Steady 2-D MHD Flow of a Second Grade Fluid in a Symmetrical Diverging Channel of Varying Width

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Abstract

Two dimensional steady flow of an incompressible second grade fluid is investigated in a symmetrically divergent channel of varying width in the presence of a transverse magnetic field. The non-linear differential equations obtained as a result are solved using ADM. The effect of dimensionless non-Newtonian elastic, cross viscosity and magnetic field parameters is shown graphically on axial and normal velocities, shear 

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stress at the curved walls and pressure gradient. It is observed that the magnitude of the axial velocity and the pressure gradient decreases with an increase in the value of each of these parameters. However, shear stress at the curved wall increased with an increase of the value of the elastic parameter. Previously published results were obtained as a special case of this present study.

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1 Introduction

The study of electrically conducting fluids in the presence of an external magnetic field through channels of varying cross-sections is very important from a theoretical as well as an applied point of view. Such flows commonly occur in mathematical modeling of many industrial and biological fluids. Some examples of theoretical applications include industrial metal casting such as the control of molten metal flows and the motion of liquid metals of alloys in the cooling systems of advanced nuclear reactors. Clearly, the motion in the region with intersecting walls could represent a local transition between two parallel channels with different cross sections, a widening or a contraction of the flow. Additionally, the study of the magnetohydrodynamic (MHD) flows could also be applied to physiological flow problems. Many modern medical diagnostic devices, especially those used in diagnosing cardiovascular diseases, make use of the interaction of magnetic fields with tissue fluids.

In 1937, Hartmann and Lazarus [1] studied the influence of a uniform transverse magnetic field on the flow of a viscous incompressible electrically conducting fluid between two infinite insulated parallel plates, in a stationary state. Since then, this work on MHD flow has received much attention. Craig et al. [2] have shown that if a magnetic field is applied to a moving and electrically conducting liquid, it will induce electric and magnetic fields. The interaction of these fields produces a body force known as the Lorentz force, which has a tendency to oppose the movement of the liquid.

Makinde [3] has studied the problem of an incompressible electrically conducting Newtonian fluid through a slowly exponentially diverging symmetrical channel. It is well known that if the cross-sectional area of a channel increases gradually with axial distance downstream, the flow separates at values of the Reynolds number above a rather moderate critical value; moreover, the separated flow is non-unique. This non-uniqueness occurs at a large Reynolds number in channels that are sufficiently slowly-varying for the flow to be gov-
cerned by the boundary layer equations, in which there is neither a transverse pressure gradient nor longitudinal viscous diffusion [4].

Most of the common fluids in the real world exhibit Newtonian behavior. There are important classes of fluids that are classified as non-Newtonian. Many industrial materials, including clay coatings, drilling muds, suspensions, certain oils and greases, polymer melts, elastomers and many emulsions, have been categorized as non-Newtonian fluids. Non-Newtonian fluids may be classified as: (i) fluids for which the shear stress depends on the shear rate (ii) fluids for which the relation between the shear stress and shear rate depends on time, and (iii) fluids which possess both elastic and viscous properties called visco-elastic fluids or elasto-viscous fluids [1]. Recommending a single constitutive equation for use in the cases described in (i), (ii) and (iii) does not seem possible because of the great diversity in the physical structure of non-Newtonian fluids. As a result many constitutive equations for non-Newtonian fluids have been proposed. Out of these constitutive equations one important type of constitutive equation is the equation of second grade fluids. The constitutive equation of a second grade fluid is a linear relation between the stress and the first Rivlin-Ericksen tensor, the square of the first Rivlin-Ericksen tensor and the second Rivlin-Ericksen tensor [5]. This constitutive equation has three coefficients and is used for the fluids of visco-elastic type. The governing differential equations of a second grade fluid are of higher order than Navier-Stokes equations.

Since it is not easy to get an exact analytical solution of a nonlinear problem, we work for approximate analytic solutions. There are many approximate analytic techniques used to solve nonlinear problems, including the perturbation method, Homotopy Perturbation Method (HPM), Homotopy Analysis Method (HAM) and Adomian Decomposition Method (ADM), which are well recognized and widely applied. The Adomian Decomposition Method [6–8] created great interest among researchers by solving both non-linear ordinary and partial differential equations. A considerable amount of research work has been invested in applying this method to a wide class of linear and non-linear ordinary differential equations, partial differential equations and integral equations. The problems arising in applied sciences and engineering can be solved by ADM and usually is characterized by its higher degree of accuracy. It provides an analytical solution in the form of an infinite series in which each term can be easily determined. Unlike the traditional methods, the ADM needs no discretization, linearization, spatial transformation or perturbation, and thus offers some significant advantages over other analytical techniques as well as numerical methods. Finding an exact solution for non-Newtonian fluid problems seems to be difficult, so a truncated number of terms are used to solve these problems. The rapid convergence of the series solution obtained by ADM has been discussed by Cherrualt and Adomian [9] and provides insight into the
characteristics and behavior of the solution as in the case with the closed form solution.

The aim of this present work is to investigate the flow through a symmetrical divergent channel of variable length for a non-Newtonian fluid of second order in the presence of a transverse magnetic field. The governing highly nonlinear differential equation representing this problem is solved by ADM. In the following section the problem is formulated and solved, then the pertinent results are discussed.

2 Basic Equations

The basic equations governing the flow of an incompressible second grade fluid in the presence of a transverse magnetic field, neglecting body forces and thermal effects, are:

\[
\text{div}\, \mathbf{V} = 0, \tag{1}
\]

\[
\rho \frac{D\mathbf{V}}{Dt} = \text{div}\, \mathbf{T} + \mathbf{J} \times \mathbf{B}, \tag{2}
\]

where \( \mathbf{V} \) is the velocity vector, \( \rho \) the density of the fluid, \( \frac{D}{Dt} \) denotes the material time derivative, \( \mathbf{T} \) the Cauchy stress tensor, \( \mathbf{J} \) the electric current density and \( \mathbf{B} \) the total magnetic field. \( \mathbf{B} = \mathbf{B}_0 + \mathbf{b} \), where \( \mathbf{B}_0 \) represents the imposed magnetic field and \( \mathbf{b} \) denotes the induced magnetic field. In the absence of displacement currents, the modified Ohm’s law and Maxwell’s equations [10–12] are

\[
\mathbf{J} = \sigma \{ \mathbf{E} + \mathbf{V} \times \mathbf{B} \}. \tag{3}
\]

\[
\text{div}\, \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \mu_m \mathbf{J}, \quad \text{Curl}\, \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{4}
\]

in which \( \sigma \) is the electrical conductivity, \( \mathbf{E} \) the electric field and \( \mu_m \) the magnetic permeability. The following assumptions are made in order to lead the discussion:

- Magnetic permeability \( \mu_m \) and electric field conductivity \( \sigma \) are constant throughout the flow field region.
- The electrical conductivity \( \sigma \) of the fluid is finite.
- Total magnetic field \( \mathbf{B} \) is perpendicular to the velocity field \( \mathbf{V} \) and the induced magnetic field \( \mathbf{b} \) is negligible compared with the applied magnetic field \( \mathbf{B}_0 \) so that the magnetic Reynolds number is small [10, 12].
- There is no energy added or extracted from the fluid by the electric field, which implies that there is no electric field present in the fluid flow region.
Under these assumptions, the magnetohydrodynamic force involved in equation (2) can be put into the form:

\[ \mathbf{J} \times \mathbf{B} = -\sigma B_0^2 \mathbf{V}. \]  

(5)

\( \mathbf{T} \) in equation (2) is the Cauchy stress tensor and for a second grade fluid is defined as:

\[ \mathbf{T} = -p \mathbf{I} + \mu \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1^2, \]  

(6)

where in equation (6) \( p \) is the pressure, \( \mathbf{I} \) is the identity tensor, \( \mu \) is the coefficient of viscosity, \( \alpha_1 \) and \( \alpha_2 \) are the normal stress modulii and \( \mathbf{A}_1 \) and \( \mathbf{A}_2 \) are the first and second Rivlin-Ericksen tensors respectively, defined as:

\[ \mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T, \quad \mathbf{L} = \nabla \mathbf{V}, \]  

(7)

\[ \mathbf{A}_2 = \dot{\mathbf{A}}_1 + \mathbf{A}_1 \mathbf{L} + \mathbf{L}^T \mathbf{A}_1, \]

and

\[ (\ast) = \frac{\partial (\ast)}{\partial t} + (\mathbf{V} \cdot \nabla)(\ast). \]  

(8)

With the help of equation (3) and (4), equation (2) becomes

\[ \rho \dot{\mathbf{V}} = -\nabla p + \mu \nabla^2 \mathbf{V} + (\alpha_1 + \alpha_2) \text{div} \mathbf{A}_1^2 + \alpha_1 \left[ \nabla^2 \mathbf{V}_1 + \nabla^2 (\nabla \times \mathbf{V}) \times \mathbf{V} \right. \]

\[ + \left. \nabla \left( \mathbf{V} \cdot \nabla^2 \mathbf{V} + \frac{1}{4} |\mathbf{A}_1|^2 \right) \right] - \sigma B_0^2 \mathbf{V}. \]  

(9)

3 Problem Formulation

Consider the steady fully developed flow of an incompressible second grade fluid through a symmetrical diverging channel of varying width in the presence of a transverse magnetic field. Using the Cartesian coordinate system such that the \( x \)-axis is in the direction of the flow and the \( y \)-axis is perpendicular to it, a transverse magnetic field is applied in the \( y \)-direction. It is assumed that the channel is symmetric with respect to the \( x \)-axis. Let \( u \) and \( v \) be the velocity components in the \( x \) and \( y \) directions respectively, and \( y = \pm b(x) \) are the rigid walls (figure 1).

![Figure 1: Geometry of the problem.](image-url)
For a two dimensional plane steady flow, we take
\[ \mathbf{V} = [u(x, y), v(x, y)] . \]  
(10)

The equations (1) and (9) in component form become:
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]  
(11)

\[ -\rho v \Omega = -\frac{\partial \hat{p}}{\partial x} - \mu \frac{\partial \Omega}{\partial y} - \alpha_1 v \nabla^2 \Omega - \sigma B'_o u, \]  
(12)

and
\[ \rho u \Omega = -\frac{\partial \hat{p}}{\partial y} + \mu \frac{\partial \Omega}{\partial x} + \alpha_1 u \nabla^2 \Omega, \]  
(13)

where
\[ \Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \]  
(14)

\[ \hat{p}(x, y) = p + \frac{\rho}{2}(u^2 + v^2) - \alpha_1 \left\{ u \nabla^2 u + v \nabla^2 v \right\} - \frac{1}{4}(3\alpha_1 + 2\alpha_2) |A_1^2|, \]  
(15)

and
\[ |A_1^2| = 8 \left| \frac{\partial u}{\partial x} \right|^2 + 2 \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2. \]  
(16)

The associated boundary conditions are [4]:

Symmetry: \[ \frac{\partial u}{\partial y} = 0, \quad v = 0 \quad \text{at} \quad y = 0, \]
(17)

No-slip: \[ u + v \frac{db}{dx} = 0, \quad v = 0 \quad \text{at} \quad b(x). \]
(18)

The flux across any cross-section of the channel is described as:
\[ Q = \int_{-b(x)}^{b(x)} udy. \]
(19)

Introducing the stream function \( \psi \) as:
\[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \]
(20)
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continuity (11) is satisfied and equations (12) to (16) after simplification become:

\[ \Omega = -\nabla^2 \psi, \]  
\[ \frac{\partial (\psi, \Omega)}{\partial (y, x)} = \nu \nabla^2 \Omega + \frac{\alpha_1}{\rho} \frac{\partial (\psi, \nabla^2 \Omega)}{\partial (y, x)} + \frac{\sigma B_o^2}{\rho} \frac{\partial^2 \psi}{\partial y^2}. \]  

The corresponding boundary conditions in terms of \( \psi \) become:

\[ \frac{\partial^2 \psi}{\partial y^2} = 0, \quad \psi = 0 \quad \text{at} \quad y = 0, \]  
\[ \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} \frac{db}{dx} = 0, \quad \psi = Q \quad \text{at} \quad y = b(x). \]  

The function \( b(x) \) is assumed to depend upon a small parameter \( \epsilon \) such that [4]:

\[ b(x, \epsilon) = a_0 S \left( \frac{\epsilon x}{a_0} \right) \]  
\[ (0 < \epsilon = \frac{a_0}{L} << 1), \]  

where \( a_0 \) is the constant characteristic half width of the channel, \( L \) is the constant characteristic length of the channel and \( S \) is the function describing the channel wall divergence geometry. In the limit \( \epsilon \to 0 \), the channel is of constant width and the velocity profile is given by the familiar Hartmann flow of axial velocity profile.

Introducing the following dimensionless variables:

\[ \bar{\Omega} = \frac{a_0^2 \Omega}{Q}, \quad \bar{x} = \frac{\epsilon x}{a_0}, \quad \bar{y} = \frac{y}{a_0}, \quad \bar{\psi} = \frac{\psi}{Q}, \]  

into the equations (21)-(22) and neglecting terms of order \( \epsilon^2 \) or higher as well as the bars for simplification, we obtain:

\[ \bar{\Omega} = -\frac{\partial^2 \bar{\psi}}{\partial y^2}, \]  
\[ \frac{\bar{\partial}^2 \bar{\Omega}}{\partial \bar{y}^2} = R_e \left( \frac{\partial (\psi, \Omega)}{\partial (y, x)} - \lambda \frac{\partial (\psi, \partial^2 \Omega)}{\partial (y, x)} - \Lambda \frac{\partial^2 \psi}{\partial y^2} \right), \]

where \( R_e = \frac{Q\epsilon}{\nu} \) is the Reynolds number, \( \Lambda = \frac{\sigma B_o^2 a_0 L}{\rho Q} \) is the magnetic field intensity parameter and \( \lambda = \frac{\alpha_1}{\rho a_0^2} \) is the dimensionless non-Newtonian elastic parameter. Boundary conditions in dimensionless form become:

\[ \frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} = 0, \quad \bar{\psi} = 0 \quad \text{at} \quad \bar{y} = 0, \]
\[ \frac{\partial \psi}{\partial y} = 0, \quad \psi = 1 \quad \text{at} \quad y = S(x), \tag{30} \]

Equation (27) is a non-linear fourth order partial differential equation subject to the boundary conditions (29) and (30), whose exact solution is difficult to obtain. In this article, we will present an approximate analytical solution using ADM.

## 4 Adomian Decomposition Method

In this section, we briefly outline the Adomian Decomposition Method (ADM) [6]. To give a clear overview of ADM, we first consider

\[ Fu = g(t), \]

where \( F \) is a differential operator with linear and non-linear terms. The linear term is written as \( Lu + Ru \), where \( L \) is invertible. To avoid difficult integration we choose \( L \) to be the highest ordered derivative. \( R \) is the remainder of the linear operator. The non-linear term is represented by \( Nu \). Thus \( Lu + Ru + Nu = g \) and we write:

\[ Lu = g - Ru - Nu. \]

Because \( L \) is invertible, an equivalent expression is

\[ L^{-1} Lu = L^{-1} g - L^{-1} Ru - L^{-1} Nu. \]

Solving for \( u \) yields:

\[ u = f - L^{-1} Ru - L^{-1} Nu, \]

where the function \( f \) represents the terms arising from the integration of the source term \( g \) and using the initial/boundary conditions to evaluate the constants of integration. According to Adomian [7], the solution \( u \) and the non-linear term \( Nu \) can be expressed respectively in the form:

\[ u = \sum_{n=0}^{\infty} u_n, \quad Nu = \sum_{n=0}^{\infty} A_n, \]

where the \( A_n \) are specially generated polynomials for the particular non-linearity, referred to as Adomian polynomials. They are defined in [8] and discussed extensively in [7]. We have now

\[ \sum_{n=0}^{\infty} u_n = f - L^{-1} R \sum_{n=0}^{\infty} u - L^{-1} \sum_{n=0}^{\infty} A_n, \]
so that
\[ u_0 = f, \]
\[ u_1 = -L^{-1}Ru_0 - L^{-1}A_0, \]
\[ u_2 = -L^{-1}Ru_1 - L^{-1}A_1, \]
\[ u_3 = -L^{-1}Ru_2 - L^{-1}A_2, \]
and so on. The practical solution will be the n-term approximation:
\[ \beta_n = \sum_{i=0}^{n-1} u_i \quad \text{and} \quad \lim_{n \to \infty} \beta_n = \sum_{i=0}^{n-1} u_i = u, \]
by definition. The convergence of this method has been established by many researchers; for examples see Cherruault and Adomian [9]. In the sequel we apply the decomposition method to our problem.

5 Solution of the Problem

Using equation (27) into (28) and writing the resulting equation in operator form according to the methodology of ADM [8]-[7] we obtain:
\[
L_{yyyy} \psi = -Re \left[ \frac{\partial \left( \frac{\partial^2 \psi}{\partial y^2}, \psi \right)}{\partial (y,x)} - \lambda \frac{\partial \left( \frac{\partial^4 \psi}{\partial y^4}, \psi \right)}{\partial (y,x)} - \Lambda \frac{\partial^2 \psi}{\partial y^2} \right],
\]
(31)
where $L_{yyyy} = \frac{\partial^4}{\partial y^4}$. Since $L_{yyyy}$ is invertible, applying $L^{-1}$ to both sides of the equation (31) yields:
\[
\psi = C_1(x) \frac{y^3}{6} + C_2(x) \frac{y^2}{2} + C_3(x)y + C_4(x) - Re L^{-1} \left[ \frac{\partial \left( \frac{\partial^2 \psi}{\partial y^2}, \psi \right)}{\partial (y,x)} - \lambda \frac{\partial \left( \frac{\partial^4 \psi}{\partial y^4}, \psi \right)}{\partial (y,x)} - \Lambda \frac{\partial^2 \psi}{\partial y^2} \right],
\]
(32)
where $C_1(x), C_2(x), C_3(x)$ and $C_4(x)$ are arbitrary functions of $x$. To solve equation (32) by ADM, we let
\[
\psi = \sum_{n=0}^{\infty} \psi_n, \quad \text{and} \quad N \psi = \sum_{n=0}^{\infty} A_n,
\]
(33)
where
\[ N_\psi = \begin{bmatrix} \frac{\partial^2(\frac{\partial^2\psi}{\partial y^2},\psi)}{\partial(y,x)} - \lambda \frac{\partial^4(\frac{\partial\psi}{\partial y^2},\psi)}{\partial(y,x)} \end{bmatrix} . \] (34)

In view of equations (33)-(34) equation (32) gives:
\[
\sum_{n=0}^{\infty} \psi_n = C_1(x)\frac{y^3}{6} + C_2(x)\frac{y^2}{2} + C_3(x)y + C_4(x) - R_\lambda L^{-1} \left( -\Lambda \sum_{n=0}^{\infty} \frac{\partial^2\psi_n}{\partial y^2} + \sum_{n=0}^{\infty} A_n \right) .
\] (35)

The zeroth component is identified as:
\[
\psi_0 = C_1(x)\frac{y^3}{6} + C_2(x)\frac{y^2}{2} + C_3(x)y + C_4(x),
\] (36)

and the remaining components as the recurrence relation:
\[
\psi_{n+1} = -R_\lambda L^{-1} \left( -\Lambda \sum_{n=0}^{\infty} \frac{\partial^2\psi_n}{\partial y^2} + \sum_{n=0}^{\infty} A_n \right) , \quad n \geq 0,
\] (37)

where \( A_n \) are the Adomian polynomials that represent the non-linear term in (34). The first few \( A_n \) are found to be:
\[
A_0 = \frac{\partial}{\partial(y,x)} \left( \frac{\partial^2\psi_0}{\partial y^2},\psi_0 \right) - \lambda \frac{\partial}{\partial(y,x)} \left( \frac{\partial^4\psi_0}{\partial y^4},\psi_0 \right),
\] (38)

\[
A_1 = \frac{\partial}{\partial(y,x)} \left( \frac{\partial^2\psi_1}{\partial y^2},\psi_0 \right) + \frac{\partial}{\partial(y,x)} \left( \frac{\partial^2\psi_0}{\partial y^2},\psi_1 \right) - \lambda \left( \frac{\partial}{\partial(y,x)} \left( \frac{\partial^4\psi_1}{\partial y^4},\psi_0 \right) + \frac{\partial}{\partial(y,x)} \left( \frac{\partial^4\psi_0}{\partial y^4},\psi_1 \right) \right),
\] (39)

and
\[
A_2 = \frac{\partial}{\partial(y,x)} \left( \frac{\partial^2\psi_2}{\partial y^2},\psi_0 \right) + \frac{\partial}{\partial(y,x)} \left( \frac{\partial^2\psi_1}{\partial y^2},\psi_1 \right) + \frac{\partial}{\partial(y,x)} \left( \frac{\partial^2\psi_0}{\partial y^2},\psi_2 \right).
\] (40)
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\[-\lambda \left( \frac{\partial \left( \frac{\partial^4 \psi_2}{\partial y^4}, \psi_0 \right)}{\partial (y, x)} + \frac{\partial \left( \frac{\partial^4 \psi_1}{\partial y^4}, \psi_1 \right)}{\partial (y, x)} + \frac{\partial \left( \frac{\partial^4 \psi_0}{\partial y^4}, \psi_2 \right)}{\partial (y, x)} \right).\]

The remaining polynomials can be easily generated. Using these polynomials in (37), the first few components can be determined recursively as:

\[
\psi_0 = C_1(x) \frac{y^3}{6} + C_2(x) \frac{y^2}{2} + C_3(x)y + C_4(x),
\]

\[
\psi_1 = -Re L^{-1} \left( -\Lambda \frac{\partial^2 \psi_0}{\partial y^2} + A_0 \right)
= Re L^{-1} \left[ -\Lambda \frac{\partial^2 \psi_0}{\partial y^2} + \frac{\partial \left( \frac{\partial^2 \psi_0}{\partial y^2}, \psi_0 \right)}{\partial (y, x)} - \lambda \frac{\partial \left( \frac{\partial^4 \psi_0}{\partial y^4}, \psi_0 \right)}{\partial (y, x)} \right],
\]

\[
\psi_2 = -Re L^{-1} \left( -\Lambda \frac{\partial^2 \psi_1}{\partial y^2} + A_1 \right)
= -Re L^{-1} \left[ -\Lambda \frac{\partial^2 \psi_1}{\partial y^2} + \frac{\partial \left( \frac{\partial^2 \psi_0}{\partial y^2}, \psi_0 \right)}{\partial (y, x)} + \frac{\partial \left( \frac{\partial^2 \psi_0}{\partial y^2}, \psi_1 \right)}{\partial (y, x)} \right]
\times \lambda \left( \frac{\partial \left( \frac{\partial^4 \psi_1}{\partial y^4}, \psi_0 \right)}{\partial (y, x)} + \frac{\partial \left( \frac{\partial^4 \psi_0}{\partial y^4}, \psi_1 \right)}{\partial (y, x)} \right),
\]

and

\[
\psi_3 = -Re L^{-1} \left( -\Lambda \frac{\partial^2 \psi_2}{\partial y^2} + A_2 \right)
= -Re L^{-1} \left[ -\Lambda \frac{\partial^2 \psi_2}{\partial y^2} + \frac{\partial \left( \frac{\partial^2 \psi_0}{\partial y^2}, \psi_0 \right)}{\partial (y, x)} + \frac{\partial \left( \frac{\partial^2 \psi_0}{\partial y^2}, \psi_1 \right)}{\partial (y, x)} \right]
\times \lambda \left( \frac{\partial \left( \frac{\partial^4 \psi_2}{\partial y^4}, \psi_0 \right)}{\partial (y, x)} + \frac{\partial \left( \frac{\partial^4 \psi_1}{\partial y^4}, \psi_1 \right)}{\partial (y, x)} \right)
\times \lambda \left( \frac{\partial \left( \frac{\partial^4 \psi_0}{\partial y^4}, \psi_1 \right)}{\partial (y, x)} \right),
\]
The corresponding boundary conditions, after using (33) in (29) and (30), become:

\[
\frac{\partial^2 \psi_0}{\partial y^2} = 0, \quad \psi_0 = 0 \quad \text{on} \quad y = 0,
\]

\[
\frac{\partial \psi_0}{\partial y} = 0, \quad \psi_0 = 1 \quad \text{on} \quad y = S(x),
\]

and for \(n \geq 0\)

\[
\frac{\partial^2 \psi_n}{\partial y^2} = 0, \quad \psi_n = 0 \quad \text{on} \quad y = 0,
\]

\[
\frac{\partial \psi_n}{\partial y} = 0, \quad \psi_n = 0 \quad \text{on} \quad y = S(x).
\]

Solving (41)-(45) subject to boundary conditions (46)-(49), we obtain:

\[
\psi_0 = \frac{1}{2} \left( \frac{3y}{S} - \frac{y^3}{S^3} \right),
\]

\[
\psi_1 = -\frac{R_e}{40S} \left( \frac{y^{11}}{4400S^{11}} - \frac{3y^9}{1120S^9} + \frac{51y^7}{4900S^7} - \frac{57y^5}{2800S^5} + \frac{4111y^3}{215600S^3} 
- \frac{115y}{17248S} \right) S_x + \Lambda S^2 \left( \frac{y^9}{2240S^9} - \frac{9y^7}{2800S^7} + \frac{47y^5}{5600S^5}
- \frac{3y^3}{1213y} + \frac{9y}{431200S^2} \right) S_{xx} + \Lambda^2 S^4 \left( \frac{y^7}{1680S^7} - \frac{y^5}{400S^5} + \frac{9y^3}{2800S^3}
- \frac{11y}{8400S} \right) + \lambda \left( \frac{y^9}{56S^{11}} + \frac{9y^7}{70S^9} - \frac{9y^5}{20S^7} + \frac{41y^3}{70S^5} - \frac{69y}{280S^3} \right) S_x
+ \lambda \left( \frac{y^9}{224S^{10}} - \frac{9y^7}{280S^8} + \frac{41y^3}{280S^4} + \frac{69y}{1120S^2} \right) S_{xx} + \Lambda \left( -\frac{3y^7}{280S^7}
+ \frac{3y^5}{40S^5} - \frac{33y^3}{280S^3} + \frac{3y}{56S} \right) S_x \right),
\]

\[
\psi_2 = \frac{1}{2} \left( \frac{3y}{S} - \frac{y^3}{S^3} \right),
\]

\[
\psi_1 = -\frac{R_e}{40S} \left( \frac{y^{11}}{4400S^{11}} - \frac{3y^9}{1120S^9} + \frac{51y^7}{4900S^7} - \frac{57y^5}{2800S^5} + \frac{4111y^3}{215600S^3} 
- \frac{115y}{17248S} \right) S_x + \Lambda S^2 \left( \frac{y^9}{2240S^9} - \frac{9y^7}{2800S^7} + \frac{47y^5}{5600S^5}
- \frac{3y^3}{1213y} + \frac{9y}{431200S^2} \right) S_{xx} + \Lambda^2 S^4 \left( \frac{y^7}{1680S^7} - \frac{y^5}{400S^5} + \frac{9y^3}{2800S^3}
- \frac{11y}{8400S} \right) + \lambda \left( \frac{y^9}{56S^{11}} + \frac{9y^7}{70S^9} - \frac{9y^5}{20S^7} + \frac{41y^3}{70S^5} - \frac{69y}{280S^3} \right) S_x
+ \lambda \left( \frac{y^9}{224S^{10}} - \frac{9y^7}{280S^8} + \frac{41y^3}{280S^4} + \frac{69y}{1120S^2} \right) S_{xx} + \Lambda \left( -\frac{3y^7}{280S^7}
+ \frac{3y^5}{40S^5} - \frac{33y^3}{280S^3} + \frac{3y}{56S} \right) S_x \right),
\]

\[
\psi_2 = -\frac{R_e^2}{40S} \left( \frac{y^{11}}{4400S^{11}} - \frac{3y^9}{1120S^9} + \frac{51y^7}{4900S^7} - \frac{57y^5}{2800S^5} + \frac{4111y^3}{215600S^3} 
- \frac{115y}{17248S} \right) S_x + \Lambda S^2 \left( \frac{y^9}{2240S^9} - \frac{9y^7}{2800S^7} + \frac{47y^5}{5600S^5}
- \frac{3y^3}{1213y} + \frac{9y}{431200S^2} \right) S_{xx} + \Lambda^2 S^4 \left( \frac{y^7}{1680S^7} - \frac{y^5}{400S^5} + \frac{9y^3}{2800S^3}
- \frac{11y}{8400S} \right) + \lambda \left( \frac{y^9}{56S^{11}} + \frac{9y^7}{70S^9} - \frac{9y^5}{20S^7} + \frac{41y^3}{70S^5} - \frac{69y}{280S^3} \right) S_x
+ \lambda \left( \frac{y^9}{224S^{10}} - \frac{9y^7}{280S^8} + \frac{41y^3}{280S^4} + \frac{69y}{1120S^2} \right) S_{xx} + \Lambda \left( -\frac{3y^7}{280S^7}
+ \frac{3y^5}{40S^5} - \frac{33y^3}{280S^3} + \frac{3y}{56S} \right) S_x \right),
\]
and

$$\psi_3 = Re^3 \left[ \left( \frac{y^{15}}{208000S^{15}} - \frac{127y^{13}}{1601600S^{13}} + \frac{603y^{11}}{1232000S^{11}} - \frac{31y^9}{19600S^9} \right) \right. $$

$$+ \left( \frac{184599y^7}{6036800S^7} - \frac{16493y^5}{4312000S^5} + \frac{439093y^3}{156956800S^3} - \frac{33897y}{39239200S} \right) S^3_x $$

$$+ \left( \frac{64279y}{7134400} - \frac{y^{15}}{320320S^{14}} + \frac{23y^{13}}{457600S^{12}} - \frac{3y^{11}}{9625S^{10}} + \frac{167y^9}{156800S^8} \right) $$

$$- \left( \frac{68609y^7}{30184000S^6} + \frac{28529y^5}{8624000S^4} - \frac{49559y^3}{1783600S^2} \right) S_xS_{xx} $$

$$+ \left( \frac{9y^5}{22422400S^{13}} - \frac{67y^{11}}{1601600S^{11}} + \frac{11y^9}{78400S^7} \right) S_{xxx} $$

$$+ \left( \frac{9y^5}{1601600S^{11}} - \frac{67y^{11}}{1724800S^9} - \frac{11y^9}{78400S^7} \right) $$

$$+ \left( \frac{19449y^7}{6036800S^5} - \frac{2000S^5}{78478400S^3} + \frac{341483y^3}{784000S^3} - \frac{58859yS}{392392000} \right) S_{xxx} $$

$$+ \left( \frac{103y^9}{156800S^7} - \frac{281y^7}{196000S^5} \right) $$

$$+ \left( \frac{31y^{13}}{1921920S^{11}} - \frac{61y^{11}}{369600S^9} + \frac{103y^9}{156800S^7} - \frac{281y^7}{196000S^5} \right) $$

$$+ \left( \frac{16153y^5}{8624000S^3} - \frac{226043y^3}{16816800S} + \frac{26693yS}{6726720} \right) S_x^2 + \left( \frac{617117y^3}{67267200} \right) $$

$$+ \left( \frac{43y^{13}}{1768780S^{16}} + \frac{83y^{11}}{1478400S^8} - \frac{437y^9}{1881600S^6} + \frac{451y^7}{78400S^4} \right) $$

$$+ \left( \frac{33829y^5}{3449600S^2} - \frac{148123yS^2}{448448000} \right) S_{xx} + \left( \frac{307yS^3}{6468000} \right) $$

$$+ \left( \frac{588000S^3}{588000S^3} - \frac{35000S}{12936000} - \frac{6468000}{16800S} $$

$$+ \left( \frac{95y^5}{5600} - \frac{19y^3S}{108000} + \frac{67yS^5}{100800} \right) \right] + \Lambda \left[ \left( \frac{103y^{13}}{80080S^{15}} \right. $$

$$+ \left. \frac{117y^{11}}{7700S^{13}} - \frac{291y^9}{3920S^{11}} + \frac{239y^7}{1225S^9} - \frac{879y^5}{2800S^7} + \frac{190053y^3}{700700S^5} \right) $$

$$+ \left( \frac{2817y^9}{5267y^9} - \frac{93y^{11}}{98560S^{14}} - \frac{93y^{11}}{98560S^{12}} + \frac{2871y^9}{62720S^{10}} - \frac{39y^7}{320S^8} \right) $$

$$+ \left( \frac{3y^{11}}{1232000S^{15}} - \frac{318939y^3}{1724800S^3} + \frac{223479y}{3449600S^2} \right) S_xS_{xx} + \left( \frac{3y^{11}}{1281280S^{13}} \right. $$

$$- \left. \frac{3y^{11}}{1232000S^{15}} - \frac{318939y^3}{1724800S^3} + \frac{223479y}{3449600S^2} \right) S_xS_{xx} + \left( \frac{127y^{11}}{246400S^{10}} - \frac{11y^9}{2688S^8} + \frac{1119y^7}{78400S^6} \right) $$

$$\right]$$
Consequently, the ADM solution up to order three is obtained as:

$$
\psi = \frac{1}{2} \left( \frac{3y}{S} - \frac{y^3}{S^3} \right) - R_e \left( \frac{3y^7}{280S^7} - \frac{3y^5}{40S^5} + \frac{33y^3}{280S^3} - \frac{3y}{56S} \right) S_x 
- R_e^2 \left( \frac{y^{11}}{4400S^{11}} - \frac{3y^9}{1120S^9} + \frac{51y^7}{4900S^7} - \frac{57y^5}{2800S^5} + \frac{4111y^3}{215600S^3} \right) S_x^2 + \left( - \frac{y^{11}}{12320S^{10}} + \frac{y^9}{1120S^8} - \frac{69y^7}{19600S^6} + \frac{3y^5}{400S^4} \right) S_x 
- \frac{3279y^3}{431200S^2} + \frac{1213y}{431200} \right) S_{xx} + \Lambda \left[ \frac{y^9}{2240S^9} - \frac{9y^7}{2800S^7} + \frac{47y^5}{5600S^5} \right] S_x 
- \frac{y^3}{1120S^3} + \frac{37y}{11200S} \right) S_x^2 \right] \Lambda S^2 \left( \frac{y^7}{1680S^7} - \frac{y^5}{400S^5} + \frac{y^3}{2800S^3} \right) S_x^2 
+ \left( - \frac{3y^7}{280S^7} + \frac{9y^5}{80S^5} - \frac{41y^3}{280S^4} + \frac{69y}{1120S^2} \right) S_{xx} + \Lambda \left( - \frac{3y^7}{280S^7} \right) 
+ \frac{3y^5}{40S^5} - \frac{33y^3}{280S^3} + \frac{3y}{56S} \right) S_x \right) \right)
+ R_e^3 \left( \frac{y^{15}}{208000S^{15}} - \frac{127y^{13}}{1601600S^{13}} + \frac{603y^{11}}{1232000S^{11}} - \frac{31y^9}{19600S^9} \right) S_x 
+ \frac{184599y^7}{60368000S^7} - \frac{16493y^5}{431200S^5} + \frac{439093y^3}{156956800S^3} - \frac{33897y}{39239200S} \right) S_x^2 
+ \left( \frac{67279y^3}{71344000} - \frac{23y^3}{457600S^{12}} - \frac{3y^{11}}{9625S^{10}} + \frac{167y^9}{156800S^8} \right) S_x 
+ \left( - \frac{3y^7}{280S^7} + \frac{9y^5}{80S^5} - \frac{41y^3}{280S^4} + \frac{69y}{1120S^2} \right) S_{xx} + \Lambda \left( - \frac{3y^7}{280S^7} \right) 
+ \frac{3y^5}{40S^5} - \frac{33y^3}{280S^3} + \frac{3y}{56S} \right) S_x \right) \right)
+ R_e^3 \left( \frac{y^{15}}{208000S^{15}} - \frac{127y^{13}}{1601600S^{13}} + \frac{603y^{11}}{1232000S^{11}} - \frac{31y^9}{19600S^9} \right) S_x
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\[
\begin{align*}
&- \frac{68609y^7}{30184000S^6} + \frac{28529y^5}{8624000S^4} - \frac{49559y^3}{17836000S^2} \left( S_xS_{xx} \right) \\
&+ \left( \frac{9y^{15}}{22422400S^{13}} - \frac{y^{13}}{160160S^{11}} + \frac{67y^{11}}{1724800S^9} - \frac{11y^9}{78400S^7} \right) \\
&+ \frac{19449y^7}{60368000S^5} - \frac{y^5}{2000S^3} + \frac{341483y^3}{78478400S^3} - \frac{58859yS}{392392000} \left( S_{xxx} \right) \\
&+ \Lambda \left[ \left( \frac{31y^{13}}{1921920S^{11}} - \frac{61y^{11}}{396000S^9} + \frac{103y^9}{156800S^7} - \frac{281y^7}{196000S^5} \right) \\
&+ \frac{16153y^5}{8624000S^3} - \frac{226043y^3}{16816800S} + \frac{26693yS}{67267200} \left( S_x^2 \right) + \left( \frac{617117y^3}{67267200} \right) \\
&- \frac{7687680S^{10}}{43y^{13}} + \frac{1478400S^8}{83y^{11}} - \frac{1881600S^6}{437y^9} + \frac{784000S^4}{451y^7} \\
&+ \frac{113y^7}{34496000S^2} - \frac{2489y^5S}{53000} + \frac{307y^3S}{1293600} - \frac{307y^5S}{6486000} \left( S_x \right) + \Lambda^2 \left[ y^{11} \left( 123200S^7 - 16800S^5 \right) \right] \\
&+ \frac{1311y^5}{588000S^3} + \frac{223479y}{16800S^5} - \frac{19y^3S^3}{56000} + \frac{19y^5S^5}{108000} + \frac{67y^7S}{1008000} + \Lambda \left[ \left( -\frac{103y^{13}}{80800S^{15}} \right) \right] \\
&- \frac{117y^{11}}{7700S^{13}} - \frac{3291y^9}{239y^7} + \frac{879y^5}{190053y^3} + \frac{1225S^9}{93y^{11}} + \frac{280S^7}{2871y^9} - \frac{700700S^5}{1280S^3} \\
&+ \frac{927y^5}{57200S^3} - \frac{281y^3}{98560S^{14}} + \frac{93y^{13}}{98560S^{12}} + \frac{2871y^9}{62720S^{10}} - \frac{39y^7}{320S^8} \\
&+ \frac{1311y^5}{6400S^5} - \frac{31939y^3}{1724800S^4} + \frac{223479y}{3449600S^2} \left( S_xS_{xx} \right) + \left( \frac{107y^{13}}{1281280S^{15}} \right) \\
&+ \frac{3y^{11}}{3200S^{11}} - \frac{281y^3}{62720S^9} - \frac{937y^7}{78400S^7} + \frac{927y^5}{431941y^3} + \frac{78400S^5}{78400S^5} \\
&+ \frac{309301y}{30484000S} \left( S_{xxx} \right) + \Lambda \left[ \left( -\frac{19y^{11}}{11200S^{11}} + \frac{13y^9}{960S^{9}} - \frac{369y^7}{7840S^7} + \frac{71y^5}{800S^5} \right) \right] \\
&+ \frac{19283y^3}{235200S^3} + \frac{9231y}{78400S} \left( S_{x}^2 \right) - \frac{246400S^{10}}{124711y^9} + \frac{11y^7}{2688S^8} + \frac{1119y^7}{78400S^6} \\
&- \frac{313y^5}{11200S^4} - \frac{27803y^3}{1034880S^2} - \frac{3317y}{344960} \left( S_xS_{xx} \right) + \Lambda^2 \left[ -\frac{y^9}{2240S^7} + \frac{9y^7}{2800S^5} \right] \\
&- \frac{47y^5}{5600S^3} - \frac{37yS}{1120} - \frac{37yS}{11200} \left( S_x \right) + \lambda^2 \left[ \left( \frac{27y^11}{440S^{13}} - \frac{27y^9}{56S^{11}} + \frac{243y^7}{140S^{11}} \right) \right] \\
&- \frac{81y^5}{810S^5} - \frac{783y}{616S^7} - \frac{440S^5}{440S^5} \left( S_x^3 \right) + \left( \frac{117y^{11}}{3520S^{14}} + \frac{117y^9}{448S^{12}} - \frac{1053y^7}{1120S^{10}} \right) \\
&+ \frac{351y^5}{160S^7} - \frac{12051y^3}{4928S^6} + \frac{3393y}{3520S^4} \left( S_xS_{xx} \right) + \left( \frac{9y^{11}}{3520S^{13}} - \frac{9y^9}{448S^{11}} + \frac{81y^7}{1120S^9} \right)
\end{align*}
\]
Using equation (54), the expression for the pressure gradient takes the form:

\[
- \frac{27y^5}{160S^7} + \frac{927y^3}{4928S^5} - \frac{261y}{3520S^3} S_{xxx} + \Lambda \left( \left( \frac{y^9}{56S^{11}} - \frac{9y^7}{70S^9} + \frac{9y^5}{20S^7} \right) \right) + \left( \left( \frac{y^9}{224S^{10}} + \frac{9y^7}{280S^8} - \frac{9y^5}{80S^6} + \frac{41y^3}{280S^4} \right) \right)
\]

where \( \lambda \) becomes:

\[
- \frac{41y^3}{70S^5} + \frac{69y}{280S^3} \sigma_x^2 + \left( \left( \frac{y^9}{224S^{10}} + \frac{9y^7}{280S^8} - \frac{9y^5}{80S^6} + \frac{41y^3}{280S^4} \right) \right)
\]

With the help of \( \psi \), higher order, we obtain:

\[
\text{Making equation (55) dimensionless and ignoring the terms of order } \epsilon^2 \text{ and higher order, we obtain:}
\]

\[
T_w = \frac{\partial \psi}{\partial y^2} + \lambda \frac{\partial \lambda}{\partial y} \left[ \frac{\partial \psi}{\partial (y,x)} + 2 \frac{\partial^2 \psi}{\partial x \partial y} \frac{\partial^2 \psi}{\partial y^2} - 4 \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \right].
\]

With the help of \( \psi \) from equation (54), the shear stress at the curved walls becomes:

\[
T_w = \frac{3}{S^2} - R_e \left( \frac{12S_x}{35S^2} - \lambda \left( \frac{36}{S^4} + \frac{18S_x}{S^4} \right) \right) + R_e^2 \left( \frac{316S_x^2}{13475S^2} - \frac{32S_{xx}}{2695S} \right) - \lambda \left( \frac{288S_x}{35S^4} + \frac{96S_x^2}{35S^4} + \frac{12S_{xx}}{35S^3} \right) + O(R_e^3).
\]

The dimensionless axial pressure gradient is obtained from equation (12) as:

\[
\frac{\partial p}{\partial x} = - \frac{\partial \Omega}{\partial y} - R_e \left[ \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial \psi}{\partial x} + \lambda \left( \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial y} \right) \right] + (3\lambda + 2\lambda_1) \frac{\partial \Omega}{\partial x} \Omega,
\]

where \( \lambda_1 = \frac{\alpha_2}{\rho a_0} \) is the dimensionless cross-viscosity parameter.

Using equation (54), the expression for the pressure gradient takes the form:

\[
\frac{\partial p}{\partial x} = \frac{3}{S^3} + R_e \left( \frac{54S_x}{35S^3} - \lambda \left( \frac{9y^2}{2S^6} - \frac{9}{2S^4} \right) - \left( \frac{81y^2}{S^7} + \frac{27y^4}{4S^7} + \frac{9y^2}{S^5} + \frac{9}{4S^3} \right) \right)
\]
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\[ \times \left( \frac{54y^2 \lambda S_x}{S_7} \right) + O(R_e^2). \]  

(59)

It may be mentioned here that setting \( \lambda = \lambda_1 = 0 \) in equations (54), (57) and (59) we recover the expression for the stream function \( \psi \), \( \Omega \), shear stress at the curved wall and pressure gradient obtained by Makinde et al [4] as our special case.

6 Results and Discussion

An attempt was made to solve the steady two dimensional flow in a diverging channel of varying gap for an incompressible second grade fluid in the presence of a transverse magnetic field using ADM. The results obtained are evaluated graphically for the elastic parameter \( \lambda \), the cross-viscosity parameter \( \lambda_1 \) and the magnetic field parameter \( \Lambda \) considering an exponentially diverging geometry i.e, \( S(x) = e^x \). Figure 2 shows the axial and normal velocity profiles for different values of \( \lambda \). It is observed that the magnitude of axial velocity decreases with increasing value of \( \lambda \). A parabolic profile is observed for both axial and normal velocities. The axial velocity has maximum value at the centre of the channel and minimum value at the walls of the channel, whereas the normal velocity has minimum value at the centre of the channel and maximum at the channel walls. The occurrence of negative axial velocity near the channel walls due to an increase of the elastic parameter \( \lambda \) indicates the possibility of flow reversal near the walls. It is also noticed that the normal velocity decreases with increasing the value of \( \lambda \). Figure 3 shows the behaviour of axial and normal velocities for different values of \( \Lambda \). It is observed that axial velocity dampens with increasing the value of \( \Lambda \). This is well known for Hartmann flow. Moreover, for a channel of varying cross section, the dampening is notable in the centre of the channel. This creates a stagnation point and consequently fluid is pushed to the walls of the channel thereby increasing the velocity in the boundary layer.

Figure 4 is provided to show the behaviour of the pressure gradient for different values of \( \lambda \) and \( \Lambda \). The pressure gradient, which is trying to accelerate the fluid, is counteracted by \( \lambda \) and \( \Lambda \). Hence we observe a pressure gradient drop in Figure 4. The drop is increasing with increasing values of \( \lambda \) and \( \Lambda \). Figure 5 is provided to show the behaviour of shear stress at the curved wall for different values of \( \lambda \) and \( \Lambda \). It is observed that the magnitude of the shear stress at the curved walls increases with increasing the values of \( \lambda \) and \( \Lambda \). However the shear stress at the curved wall decreases as the axial distance increases.
Figure 2: Axial (left) and normal (right) velocity profiles of the flow in an exponentially diverging channel for different values of $\lambda$ at $Re = 0.5$.

Figure 3: Axial velocity profile of the flow in an exponentially diverging channel for different values of $\lambda$ at $Re = 0.5$.

7 Conclusion

Two dimensional steady flow of an incompressible second grade fluid in the presence of a transverse magnetic field in a symmetrical divergent channel of varying width is solved using ADM. Expressions for velocity, shear stress at the curved walls and pressure gradient are calculated in terms of the stream function $\psi$. Our results are general as we can recover the solution obtained by Makinde et al [3] by setting $\lambda = \lambda_1 = 0$. The results obtained are analysed graphically for different values of non-Newtonian parameters $\lambda$, $\lambda_1$ and magnetic field parameter $\Lambda$, taking the channel to be diverging exponentially. It is noticed that increasing the value of $\lambda$ and $\Lambda$ dampens the axial velocity and
Figure 4: Axial pressure gradient of the flow in an exponentially diverging channel for different values of $\lambda$ and $\lambda_1$ at $R_e = 0.5$.

Figure 5: Shear stress at the curved wall for different values of $\lambda$ at $R_e = 0.5$

axial pressure gradient, whereas a rise in the magnitude of shear stress at the curved wall is observed as the value of $\lambda$ and $\Lambda$ is increased. In general it can be concluded that $\lambda$ and $\Lambda$ have influence on the velocity, the shear stress and the pressure gradient. As most industrial fluids are non-Newtonian in nature we hope that this investigation may be helpful to further research in industrial and other fields where fluid flow through diverging channels in the presence of magnetics field is observed.

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References


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