Applied Mathematical Sciences, Vol. 8, 2014, no. 114, 5689 - 5696
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ams.2014.47550

Control of Multi Flying Vehicle which Move in 3 Dimensions to Save Fuel

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Abstract

The world of commercial and military aviation, as well as the world of transportation in general are unable to break away from the fuel. With the lack of fuel and increasingly expensive fuel prices in the world market, airlines are required to make innovations that fuel can be saved. The idea that an aircraft is flying with another aircraft to fly together in the save formation, sure is one of the alternative methods to save fuel. This paper is presented a moving plane space a real motion in three dimensions or spaces. Designing the control for each flying vehicle is carried out by optimal control theory and the simulation is done with matlab software. The results of this research are expected to provide the basics in trying to fly some airplanes together. How to fly an airplane simulations together with the airplane in the special formation, not always together from beginning to end, but from the initial position, the aircraft fly singly, and then start at a position already agreed do flew together in formation until the point a particular purpose, then the aircraft spread towards the ultimate goal of each plane.

Keywords: flying vehicle, optimal control theory, save fuel

1. Introduction

This paper begins from the exposition multi flying vehicle research which have been published in recent years, for examples, Michael, et al. (2011), which uses multiple spacecraft to fly to the surveillance activities. The next, Kaliappan, et al. (2011) examining models of mini multi helicopter is used for the purposes of search and surveillance. The next publication was done by Fink, et al. (2011) who are working on controlling a flying robot picked up the goods together. While Turpin, et al. (2012) published his work on formation flying spacecraft quadrotors were used on the opponent's attack. Other publications done by Lindsey, et al. (2012) also explores the quadrotors team together to build three dimensional objects. Enhanced studies involving multiple spacecraft flying published in 2013, among others, performed by Duan, et al. (2013a) which utilizes multiple probe Trophallaxis in controlling the fly. Subsequently, Duan et al. (2013b) using the discretization method and the parameterization of time Control in controlling the spacecraft formation flying. The subsequent Yuan, et al. (2013) controlling multi flying vehicles hooks or aerodynamic coupling of each vehicle. Won, et al. (2013) publicized the use of multi-rotor unmanned in the tasks present a picture of 3-dimensional (3D) with high acquisition. Perez, et al. (2013) serves a Ground Control Station which was developed for a number of unmanned aerial vehicles for reconnaissance missions. Ortiz, et al. (2013) discussed something that allows one person to be able to check the target operator for a fixed amount of time in reconnaissance missions. The task of the operator is to classify the target as friends or enemies in real time, as it appears in the video dirimkan by several spacecraft to fly. Papers published in these last years, most still revolves around the issue of control of multiple spacecraft to fly alone, if it is associated with the mission or task of the probe, but generally not to expose the problem of fuel the vehicle flying. As for the paper, written by the same author team with this paper, which was published in 2014, expose the control of multi flying vehicles for saving fuel, but the vehicles are assumed to move without changing the height or move without rising or falling (Tjahjana and Sasongko, 2014). Furthermore, the main purpose of this paper is to show that with the formation move in three dimensions, the required fuel the vehicle becomes more efficient.

2. The model

The model of a single flying vehicle which done in this paper is Airplane model follows the Dubins Airplane contained in the paper authored by Randal D. Beard and Timothy W. McLain (Beard and McLain, 2014). Explicitly, the single model of the flying vehicle is described as follwos

$$\begin{aligned} \dot{x} &= \cos\psi \\ \dot{y} &= \sin\psi \\ \dot{z} &= u_1 \\ \dot{\psi} &= u_2. \end{aligned} \tag{1.a}$$

Mathematically, the dynamics of the motion of a single flying vehicle can be modelled in general form as a model of non-linear dynamic system as follows

$$\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x}(t),\mathbf{u}(t)). \tag{1.b}$$

In equation (1.b), the state vector contains the **x** variable circumstances that determine the dynamics of motion of the vehicle to fly. In General, here is the state vector that describes the coordinate position and orientation of the single flying vehicles Vector fields **f** in equation (1.b) is something that explains the link between the state of the vector **x** and input or control **u**. Mathematically, equation (1.b) can be viewed as a system of non linear differential equations order 1. Equation (1.b) is a general model of the dynamics of a single flying vehicle. In this paper, the special form of equation (1.b) is presented in equation (1.a), so the state $\mathbf{x} = [x, y, z, \psi]^T$ and the contol vector $\mathbf{u} = [u_1, u_2]^{\boldsymbol{T}}$. Consider that the state variables and the control vectors in this paper are diferent from the previous paper authored by the same author which published in Tjahjana and Sasongko (2014). This paper is continuation of the previous paper, the number of state varibles are four and the number of control variables are two.

If the equation (1.b) is presented as a model of multiple vehicle, then from equation (1.b) can be described general multi flying vehicle model as

$$\begin{aligned}
\dot{\mathbf{x}}_1(t)&=\mathbf{f}_1(\mathbf{x}_1(t),\mathbf{u}_1(t))\\
\dot{\mathbf{x}}_2(t)&=\mathbf{f}_2(\mathbf{x}_2(t),\mathbf{u}_2(t))\\
&\vdots\\
\dot{\mathbf{x}}_k(t)&=\mathbf{f}_k(\mathbf{x}_k(t),\mathbf{u}_k(t)).
\end{aligned} \tag{2}$$

In the system of equations which presented in (2), the dynamics of the first flying vehicle modeled by the first line of the equation system (2), the dynamics of the second flying vehicle modeled by the second line of equation system (2), and so on until the *k*-th flying vehicle modeled by line *k*-th of equation system (2).

Next, the common functional model discussed fare that makes the vehicle can move along each other away and not move together but donot bump into each other. This cost includes a functional model which represents the fuel from the spacecraft that fly together, which became the subject of major concern in this research. If we denote that $\mathbf{y}_i = [x_i, \quad y_i, \quad z_i]^T$ then the functional model of a fare that is meant, in general can be presented as follows

$$J = \int_0^T \{g(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k) + h(\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k)\}dt. \tag{3}$$

Consider the cost functional (3), functional J is contain two terms g and h. The terms g load of repulsion and attraction, which makes the vehicle do not fly conflicting mutually and not too far away from each other. The term h contains the tribe stating the total cost of the vehicle control or may be meant as a special energy is directly proportional to fuel the vehicle. The cost functional (3) is the most common functional expenses. The more specific term g which contains the repulsion and attraction can be presented as

$$g(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k) = \sum_{i=1, i\neq j}^{k} \frac{\gamma}{\left\|\mathbf{y}_i - \mathbf{y}_j\right\|^2} + \sum_{i=1,\ \ i\neq j}^{k} \mu\left(\left\|\mathbf{y}_i - \mathbf{y}_j\right\| - q\right)^2. \tag{4}$$

In the equation (4), the first term is the repulsion, with γ is a repulsion constant, this term will make the vehicles do not collide with each other. Next, the second term in equation (4) is the attraction term, with μ is an attraction constant, with the existence of the tribe was not flying probe makes its move away from each other. Thus, in more detail the cost functional is generally presented in (3) can be expressed in the following equation

$$J = -\frac{1}{2}\int_0^T \sum_{i=1, i\neq j}^{k} \frac{\gamma}{\left\|\mathbf{y}_i - \mathbf{y}_j\right\|^2} + \sum_{i=1,\ \ i\neq j}^{k} \mu\left(\left\|\mathbf{y}_i - \mathbf{y}_j\right\| - q\right)^2 + \sum_{i=1}^{k} \theta\|\mathbf{u}_i(t)\|^2\, dt\,. \tag{5}$$

In equation (5), actually still be common also, because of the equation (5) set for k vehicles. In this paper, the equation (5) applied for 3 vehicles, so the model of a fare that used in this paper is described in the following equation

$$J = -\frac{1}{2}\int_0^T \sum_{i=1, i\neq j}^{3} \frac{\gamma}{\left\|\mathbf{y}_i - \mathbf{y}_j\right\|^2} + \sum_{i=1,\ \ i\neq j}^{3} \mu\left(\left\|\mathbf{y}_i - \mathbf{y}_j\right\| - q\right)^2 + \sum_{i=1}^{3} \theta\|\mathbf{u}_i(t)\|^2\, dt\,. \tag{6}$$

Consider the equations (3),(4),(5) and equation number (6), the distance between two vehicles in these equations is distance in space and the input vector is input vector in two dimentions. So, the materials which contained are different from the similar equations in the previous paper which authored by the same authors (Tjahjana and Sasongko, 2014), because in the previous paper the distance between two vehicles just in the plane or in two dimentions.

3. Scenario of Simulation

Each vehicle flying moves from the position and orientation of a particular requirement are presented as the beginning. After all this time, given as T, the spacecraft was flying had to stop at a particular position and orientation. As long as the spacecraft moves away they should not and must not collide with each other. This simulation scenario can be seen as part of the trip, each vehicle has the origin and destination of its own. For the purposes of aerodynamic effects in the form of getting the fuel savings, the spacecraft towards a particular location, then from that location to the specific location of the spacecraft do the task together is to fly to save fuel. Next, the vehicles fly to the purpose of each. In this paper, the simulation is shown that along of the moving vehicles fly together to make fuel savings.

4. Simulation Result and Discussion

The simulation result is obtained with initial condition
$\mathbf{x}$ (0) = [x_1(0), x_2(0), x_3(0), y_1(0), y_2(0), y_3(0), z_1(0), z_2(0), z_3(0), ψ_1(0), ψ_2(0) ψ_3(0)] = [7, 3, 9, 7, 5, 3, 0, 0, 0, pi/4, pi/4, pi/4]
and the boundary condition

$$\mathbf{x}\ (T) = [x_1(T),\ x_2(T),\ x_3(T),\ y_1(T),\ y_2(T),\ y_3(T),\ z_1(T),\ z_2(T),\ z_3(T),\ \psi_1(T),\ \psi_2(T)\ \ \psi_3(T)] = [11.5, 10, 9.5, 11.5, 14.5, 12.5, 11, 10, 10, 10, 2, 0.5, 2],$$

with is T =10 is given in Figure 1. In this paper, the unit of the position (absis x-ordinat y- applicat z) are km and the unit of orientation (ψ) is radian. The initial condition $\mathbf{x}$(0) and the boundary condition $\mathbf{x}$(10) are determined first, the initial condition is the condition when the vehicles start flying together and the boundary condition is the final conditions that the vehicles stop flying together. In natural, two conditions are determined first. The result of simulation are carried out with the of software Matlab, after structured process in control design through optimal control theory. In applying the theory of optimal control, from the multi flying vehicle model (2) and the cost functional (6) can be formed to the function of Hamiltonian. Mathematically, from the function of Hamiltonian, can be obtained the system of Hamiltonian. The system of Hamiltonian is a non linear system of differential equations that are over determined requirements. From the mathematical side, settlement of non linear system of differential equations that are over determined is still an open field that can be explored extensively. Researchers, proposed method used to solve this problem. The result is a method of solving systems of differential equations that this over determined, is planned to be published in another publication. From this over determined system of settlement, will lead to a settlement in order to get the optimum trajectory of the spacecraft. Mathematically, the Hamiltonian system solutions which is the solution of the over determined system, physically can be viewed as the optimum trajectory of the spacecraft fly.
Consider Figure 1, from this figure the optimal trajectory which satisfy the given initial and boundary conditions can be viewed. Along the motion, the vehicles do not collide one and the other, but the vehicles also do not move away from each other. So, the first objective of the simulation which described in functional (4) is satisfied.

Next, the calculations the fuel spent by each vehicle is done. The fuel calculation uses the cost functional (6). The result will be compared to the fuel used by the spacecraft flying in tandem, with the spacecraft that fly singly (solo flying). The results obtained are for origin and destination are the same, the fuel that is used when flying together more frugal than when the spacecraft flew singly. The simulation results give the information for the fuel that for the same origin and destination, each vehicle takes 1.2050e003 unit of volume, whereas when the vehicles fly together, each vehicle enough fuel pulled out of 9.6400e002 unit volume. So, the second objective of the simulation is shown, the fuel is more efficient if the vehicles fly together in safety formation.

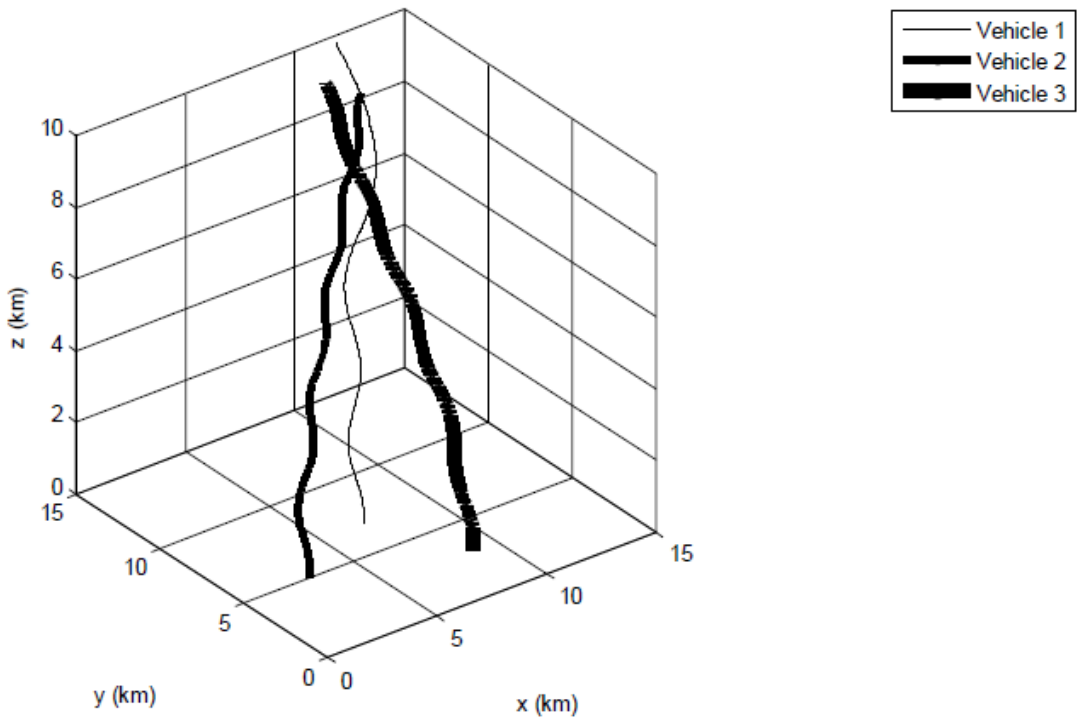

Figure 1. Optimal trajectory of simulation result

5. Conclusion

Based on the results of the simulations which provided in the previous section can be inferred that the research has been successfully carried out. Having regard to the research objectives has been granted, the method of research which used in this paper, the results of previous research, as well as a description of the method of design control, It was concluded that the three flying vehicles control simulation has been run. The results obtained from the mathematical sciences that alternative methods of solving a system of non-linear differential equations are overtermined. Determination of optimum trajectories of the spacecraft to fly to fly from the region of origin to region of destination it can be determined. To reproduce the iteration, will a better path is obtained. The mastery of the research, especially the comparison of fuel use between the spacecraft that fly singly with the spacecraft that fly together get the conclusion that the fuel more efficient, if the vehicles flying together.

Acknowledgements

This paper based on the research was funded by The Government of the Republic of Indonesia, through Fundamental Research Grant, under contract number 183-24/UN7.5.1/PG/2014.

References

[1]A. Ortiz, K. Derek and L. Cédric, Multi-UAV velocity and trajectory Scheduling strategies for target classification by a Single Human Operator, *Journal of Intelligent & Robotic Systems* , 70, (2013), 255-274.

[2]D. Perez, M. Ivan , C. Fernando, , S. David, C. Enrique and O. Anibal, A ground control station for a Multi-UAV surveillance system, *Journal of Intelligent & Robotic Systems*, 69, (2013), 119-130.

[3]D.Y. Won, A.H. Göktoğan, S. Sukkarieh and M.J. Tahk, View planning of a multi-rotor unmanned air vehicle for tree modeling using silhouette-based shape estimator, *Frontiers of Intelligent Autonomous Systems Studies in Computational Intelligence*, 46, (2013), 193-207.

[4]H. Duan, Q. Luo and Y. Yu, Trophallaxis network control approach to formation flight of multiple unmanned aerial vehicles, *Science China Technological Sciences*, 56(5), (2013a), 1066-1074.

[5]H. Duan, Q. Luo, Y. Shi and G. Ma, Hybrid particle swarm pptimization and genetic algorithm for Multi-UAV formation reconfiguration, *IEEE Computational Intelligence Magazine*, 8, (2013b), 16-27.

[6]J. Fink, N. Michael, S. Kim, and V. Kumar, Planning and control for cooperative manipulation and transportation with aerial robots, *The International Journal of Robotics Research*, 30 (3), (2011), 324-334.

[7]M. Turpin, N. Michael, and V. Kumar, Trajectory design and control for aggressive formation flight with quadrotors, *Autonomous Robots*, 33, (2012), 143-156.

[8]N. Michael, E. Stump, and K. Mohta, Persistent surveillance with a team of MAVs, *Proceeding of International Conference on Intelligent Robots and System IROS 2011 San Francisco CA USA*, (2011), 2708-2714.

[9]Q. Lindsey, D. Mellinger and V. Kumar, Construction with quadrotor teams, *Autonomous Robots* , 33, Issue 3, (2012), 323-336.

[10] R.D. Beard and T.W. McLain, , Implementing Dubins airplane fixed wing UAVs on paths, in *Handbook of Unmanned Aerial Vehicles*, Kimon P. Valavanis and George Vachtsevanos (editor), Springer Verllag, (2014).

[11] R.H. Tjahjana and P.S. Sasongko, Multi flying vehicle control for saving fuel, *Applied Mathematical Sciences*, 8, (2014) 733–742.

[12] V.K. Kaliappan, H. Yong, D. Min and A. Budiyono, Behavior-based decentralized approach for cooperative control of a multiple small scale unmanned helicopter, *Proceeding of International Conference on Intelligent Systems Design and Applications, Córdoba, Spain*, (2011), 196-201.

[13] Y. Yuan, J.G. Yan, and Y.H. Qu, The aerodynamic coupling simulation in formation flight of multi-UAV, *Flight Dynamics*, 31, (2013), 29-32.

Received: July 19, 2014