Realistic Method for Solving Fully Intuitionistic Fuzzy Transportation Problems

P. Pandian

Department of Mathematics, School of Advanced Sciences
VIT University, Vellore-14, Tamilnadu, India

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Abstract

A new method namely, realistic method is proposed for finding an optimal intuitionistic fuzzy (IF) solution to a fully intuitionistic fuzzy transportation (IFT) problem in which ranking functions are not used. The proposed method is based on the crisp transportation algorithm and provides that the optimal IF solution and the optimal objective IF value of the fully IFT problem do not contain any negative part. For illustrating, a fully IFT problem is solved by using the proposed method. The proposed method is an appropriate method to apply for finding an optimal solution of IFT problems occurring in real life situations.

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1. Introduction

The transportation problem is a special class of linear programming problem which has applications in Science, Engineering and Technology. An uncertain transportation problem is a transportation problem in which at least one of the parameters is uncertainty. The theory of fuzzy set introduced by Zadeh [9] has achieved successful applications in various fields including Mathematical Programming. In the literature, many researcher have developed various methods for solving different types of fuzzy transportation (FT) problems. Pandian and Natrajan [6] proposed the dynamic method for solving fully FT problems which provides that optimal fuzzy decision variables and the optimal objective fuzzy value of the FT problems do not contain any negative part. Atanassov [1] introduced the concept of IF

In the existing methods [2,3,4,8], the optimal solution of some of the IF decision variables and the optimal objective IF value of the IFT problem have negative part which depicts that quantity of the product and transportation cost may be negative. But the negative quantity of the product and negative transportation cost have no physical meaning. Therefore, the solution obtained in [2,3,4,8] for IFT problems are not realistic and not applicable.

In this paper, we develop a new method namely, realistic method for obtaining an IF optimal solution to a fully IFT problem where all parameters are IF triangular numbers. To overcome the shortcomings of the existing methods [2,3,4,8], the proposed method based on algorithm of the crisp transportation problem and provides non-negative optimal IF solution and non-negative optimal objective IF value of the fully IFT problem. Ranking functions and IF arithmetic operations are not used. By means of a numerical example, the proposed method of solving the fully IFT problem is illustrated. The realistic method is an appropriate method for solving IFT problems of the real life situations and provides an applicable optimal solution.

2. Intuitionistic Fuzzy Transportation Problem

We need the following mathematical orientated definitions of IF set, triangular IF number and membership function and non-membership function of a IF set/number which can be found in [1,2,3,4,8].

**Definition 2.1.** Let X denote a universe of discourse and \( A \subseteq X \). Then, an IF set of \( A \) in X, \( \tilde{A} \) is defined as follows:

\[
\tilde{A} = \{(x, \mu_A(x), \vartheta_A(x)); x \in X\}
\]

where \( \mu_A(x), \vartheta_A(x) : X \to [0,1] \) are functions such that \( 0 \leq \mu_A(x) + \vartheta_A(x) \leq 1 \) for all \( x \in X \). For each \( x \) in \( X \), \( \mu_A(x) \) and \( \vartheta_A(x) \) represent the membership the degree of non-membership values of \( x \) in the set \( A \subseteq X \). \( \mu_A(x) \) and \( \vartheta_A(x) \).
Definition 2.2: A fuzzy number $\tilde{a}^i$ is a triangular IF number denoted by $(a_2, a_3, a_4)(a_1, a_3, a_5)$ where $a_1, a_2, a_3, a_4$ and $a_5$ are real numbers such that $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$ and its membership function $\mu_{\tilde{a}^i}(x)$ and its non-membership function $\nu_{\tilde{a}^i}(x)$ are given below.

$$
\mu_{\tilde{a}^i}(x) = \begin{cases} 
\frac{x - a_2}{a_3 - a_2} : a_2 \leq x \leq a_3 \\
\frac{a_4 - x}{a_4 - a_3} : a_3 \leq x \leq a_4 \\
0 : \text{otherwise}
\end{cases}
$$

$$
\nu_{\tilde{a}^i}(x) = \begin{cases} 
\frac{a_3 - a_5}{a_5 - a_3} : a_3 \leq x \leq a_5 \\
\frac{x - a_5}{a_5 - a_3} : \text{otherwise}
\end{cases}
$$

Let $IF(R)$ be a set of all triangular IF numbers over $R$, a set of real numbers. Based on ordering relation in interval theory/fuzzy set theory, we define the following:

Definition 2.3: Let $\tilde{a}^i = (a_2, a_3, a_4)(a_1, a_3, a_5)$ and $\tilde{b}^i = (b_2, b_3, b_4)(b_1, b_3, b_5)$ be in $IF(R)$. Then,

(a) $\tilde{a}^i$ and $\tilde{b}^i$ are said to be equal if $a_i = b_i$, $i = 1, 2, 3, 4, 5$ and

(b) $\tilde{a}^i$ is said to be less than or equal $\tilde{b}^i$ if $a_i \leq b_i$, $i = 1, 2, 3, 4, 5$.

Definition 2.4: Let $\tilde{a}^i = (a_2, a_3, a_4)(a_1, a_3, a_5)$ be in $F(R)$. Then, $\tilde{a}^i$ is said to be positive ($\tilde{a}^i \geq \tilde{0}^i$) if $a_i \geq 0$.

Definition 2.5: Let $\tilde{a}^i = (a_2, a_3, a_4)(a_1, a_3, a_5)$ be in $F(R)$. Then, $\tilde{a}^i$ is said to be integer if $a_i \geq 0$, $i = 1, 2, 3, 4, 5$ are integers.

Consider the following fully IFT problem in which all parameters, that is, decision variables, transportation costs, supplies and demands, are triangular IF numbers:

(P) Minimize $\tilde{Z}^i = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$

subject to

$$
\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_{ij}^i, \text{ for } i = 1, 2, \ldots, m
$$

$$
\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_{ij}^i, \text{ for } j = 1, 2, \ldots, n
$$

$$
\tilde{x}_{ij} \leq \tilde{0}^i, \text{ for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n \text{ and integers}
$$
where \( m \) = the number of supply points; \( n \) = the number of demand points; \( \tilde{x}_{ij} \approx (x_{ij}^2, x_{ij}^3, x_{ij}^4) \) (\( x_{ij}^1 \), \( x_{ij}^3 \), \( x_{ij}^5 \)) is the uncertain number of units shipped from supply point \( i \) to demand point \( j \); \( \tilde{c}_{ij} \approx (c_{ij}^2, c_{ij}^3, c_{ij}^4) \) (\( c_{ij}^1 \), \( c_{ij}^3 \), \( c_{ij}^5 \)) is the uncertain cost of shipping one unit from supply point \( i \) to the demand point \( j \); \( \tilde{a}_i \approx (a_i^2, a_i^3, a_i^4) \) (\( a_i^1 \), \( a_i^3 \), \( a_i^5 \)) is the uncertain supply at supply point \( i \) and \( \tilde{b}_j \approx (b_j^2, b_j^3, b_j^4) \) (\( b_j^1 \), \( b_j^3 \), \( b_j^5 \)) is the uncertain demand at demand point \( j \).

A set of triangular IF numbers \( \tilde{X}^I = \{\tilde{x}_{ij}^I = (x_{ij}^{I_2}, x_{ij}^{I_3}, x_{ij}^{I_4}) (x_{ij}^{I_1}, x_{ij}^{I_3}, x_{ij}^{I_5}), i = 1,2,\ldots,m \) and \( j = 1,2,\ldots,m \} \) is said to be a feasible IF solution to the problem \( (P) \) if \( \tilde{X}^I = \{\tilde{x}_{ij}^I = (x_{ij}^{I_2}, x_{ij}^{I_3}, x_{ij}^{I_4}) (x_{ij}^{I_1}, x_{ij}^{I_3}, x_{ij}^{I_5}), i = 1,2,\ldots,m \) and \( j = 1,2,\ldots,m \} \) satisfies the conditions (1), (2) and (3).

A feasible IF solution \( \tilde{X}^I = \{\tilde{x}_{ij}^I = (x_{ij}^{I_2}, x_{ij}^{I_3}, x_{ij}^{I_4}) (x_{ij}^{I_1}, x_{ij}^{I_3}, x_{ij}^{I_5}), i = 1,2,\ldots,m \) and \( j = 1,2,\ldots,m \} \) of the problem \( (P) \) is said to be an optimal IF solution to the problem \( (P) \) if \( Z(\tilde{X}^I) \leq Z(\tilde{U}^I) \) for all feasible \( \tilde{U}^I \) of the problem \( (P) \).

### 3. The Realistic Method

We need the following theorem which is used in the proposed method, namely realistic method for solving the fully IFT problems.

**Theorem 3.1:** Let \( \{x_{ij}^{I_5}, i = 1,2,\ldots,m \) and \( j = 1,2,\ldots,n \} \) be an optimal solution to the problem \( (P_2) \) and \( \{x_{ij}^{k}, i = 1,2,\ldots,m \) and \( j = 1,2,\ldots,n \} \) be an optimal solution to the problem \( (P_k) \), \( k = 4,3,2,1 \) where

\[
(P_2) \text{ Minimize } Z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^5 x_{ij}^5 \\
\text{subject to} \\
\sum_{j=1}^{n} x_{ij}^5 = a_i^5 \text{, for } i = 1,2,\ldots,m \\
\sum_{i=1}^{m} x_{ij}^5 = b_j^5 \text{, for } j = 1,2,\ldots,n \\
x_{ij}^5 \geq 0 \text{, for } i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n \text{ and integers,}
\]

and for \( k = 5, 4, 3, 2, \)

\[
(P_{k-1}) \text{ Minimize } Z_{k-1} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{k-1} x_{ij}^{k-1}
\]
subject to
\[ \sum_{j=1}^{n} x_{ij}^{k-1} = a_i^{k-1}, \text{ for } i = 1, 2, \ldots, m \]
\[ \sum_{i=1}^{m} x_{ij}^{k-1} = b_j^{k-1}, \text{ for } j = 1, 2, \ldots, n \]
\[ x_{ij}^{k-1} \leq x_{ij}^{*k}, \text{ for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n \]
\[ x_{ij}^{k-1} \geq 0, \text{ for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n \text{ and integers.} \]

Then, \( [\tilde{x}_{ij}^{k}] = \{ \tilde{x}_{ij}^{k} = (x_{ij}^{x_2}, x_{ij}^{x_3}, x_{ij}^{x_4})(x_{ij}^{x_1}, x_{ij}^{x_3}, x_{ij}^{x_5}), i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, m \} \) is an optimal IF solution to the given problem (P).

**Proof:** Now, since \([x_{ij}^{*1}],[x_{ij}^{*2}],[x_{ij}^{*3}],[x_{ij}^{*4}]\) and \([x_{ij}^{*5}]\) are feasible solutions of \((P_1),(P_2),(P_1)\) \((P_4)\) and \((P_5)\) respectively, \([\tilde{x}_{ij}^{k}]\) is a feasible IF solution to the problem (P) where
\[ [\tilde{x}_{ij}^{k}] = \{ \tilde{x}_{ij}^{k} = (x_{ij}^{x_2}, x_{ij}^{x_3}, x_{ij}^{x_4})(x_{ij}^{x_1}, x_{ij}^{x_3}, x_{ij}^{x_5}), i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, m \}. \]

Let \([\tilde{y}_{ij}^{1}], [\tilde{y}_{ij}^{2}], [\tilde{y}_{ij}^{3}], [\tilde{y}_{ij}^{4}], [\tilde{y}_{ij}^{5}]\) be a feasible IF solution of the problem (P). Clearly, \([y_{ij}^{1}], [y_{ij}^{2}], [y_{ij}^{3}], [y_{ij}^{4}]\) and \([y_{ij}^{5}]\) are feasible solutions of \((P_1),(P_2),(P_3), (P_4)\) and \((P_5)\) respectively.

Now, since \([x_{ij}^{*1}],[x_{ij}^{*2}],[x_{ij}^{*3}],[x_{ij}^{*4}]\) and \([x_{ij}^{*5}]\) are optimal solutions of \((P_1),(P_2),(P_3), (P_4)\) \((P_4)\) and \((P_5)\) respectively, we have
\[ Z_i([x_{ij}^{*1}]) \leq Z_i([y_{ij}^{*1}]): Z_2([x_{ij}^{*2}]) \leq Z_2([y_{ij}^{*2}]); Z_3([x_{ij}^{*3}]) \leq Z_3([y_{ij}^{*3}]); Z_4([x_{ij}^{*4}]) \leq Z_4([y_{ij}^{*4}]) \text{ and } Z_5([x_{ij}^{*5}]) \leq Z_5([y_{ij}^{*5}]) \]
This implies that \( Z([\tilde{x}_{ij}^{k}]) \leq Z([\tilde{y}_{ij}^{k}]), \) for all feasible \([\tilde{y}_{ij}^{k}]\) of the problem (P).

Thus, \([\tilde{x}_{ij}^{k}] = \{ \tilde{x}_{ij}^{k} = (x_{ij}^{x_2}, x_{ij}^{x_3}, x_{ij}^{x_4})(x_{ij}^{x_1}, x_{ij}^{x_3}, x_{ij}^{x_5}), i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, m \} \) is an optimal IF solution to the given problem (P).

Hence the theorem is proved.

**Remark 3.1:** The optimal IF solution \([\tilde{x}_{ij}^{k}] = \{ \tilde{x}_{ij}^{k} = (x_{ij}^{x_2}, x_{ij}^{x_3}, x_{ij}^{x_4})(x_{ij}^{x_1}, x_{ij}^{x_3}, x_{ij}^{x_5}), i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, m \} \) and the total minimum IF transportation cost
\[ Z([\tilde{x}_{ij}^{k}]) = (Z_2([x_{ij}^{x_2}]), Z_2([x_{ij}^{x_3}]), Z_3([x_{ij}^{x_4}]))(Z_1([x_{ij}^{x_1}]), Z_3([x_{ij}^{x_3}]), Z_5([x_{ij}^{x_5}])) \]
are positive because for each \( k = 1, 2, 3, 4, 5, \) \( x_{ij}^{*k} \) and \( Z_i([x_{ij}^{*k}]) \) are positive for all \( i \) and \( j \).

We now propose a new method namely, realistic method to obtain an optimal solution of fully IFT problem based on the crisp transportation algorithm.

The realistic method is as follows:
Algorithm:

Step 1: Construct the problem \((P_5)\) from the given IFT problem, \((P)\) and solve it by the zero point method [5]/ a classical transportation algorithm. Let \(x_{ij}^5, i = 1,2,\ldots,m \quad \text{and} \quad j = 1,2,\ldots,n\) be an optimal solution of the problem \((P_5)\).

Step 2: Construct the problem \((P_{k-1})\) from the given problem \((P)\) and solve it by the zero point method [5] / a classical transportation algorithm for each \(k=5,4,3,2\). Let \(x_{ij}^{k-1}, i = 1,2,\ldots,m \quad \text{and} \quad j = 1,2,\ldots,n\) be an optimal solution of \((P_{k-1})\), \(k=5,4,3,2\).

Step 3: \(\bar{x}_{ij} = (x_{ij}^5,x_{ij}^4,x_{ij}^3,x_{ij}^2), i = 1,2,\ldots,m \quad \text{and} \quad j = 1,2,\ldots,n\) is an optimal IF solution to the given IFT problem, \((P)\) by the Theorem 3.1.

4. Numerical Example

The proposed method is illustrated by the following example.

Example 4.1: Consider the following fully IFT problem.

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>(3,16,18)</td>
<td>(1,16,20)</td>
<td>(0,1,13)</td>
<td>(2,4,5)</td>
</tr>
<tr>
<td>S2</td>
<td>(5,11,16)</td>
<td>(3,11,18)</td>
<td>(2,4,6)</td>
<td>(3,6,8)</td>
</tr>
<tr>
<td>S3</td>
<td>(2,8,10)</td>
<td>(1,8,12)</td>
<td>(3,15,16)</td>
<td>(3,7,12)</td>
</tr>
<tr>
<td>Demand</td>
<td>(3,4,6)</td>
<td>(2,5,7)</td>
<td>(4,8,12)</td>
<td>(9,17,25)</td>
</tr>
</tbody>
</table>

The given IFT problem is a balanced one since total IF demand = total IF supply = (9,17,25)(6,17,30).

Now, by solving the problems \((P_5)\), \((P_4)\), \((P_3)\), \((P_2)\) and \((P)\) using the zero point method / any one of the existing crisp transportation algorithms, we obtain the following optimal solutions of the crisp problems \((P_5)\), \((P_4)\), \((P_3)\), \((P_2)\) and \((P)\) respectively:

\((P_5): x_{i1}^5 = 6; x_{i2}^5 = 2; x_{i3}^5 = 8; x_{j1}^5 = 8; x_{j2}^5 = 6 \quad \text{and the minimum transportation cost is 318;}

\(P_4): x_{i1}^4 = 5; x_{i2}^4 = 2; x_{i3}^4 = 1; x_{j1}^4 = 6; x_{j2}^4 = 6 \quad \text{and the minimum transportation cost is 208;}

\((P_3): x_{i1}^3 = 2; x_{i2}^3 = 1; x_{i3}^3 = 5; x_{j1}^3 = 4; x_{j2}^3 = 3 \quad \text{and the minimum transportation cost is 102;}

\((P_2): x_{i1}^2 = 2; x_{i2}^2 = 0; x_{i3}^2 = 4; x_{j1}^2 = 3; x_{j2}^2 = 3; x_{j3}^2 = 0 \quad \text{and the minimum transportation cost is 18;}

\((P): x_{i1}^1 = 1; x_{i2}^1 = 0; x_{i3}^1 = 3; x_{j1}^1 = 2 \quad \text{and the minimum transportation cost is 5.}

Thus, the optimal IF solution of the given IFT problem is \(\bar{x}_{i1} = (2,4,5)(1,4,6); \bar{x}_{i2} = (0,1,2)(0,1,2); \bar{x}_{i3} = (4,5,6)(3,5,8); \bar{x}_{j1} = (3,4,6)(2,4,8)\) and \(\bar{x}_{j3} = (0,3,6)(0,3,6)\) and the total minimum IFT cost is \((18,102,208)(5,102,318)\).
5. Conclusion

The main advantage of the realistic method is that both the optimal IF solution and the optimal objective IF value of the fully IFT problem are non-negative IF numbers. Since the proposed method is based on the classical transportation algorithm so it can be easy to compute and to apply. Since the optimal solution obtained by the proposed method does not contain negative part, the solution is meaningful and can be applied in real life situations. Thus, the realistic method provides an applicable optimal solution which helps the decision makers while they are handling real life transportation problems having IF parameters.

References


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