Modelling Dependence between the Equity and Foreign Exchange Markets Using Copulas

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Abstract

Dependence between financial variables is a key consideration for portfolio diversification and risk management. Linear correlation as a measure of dependence is inadequate in capturing dependence of financial variables. In this paper we apply the semi parametric copula based multivariate dynamical model to estimate dependence structure between the equity and foreign exchange markets in Kenya. Several parametric copula models are fitted into the data and their performance in capturing the dependence compared. We find that there exists significant symmetric dependence between the variable. Besides, we find evidence of tail dependence amongst the variables. The findings of this paper are significant to global investors in their pursuit to diversify their portfolios and manage their risks.
1. Introduction

Dependence modeling of financial variables is important for investor because of the needs of portfolio diversification and risk management. Theoretical and empirical models have been used to explain the relationship between the exchange rate and equity markets. The theoretical models include “flow oriented” model of Dornbusch, and Fisher [5] and the “stock oriented” model of Branson [2]. In using these models, the Pearson’s linear correlation coefficient has been used to capture dependence between the variables. Embrechts et al.[6] highlighted the limitations of using linear correlation to capture dependence of heavy tailed financial data. Key among these limitations is that to obtain linear correlation, the variables must have finite first and second order moments, which is not always feasible when dealing with heavy tailed data. Other common measures of dependence include the Spearman’s and Kendall’s rank correlation and the coefficient of tail dependence.

Application of copula theory in finance has been popular over the past couple of years. [12] and [13] are some of the literature that have used copula to study dependence between financial markets. Sklar [14] created a new class of functions known as copulas, which separate a joint distribution function to its univariate marginal distributions and the dependence structure. Some appealing copula properties include: copulas allow the separation of the marginal distributions of variables from their dependence and this increases the flexibility in model estimation and specification. In addition, copula function enables us to describe not only the degree of dependence but also the nature of dependence (whether symmetric or asymmetric). Therefore, copulas can be used to model extreme events. Besides, copulas do not require the assumption of joint normality akin to linear correlations. This property enables copulas to be suitable for use in financial data. Finally, copulas are invariant to strictly increasing nonlinear transformations.

Estimation of copula parameters is key in dependence modeling. Copula estimation techniques include the maximum likelihood estimation (MLE), inference function for margins (IFM) of Joe and Xu [9] and canonical maximum likelihood (CML) of Genest et al. [7]. Applying CML for time series data, Chen and Fan [4] established the large sample properties of the semi parametric copula based multivariate dynamical models (SCOMDY). Chan et al. [3] established the statistical inferences methodologies for the semi parametric copula based multivariate GARCH models.

In this paper we apply the SCOMDY model to estimate dependence between the equity and exchange markets in Kenya. Using the parametric bootstrapping procedure proposed by Genest et al. [8], copula goodness of fit testing is carried out in order to obtain the “best” copula model for capturing the dependence inherent in the data.
One of the findings of this paper is that the dependence among the financial markets is significant. Upper and lower tail dependence is present hence the Gaussian copula is ill-suited for the data. The \( t \) copula with 10 degrees of freedom is found to be the best model. This is in line with the empirical copula tests done on the bivariate return data which points to symmetric dependence. These findings will improve risk management efforts of international investors who consider the Kenyan market as a suitable investment destination within the region.

The rest of this paper is organized as follows. In section 2, we review the theory of copula and dependence measures. In section 3 we present the SCOMDY model. In section 4 we describe the data and analyze results. Conclusion is in section 5.

2. Copula Theory and Estimation techniques

A two dimensional copula \( C(u,v) \) is a real valued function with the following properties: \( \text{dom}(C) = [0,1]^2 \), \( C \) is both 2-increasing and grounded and for every \( u,v \in [0,1] \), \( C(u,1) = u \) and \( C(1,v) = v \).

**Sklar’s theorem**

Let \( H \) be a bivariate joint distribution function with marginal distributions \( F \) and \( G \) respectively. Then there exists a copula \( C \) such that for all \( x,y \in \mathbb{R} \),

\[
H(x,y) = C(F(x),G(y))
\]

(1)

If the marginals \( F \) and \( G \) are continuous then \( C \) is unique. Otherwise \( C \) is uniquely defined on \( \text{ran}(F) \times \text{ran}(G) \). Conversely, if \( C \) is a copula and \( F \) and \( G \) are distribution functions, the function \( H \) defined above, is a joint distribution function with margins \( F \) and \( G \). Sklar’s theorem enables the description of multivariate distributions without constraints on the univariate margins. Several dependence measures including Spearman’s rho, Kendall’s tau and tail dependence coefficients can be expressed as functions of copulas. The tail dependence coefficients are particularly useful in capturing extreme dependence of variables. Some dependence measures are defined below:

i) Kendall’s tau \( \tau_{X,Y} \) - for two independent and identically distributed random vectors \( (X,Y) \) and \( (X',Y') \) with joint distribution \( H(x,y) = C(F(x),G(y)) \)

\[
\tau_{X,Y} = \frac{\text{P}\left(\left(X-X'\right)\left(Y-Y'\right) > 0\right) - \text{P}\left(\left(X-X'\right)\left(Y-Y'\right) < 0\right)}{\text{P}\left(\left(X-X'\right)\left(Y-Y'\right) > 0\right) + \text{P}\left(\left(X-X'\right)\left(Y-Y'\right) < 0\right)}
\]

(2)

ii) Spearman’s rho \( \rho_{X,Y} \) - for three independent and identically distributed
random vectors \((X,Y), (X', Y')\) and \((X', Y')\) with joint distribution

\[
H(x, y) = C(F(x), G(y))
\]

\[
\rho_{x,y} = 3\left( P\left( (X - X')(Y - Y') > 0 \right) - P\left( (X - X')(Y - Y') < 0 \right) \right)
\]

\[
= 12 \iint_{[0,1]^2} C(u,v) \, du \, dv - 3
\]

iii) The coefficient of upper tail dependence is defined as

\[
\lambda_u = \lim_{q \to 1} P(Y > G^{-1}(q)|X > F^{-1}(q)) = 2 - \lim_{q \to 1} \frac{1 - C(t,t)}{1 - t}
\]

for \(0 < q < 1\)

iv) The coefficient of lower tail dependence is defined as

\[
\lambda_l = \lim_{q \to 0} P(Y \leq G^{-1}(q)|X \leq F^{-1}(q)) = \lim_{q \to 0} \frac{C(q,q)}{q}
\]

for \(0 < q < 1\)

The above dependence measures being functions of copulas are invariant to strictly increasing non-linear transformations. Some copula families include the elliptical (Gaussian and \(t\) copulas), Archimedean (Clayton, Gumbel, Joe, Frank), and Marshall-Olkin copulas. The copula models have varying tail dependence behavior e.g. Gaussian copula does not capture tail dependence whilst the \(t\) copula captures symmetric tail dependence. Clayton and Gumbel copulas capture lower and upper tail dependence respectively. Nelsen [11] offers a concise review of copula models and dependence measures.

3. SCOMDY Model Specification

SCOMDY model is a semi parametric estimation procedure proposed by Chen and Fan [4]. It is an extension to time series data of the CML method proposed be Genest et al. [7]. The model is specified below for the bivariate case

1. Let \(\{X_t\}_{t \geq 1}\) be a vector valued process such that \(X_t = \sum_{i} \varepsilon_i\) with vector valued residuals \(\varepsilon_i = (\varepsilon_{i1}, \varepsilon_{i2})\), conditional variance matrices \(\sum = (\sigma_{ij}^2)_{i,j=1,2}\).

2. Let \(\sum = diag(\sigma_{11}^2, \sigma_{22}^2)\) be such that for every \(j = 1, 2\) a univariate GARCH (p, q) specification holds i.e.

\[
\sigma_{jj}^2 = \alpha_{0j} + \sum_{i=1}^{p} \alpha_{ij} X_{j-i}^2 + \sum_{k=1}^{q} \beta_{jk} \sigma_{j-k}^2
\]

with \(\gamma_j = (\alpha_{0j}, \alpha_{1j}, \ldots, \alpha_{pj}, \beta_{1j}, \ldots, \beta_{pj}, \beta_{pj+1}, \ldots, \beta_{pj+q}) \in (0, \infty)^{p+q}\)

Also, let \(\{\varepsilon_i\}_{i \geq 1}\) be independent and identically distributed according to a bivariate cumulative distribution function \(H(\varepsilon) = C(F_{\varepsilon_1}, F_{\varepsilon_2} : \theta)\) where
Dependence between the equity and foreign exchange markets

$F_{e,j}$ are marginal distributions for the univariate residuals $e_{j,t}$ and $C(.; \theta)$ belongs to a parametric family of copulas with parameter $\theta \in \Theta \subseteq \mathbb{R}$. Let $E(e_i)=0$ and $Cov(e_i)=(r_{i,j})_{i,j=1,2}$ such that for all $i=1,2$ $r_{i,j}=Var(e_{i,j})=1$ and $r_{i,j}=Cov(e_{i,j})$ for $i \neq j$

3. Let $\tilde{\gamma}_j$ be the quasi maximum likelihood estimator of $\gamma_j$ as proposed by Berkes et al. [1] and $\tilde{e}_{j,t}=X_{j,t}/\tilde{\sigma}_{j,t}$ be the estimated standardized residual in every component $j=1,2$. With $v=v(T)$, we can estimate the true marginal distribution of $\tilde{e}_{j,t}$, $F_{e,j}$ by

$$\tilde{F}_{e,j}(\tilde{e}_{j,t}) = \frac{1}{T-v+1} \sum_{s=v}^{T} 1_{(\tilde{e}_{j,s} \leq \tilde{e}_{j,t})} \quad (7)$$

which is the modified empirical distribution function. The residual copula parameter $\theta$ is estimated by $\tilde{\theta}$ which is the pseudo MLE based on the pseudo sample $(\tilde{U}_1, \ldots, \tilde{U}_T)=\{\tilde{U}_t=(\tilde{F}_{e,1}(\tilde{e}_{1,t}), \tilde{F}_{e,2}(\tilde{e}_{2,t}))\}_{t=v}^{T}$ i.e

$$\tilde{\theta} = \arg \max_{\theta} \frac{1}{T-v+1} \sum_{s=v}^{T} \log c(\tilde{F}_{e,1}(\tilde{e}_{1,t}), \tilde{F}_{e,2}(\tilde{e}_{2,t}); \theta) \quad (8)$$

where $c(u,v; \theta) = \frac{\partial^2 C(u,v)}{\partial u \partial v}$ is the copula density. The pseudo MLE estimator is similar to the canonical MLE but offset by a factor $v=v(T)$ such that $v=0(T)$. Chan et al. [3] provided the conditions necessary for consistency and asymptotic normality of the SCOMDY parameter estimator. The estimator is robust to marginal distribution misspecification as shown in simulation studies by Kim et al. [10] for CML estimators.

4. Data Analysis

4.1 Data Description

The data consists of daily prices from 02/01/2001 to 31/12/2013 for both the 20-share stock price index and the exchange rate for the Kenya Shilling (KES) versus the United States dollar (USD). The exchange rate data is the daily spot exchange rate of the units of the USD against one unit of KES. The equity and exchange rate data are available from the Nairobi Securities Exchange (NSE) and Central Bank of Kenya (CBK) websites respectively. The dataset offers a snapshot of Kenya’s financial markets activity over the period under study. For each price series, the raw price data is converted to percentage log returns $R_v = \{\log P_v - \log P_{v-1}\} \times 100$ where $R_v =$ return, $P_v =$ price for $i=1,2$ and $t=1,3255$.
The summary statistics of the returns data are presented in table 1 below.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Exchange returns</th>
<th>Equity returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.0031</td>
<td>0.0292</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.4914</td>
<td>0.8631</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.3549</td>
<td>0.5425</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>20.5783</td>
<td>7.4534</td>
</tr>
<tr>
<td>Jarque Bera statistic</td>
<td>57564.3336</td>
<td>7704.8077</td>
</tr>
<tr>
<td>p value</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Table 1: Summary Statistics**

Both series have positive skewness values and excess kurtosis which signifies their non-normality and heavy tailed nature. The Jarque-Bera test strongly rejects the normality of both returns series which is in line with the stylized facts of financial time series. Correlation tests were carried out on the returns series and the results are presented in table 2 below.

<table>
<thead>
<tr>
<th>Correlation Measures</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson</td>
<td>0.1562</td>
</tr>
<tr>
<td>Spearman</td>
<td>0.063</td>
</tr>
<tr>
<td>Kendall</td>
<td>0.0424</td>
</tr>
</tbody>
</table>

**Table 2: Correlation measures**

All the correlation measures are positive and significant at 5% significance level. The positive value of Kendall’s tau is indicative that the probability of concordance is slightly higher than the probability of discordance in the bivariate series. The bivariate empirical copula grid was constructed to view the dependence structure inherent in the data. Table 3 below presents the empirical copula grid of frequencies. The ranks for the exchange rate returns are in the vertical axis in ascending order while the ranks for the equities returns are on the horizontal axis in ascending order.

<table>
<thead>
<tr>
<th>36</th>
<th>23</th>
<th>32</th>
<th>39</th>
<th>28</th>
<th>27</th>
<th>31</th>
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<th>46</th>
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<tr>
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<td>30</td>
<td>28</td>
<td>31</td>
<td>24</td>
<td>30</td>
<td>24</td>
<td>28</td>
</tr>
</tbody>
</table>

**Table 3: Empirical copula grid table. The frequencies show how the pairwise returns relate. On each axis the step size used is 0.1**
From table 3, cell (1,1) has 48 observations indicating that out of 3254 observations, there are 48 occurrences when the exchange rate and equity returns lie in their respective 1/10 percentile. Cell (10,10) has 46 observations indicating that there are 46 occurrences out of the 3254 observations when both the equity and exchange rate returns lie in their respective 9/10 percentile. There is presence of upper and lower tail dependence. The distribution of frequencies across the grid indicates the symmetric dependence of the returns with no obvious presence of perfect positive or negative dependence.

4.2 Marginal Models

We filter each returns series using the AR(k)-GARCH(p,q) models while applying the quasi maximum likelihood estimation technique of Berkes et al. [1]. Table 4 below presents the parameter estimates of the AR(k)-GARCH(p,q) models for each series.

<table>
<thead>
<tr>
<th></th>
<th>AR (1)</th>
<th>AR (2)</th>
<th>μ</th>
<th>ω</th>
<th>α</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange Rate</td>
<td>0.17204 (0.029290)</td>
<td>0.011314 (0.006708)</td>
<td>0.004408 (0.002192)</td>
<td>0.348978 (0.107875)</td>
<td>0.724490 (0.058241)</td>
<td></td>
</tr>
<tr>
<td>Stock market</td>
<td>0.32115 (0.02146)</td>
<td>0.14537 (0.01917)</td>
<td></td>
<td></td>
<td>0.19347 (0.06251)</td>
<td>0.66984 (0.08453)</td>
</tr>
</tbody>
</table>

Table 4: Parameter estimates for the marginal models. Number in parenthesis are the asymptotic standard errors.

All the parameter estimates are statistically significant at 5% significance level. The GARCH parameter estimates for both series indicate persistence of shocks. The time varying means for each series are significant thus indicating long memory. To check the suitability of the filtering models, we subject the standardized residuals to test of serial independence using the Lagrange Multiplier and randomness test using the Ljung-Box statistic. Table 5 presents the independence tests carried out on the standardized residuals from each series. For both series, the tests were carried out up to the twentieth order. The null hypothesis of no ARCH effects could not be rejected at 5% level of significance. The Ljung-Box p value for exchange rate residuals pointed to presence of residual serial dependence in the data. On the other hand, the equity data lacked any signs of residual serial dependence. We opt to use the bivariate standardized residuals assuming that they are independent and identically distributed.

<table>
<thead>
<tr>
<th></th>
<th>LM test</th>
<th>p value</th>
<th>L-B test</th>
<th>p value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange rate</td>
<td>4.5837</td>
<td>0.9999</td>
<td>36.3972</td>
<td>0.01381</td>
</tr>
<tr>
<td>Stock market</td>
<td>18.1945</td>
<td>0.5746</td>
<td>25.1315</td>
<td>0.1964</td>
</tr>
</tbody>
</table>

Table 5: Results of the independence tests carried out on each residual series.
4.3 Copula Modelling

The empirical distribution function in equation (7) is applied on each residual series to obtain the probability integral transforms. Parametric copula models including the Gaussian, $t$ and Clayton copulas are fitted into the transformed bivariate series. Applying the parametric bootstrapping procedure proposed by Genest et al. [8], goodness of fit testing on the fitted copula models is carried out to determine the “best” copula model among the selected models. The results of the parameter estimates, asymptotic standard errors and goodness of fit testing are presented in the table 6 below.

<table>
<thead>
<tr>
<th>Copula</th>
<th>$\hat{\theta}$</th>
<th>std error</th>
<th>LL</th>
<th>CvM</th>
<th>p value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>0.0660</td>
<td>0.0169</td>
<td>0.0155</td>
<td>0.0257</td>
<td>0.4510</td>
</tr>
<tr>
<td>$t$ (4df)</td>
<td>0.0430</td>
<td>0.0217</td>
<td>-42.4200</td>
<td>0.0398</td>
<td>0.0225</td>
</tr>
<tr>
<td>$t$ (10 df)</td>
<td>0.0489</td>
<td>0.0196</td>
<td>1.8360</td>
<td>0.0168</td>
<td>0.5509</td>
</tr>
<tr>
<td>Clayton</td>
<td>0.0381</td>
<td>0.0196</td>
<td>2.0780</td>
<td>0.0423</td>
<td>0.0405</td>
</tr>
</tbody>
</table>

Table 6: Copula models fitted to the data. $\hat{\theta}$ is the parameter estimate, std error is the asymptotic standard error of the estimate, LL is the maximized log likelihood value, CvM is the value of the Cramer-von Mises statistic.

All the parameter estimates are statistically significant at 5% level of significance. The correlation coefficients for both the Gaussian and Student copulas are positive which is in line with the sample correlation estimates. The $t$ copula with 4 degrees of freedom and the Clayton copula were rejected at 5% significance level. The remaining two copula models could not be rejected at the said significance level. However, since the empirical copula model suggests presence of both lower and upper tail dependences, the Gaussian copula which captures zero tail dependence is discarded. The $t$ copula with 10 degrees of freedom is thus found to be a good fit for the bivariate transformed series.

5. Conclusion

This paper examines the dependence structure between the equity and exchange rate markets in Kenya. We first filter the univariate returns series using AR(k)-GARCH (p,q). Empirical distribution function is applied on the univariate standardized residuals series to transform them to standard uniform margins. Parametric copula models are fitted to the transformed series and copula parameter estimation is done using the SCOMDY model technique. One major finding of this paper is that the $t$ copula is the best model to capture the dependence structure in the data. This is a departure from the Gaussian copula with normal margins which is quite popular in modeling multivariate financial data. The finding of significant positive dependence in the bivariate return series is in line with market expectations which point to investors flocking a country when it is viewed as a
favorable investment destination leading an appreciation of its currency against the international currencies and rise in stock levels. On the other hand, investors divest from an economy whenever the country’s investment climate deteriorates leading to a decline in equity prices, equity index levels and depreciation in the country’s currency. Lastly, the finding of the presence of tail dependence in the bivariate series as captured by the empirical copula points to the possibility of the exchange and equity markets to rise and fall together during periods of economic boom and bust. These findings are vital to global investors in their pursuit to diversify their portfolios in the country’s economy and manage their risks.

References


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