{23} = 7.40\) ; \(x_{31} + x_{32} + x_{33} = 4.74\) .

\(x_{ij} \geq 0, \text{ for all } i=1,2,3. \text{ and } j=1,2,3.\)

Step 6: Solve the crisp linear programming problem, obtained in step 5, by using TORA software the optimal solution obtained is:

\(x_{12} = 1.97, x_{13} = 4.44, x_{21} = 5.43, x_{23} = 1.97, x_{32} = 4.74.\)

Step 7: Using step 8 of the proposed model, the minimum fuzzy transportation cost and time respectively are (110.6, 156.01, 36.11, 43.01) and (107.05, 145.14, 28.9, 35.13).

Case (iii) \(L(x) = \max \{0, 1 - |x|\} \text{ and } R(x) = e^{-x}\)

Step 1: The given fuzzy multiobjective transportation problem is a balanced one.
Step 2: Using step3 of the proposed model, the given fuzzy multiobjective transportation problem is converted into a single fuzzy objective problem as follows
Minimize \( \tilde{z} = 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} (\tilde{c}_{ij})x_{ij} + 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} (\tilde{t}_{ij})x_{ij} \)

subject to \( \sum_{j=1}^{3} x_{ij} = \tilde{a}_{i} \quad i=1,2,3 \)
\( \sum_{i=1}^{3} x_{ij} = \tilde{b}_{j} \quad j=1,2,3 \)

Here, \( w_1 = 0.5, w_2 = 0.5. \)

**Step 3:** The fuzzy multiobjective transportation problem is converted into the following crisp linear programming problem

Minimize \( z = 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} R(\tilde{c}_{ij})x_{ij} + 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} R(\tilde{t}_{ij})x_{ij} \)

subject to \( \sum_{j=1}^{3} x_{ij} = R(\tilde{a}_{i}) \quad i=1,2,3 \)
\( \sum_{i=1}^{3} x_{ij} = R(\tilde{b}_{j}) \quad j=1,2,3 \)

**Step 4:** Using section 3.1, the values of \( R(\tilde{c}_{ij}), R(\tilde{t}_{ij}), R(\tilde{a}_{i}) \) and \( R(\tilde{b}_{j}) \) \( \forall i, j \) are calculated and given in Table VIII.

**TABLE VIII:** Ranks of Fuzzy Cost and Fuzzy Time of case (iii)

<table>
<thead>
<tr>
<th>Source/ Destination</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.02</td>
<td>5.82</td>
<td>8.34</td>
<td>5.62</td>
</tr>
<tr>
<td></td>
<td>9.28</td>
<td>8.64</td>
<td>4.24</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>7.01</td>
<td>6.81</td>
<td>6.86</td>
</tr>
<tr>
<td></td>
<td>8.14</td>
<td>12</td>
<td>6.22</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>8.39</td>
<td>9.28</td>
<td>7.85</td>
<td>6.46</td>
</tr>
<tr>
<td></td>
<td>4.29</td>
<td>5.77</td>
<td>7.55</td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>6.51</td>
<td>7.60</td>
<td>4.83</td>
<td></td>
</tr>
</tbody>
</table>

Note: “\( \square \)” indicates rank of fuzzy time

**Step 5:** Using step 6 of the proposed method convert the chosen fuzzy multiobjective transportation problem into the following crisp linear programming

Minimize: \((7.65)x_{11} + (7.23)x_{12} + (6.29)x_{13} + (6.07)x_{21} + (9.50)x_{22} + (6.51)x_{23} + \)
(6.34)x_{31} + (7.52)x_{32} + (7.7)x_{33}.

subject to: x_{11} + x_{21} + x_{31} = 5.62; \quad x_{12} + x_{22} + x_{32} = 6.86; \\
x_{13} + x_{23} + x_{33} = 6.46; \quad x_{11} + x_{12} + x_{13} = 6.51; \\
x_{21} + x_{22} + x_{23} = 7.60; \quad x_{31} + x_{32} + x_{33} = 4.83.

x_{ij} \geq 0, \text{ for all } i=1,2,3. \text{ and } j=1,2,3.

**Step 6:** Solve the crisp linear programming problem, obtained in step 5, by using TORA software the optimal solution obtained is:

\begin{align*}
x_{12} &= 2.03, \\
x_{13} &= 4.48, \\
x_{21} &= 5.62, \\
x_{23} &= 1.98, \\
x_{32} &= 4.83.
\end{align*}

**Step 7:** Using step 8 of the proposed model, the minimum fuzzy transportation cost and time respectively are (112.5, 158.93, 36.74, 43.87) and (109.58, 148.55, 29.46, 35.9).

**Case (iv)\right\{\begin{align*}
L(x) &= e^{-x} \text{ and } R(x) = \max \{0, 1 - |x|\} \\

**Step 1:** The given fuzzy multiobjective transportation problem is a balanced one.

**Step 2:** Using step 3 of the proposed model, the given fuzzy multiobjective transportation problem is converted into a single fuzzy objective problem as follows

\text{Minimize } \tilde{z} = 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} (\tilde{c}_{ij})x_{ij} + 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} (\tilde{t}_{ij})x_{ij}

\text{subject to } \sum_{j=1}^{3} x_{ij} \equiv \tilde{a}_{i}, \quad i=1,2,3

\sum_{i=1}^{3} x_{ij} \equiv \tilde{b}_{j}, \quad j=1,2,3

Here, \( w_{1} = 0.5, w_{2} = 0.5.\)

**Step 3:** The fuzzy multiobjective transportation problem is converted into the following crisp linear programming problem

\text{Minimize } z = 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} R(\tilde{c}_{ij})x_{ij} + 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} R(\tilde{t}_{ij})x_{ij}

\text{subject to } \sum_{j=1}^{3} x_{ij} \equiv R(\tilde{a}_{i}), \quad i=1,2,3

\sum_{i=1}^{3} x_{ij} \equiv R(\tilde{b}_{j}), \quad j=1,2,3

**Step 4:** Using section 3.1., the values of \( R(\tilde{c}_{ij}), R(\tilde{t}_{ij}), R(\tilde{a}_{i})\text{ and } R(\tilde{b}_{j}) \forall i, j \) are calculated and given in Table IX.
TABLE IX: Ranks of Fuzzy Cost and Fuzzy Time of case (iv)

<table>
<thead>
<tr>
<th>Source/Destination</th>
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<th>2</th>
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<th>Supply</th>
</tr>
</thead>
<tbody>
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<td>6.22</td>
<td>5.97</td>
<td>8.69</td>
<td>6.12</td>
</tr>
<tr>
<td></td>
<td>9.48</td>
<td>8.98</td>
<td>4.44</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>4.04</td>
<td>7.16</td>
<td>6.91</td>
<td>7.20</td>
</tr>
<tr>
<td></td>
<td>8.19</td>
<td>12.29</td>
<td>6.46</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>8.59</td>
<td>9.48</td>
<td>7.90</td>
<td>6.46</td>
</tr>
<tr>
<td></td>
<td>4.34</td>
<td>5.82</td>
<td>7.60</td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>6.61</td>
<td>8.14</td>
<td>5.03</td>
<td></td>
</tr>
</tbody>
</table>

Note: “     ” indicates rank of fuzzy time

**Step 5:** Using step 6 of the proposed method convert the chosen fuzzy multiobjective transportation problem into the following crisp linear programming.

Minimize: 
\[
(7.85)x_{11} + (7.47)x_{12} + (6.56)x_{13} + \\
(6.11)x_{21} + (9.72)x_{22} + (6.68)x_{23} + \\
(6.46)x_{31} + (7.65)x_{32} + (7.75)x_{33}.
\]

subject to: 
\[
x_{11} + x_{21} + x_{31} = 6.12 \text{; } x_{12} + x_{22} + x_{32} = 7.20 \text{; } \\
x_{13} + x_{23} + x_{33} = 6.46 \text{; } x_{11} + x_{12} + x_{13} = 6.61 \text{; } \\
x_{21} + x_{22} + x_{23} = 8.14 \text{; } x_{31} + x_{32} + x_{33} = 5.03.
\]

\[x_{ij} \geq 0, \text{ for all } i=1,2,3, \text{ and } j=1,2,3.\]

**Step 6:** Solve the crisp linear programming problem, obtained in step 5, by using TORA software the optimal solution obtained is: 
\[x_{12} = 2.17, x_{13} = 4.44, x_{21} = 6.12, x_{23} = 2.02, x_{32} = 5.03.\]

**Step 7:** Using step 8 of the proposed model, the minimum fuzzy transportation cost and time respectively are (116.1, 164.69, 37.88, 45.77) and (115.38, 156.47, 30.58, 37.54).

**Case (v)** \(L(x) = e^{-px}\) and \(R(x) = \max \{0, 1 - |x|^p\}\)

**Step 1:** The given fuzzy multiobjective transportation problem is a balanced one.

**Step 2:** Using step 3 of the proposed model, the given fuzzy multiobjective transportation problem is converted into a single fuzzy objective problem as follows:

Minimize \[\bar{z} = 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} (\bar{c}_{ij})x_{ij} + 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} (\bar{t}_{ij})x_{ij}\]
subject to $\sum_{j=1}^{3} x_{ij} \cong \tilde{a}_i$ $i = 1, 2, 3$
$\sum_{i=1}^{3} x_{ij} \cong \tilde{b}_j$ $j = 1, 2, 3$

Here, $w_1 = 0.5$, $w_2 = 0.5$.

**Step 3:** The fuzzy multiobjective transportation problem is converted into the following crisp linear programming problem

Minimize $z = 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} R(\tilde{c}_{ij}) x_{ij} + 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} R(\tilde{t}_{ij}) x_{ij}$

subject to $\sum_{j=1}^{3} x_{ij} \cong R(\tilde{a}_i)$ $i = 1, 2, 3$
$\sum_{i=1}^{3} x_{ij} \cong R(\tilde{b}_j)$ $j = 1, 2, 3$

**Step 4:** Using section 3.1., the values of $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$, $R(\tilde{a}_i)$ and $R(\tilde{b}_j)$ $\forall i, j$ are calculated and given in Table X.

**TABLE X: Ranks of Fuzzy Cost and Fuzzy Time of case (v)**

<table>
<thead>
<tr>
<th>Source/ Destination</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.97</td>
<td>5.75</td>
<td>8.29</td>
<td>5.58</td>
</tr>
<tr>
<td></td>
<td>9.23</td>
<td>8.59</td>
<td>4.19</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>3.95</td>
<td>6.93</td>
<td>6.71</td>
<td>6.81</td>
</tr>
<tr>
<td></td>
<td>8.09</td>
<td>11.92</td>
<td>6.19</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>8.34</td>
<td>9.23</td>
<td>7.80</td>
<td>6.39</td>
</tr>
<tr>
<td></td>
<td>4.24</td>
<td>5.72</td>
<td>7.50</td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>6.41</td>
<td>7.58</td>
<td>4.79</td>
<td></td>
</tr>
</tbody>
</table>

Note: “ ” indicates rank of fuzzy time

**Step 5:** Using step 6 of the proposed method convert the chosen fuzzy multiobjective transportation problem into the following crisp linear programming

Minimize: $(7.6)x_{11} + (7.17)x_{12} + (6.24)x_{13} +$
$(6.02)x_{21} + (9.42)x_{22} + (6.45)x_{23} +$
$(6.29)x_{31} + (7.47)x_{32} + (7.65)x_{33} .

subject to: $x_{11} + x_{21} + x_{31} = 5.58$ ; $x_{12} + x_{22} + x_{32} = 6.81$ ;
$x_{13} + x_{23} + x_{33} = 6.39$ ; $x_{11} + x_{12} + x_{13} = 6.41$ ;
$x_{21} + x_{22} + x_{23} = 7.58$ ; $x_{31} + x_{32} + x_{33} = 4.79$ .

Fuzzy multi objective transportation model
\(x_{ij} \geq 0\), for all \(i=1,2,3\). and \(j=1,2,3\).

**Step 6:** Solve the crisp linear programming problem, obtained in step 5, by using TORA software the optimal solution obtained is:

\(x_{12} = 2.02, x_{13} = 4.39, x_{21} = 5.58, x_{23} = 2.00, x_{32} = 4.79\).

**Step 7:** Using step 8 of the proposed model, the minimum fuzzy transportation cost and time respectively are (111.49, 157.42, 36.37, 43.58) and (108.75, 147.48, 29.21, 35.56)

<table>
<thead>
<tr>
<th>Linear and non-linear functions</th>
<th>Optimal solution</th>
<th>Total Optimal Fuzzy Transportation Cost and Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L(x) = R(x) = [0,1-</td>
<td>x</td>
<td>])</td>
</tr>
<tr>
<td>(L(x) = R(x) = e^x)</td>
<td>(x_{12} = 1.97, x_{13} = 4.44, x_{21} = 5.43, x_{23} = 1.97, x_{32} = 4.74).</td>
<td>(110.6, 156.01, 36.11, 43.01) and (107.05, 145.14, 28.9, 35.13)</td>
</tr>
<tr>
<td>(L(x) = \max{0,1-</td>
<td>x</td>
<td>}, R(x) = e^{-x})</td>
</tr>
<tr>
<td>(L(x) = e^{-x}, R(x) = \max{0,1-</td>
<td>x</td>
<td>})</td>
</tr>
<tr>
<td>(L(x) = \max{0,1-</td>
<td>x'</td>
<td>}, R(x) = e^{-x})</td>
</tr>
</tbody>
</table>

Note: “ ” denotes total optimal fuzzy transportation time.

**6. Conclusion**

In this paper, we have present a new method for ranking generalized LR fuzzy numbers to various linear and non-linear functions of generalized LR fuzzy numbers from its \(\lambda\)-cut based on area, mode and spread. The proposed ranking method can efficiently rank various LR fuzzy numbers, their images and crisp numbers which are consider to be a special case of fuzzy numbers and can overcome the drawbacks of the existing ranking methods. We also have presented a fuzzy multi objective
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transportation model using a linear programming to deal with the fuzzy multi objective transportation problem based on the proposed ranking method. The proposed fuzzy multiobjective transportation problem has the advantage of allowing the evaluating values to be represented by generalized LR fuzzy numbers for dealing dealing with fuzzy multiobjective transportation problem.

References


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