Comments on “Modeling Dynamical Interactions between Leptospirosis Infected Vector and Human Population”

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Abstract

It is pointed out here that some stability conditions presented by Zaman et al in reference [2] are incorrect, as they depend upon the erroneously obtained eigenvalues of some matrices. The correct eigenvalues are also given here.

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In reference [2] Zaman et al first combined two non-linear models of human and vector population. Then they discussed: (i) the local and the global asymptotic stability of disease-free equilibria, (ii) the endemic equilibrium and background bi-function for several parameters, and (iii) the numerical stimulations of real leptospirosis epidemic. The stability conditions presented as Theorems 3.1 and 3.2 are incorrect as their proves depend on the eigenvalues of the matrices \( J_1 \) and \( J_2 \) of Theorem 3.1 and \( J_0 \) of Theorem 3.2. But the eigenvalues for these matrices given and used in [2] are not correct, as these values do not satisfy the well known properties of the eigenvalues [1] that if \( \lambda_1, \lambda_2, ..., \lambda_n \) are \( n \) eigenvalues of a \( n \times n \) matrix \( A \), then trace of the matrix is

\[
tr (A) = \lambda_1 + \lambda_2 + ... + \lambda_n, \quad (1)
\]
and determinant of the matrix is

\[ \text{det}(A) = \lambda_1 \lambda_2 \ldots \lambda_n. \]  

(2)

Now, to justify our claim that the given eigenvalues are incorrect we consider each matrix separately.

**The matrix \( J_1 \) of Theorem 3.1 is**

\[
J_1 = \begin{bmatrix}
-\mu_h & -\beta_1 S_h^0 & \lambda_h & 0 & -\beta_2 S_h^0 \\
0 & -Q_2 + \beta_1 S_h^0 & 0 & 0 & \beta_2 S_h^0 \\
0 & \gamma_h & -Q_3 & 0 & 0 \\
0 & -\beta_3 S_\nu^0 & 0 & -\gamma_\nu & 0 \\
0 & \beta_3 S_\nu^0 & 0 & 0 & -Q_1
\end{bmatrix}.
\]

(3)

The eigenvalues given in the reference [2] for \( J_1 \) are

\[
\lambda_1 = -\mu_h, \quad \lambda_2 = -\gamma_\nu, \quad \lambda_3 = -S_\nu^0 Q_3 \beta_3, \\
\lambda_4 = -Q_2 + \beta_1 S_h^0, \quad \lambda_5 = -Q_1 (-Q_2 + \beta_1 S_h^0) - S_h^0 S_\nu^0 \beta_2 \beta_3.
\]

(4)

For these values, we have

\[
\begin{align*}
\lambda_1 + \lambda_2 + \ldots + \lambda_5 & = -\mu_h - \gamma_\nu - S_\nu^0 Q_3 \beta_3 - Q_2 + \beta_1 S_h^0 - Q_1 (-Q_2 + \beta_1 S_h^0) \\
& - S_h^0 S_\nu^0 \beta_2 \beta_3, \\
\lambda_1 \lambda_2 \ldots \lambda_5 & = -S_h^0 S_\nu^0 Q_3 \beta_3 (-Q_2 + \beta_1 S_h^0) [-Q_1 (-Q_2 + \beta_1 S_h^0)] \\
& - S_h^0 S_\nu^0 \beta_2 \beta_3.
\end{align*}
\]

(6)

(7)

Now, one can easily obtain the trace and the determinant of the above matrix given by eq.(3) as

\[
\begin{align*}
\text{tr}(J_1) & = S_h^0 \beta_1 - Q_2 - Q_3 - \mu_h - Q_1 - \gamma_\nu, \\
\text{det}(J_1) & = S_h^0 Q_1 Q_3 \beta_1 \mu_h \gamma_\nu - Q_1 Q_2 Q_3 \mu_h \gamma_\nu + S_h^0 S_\nu^0 Q_3 \beta_2 \beta_3 \mu_h \gamma_\nu.
\end{align*}
\]

(8)

(9)

It can be seen by comparing eq.(6) and eq.(8) that \( \text{tr}(J_1) \neq \lambda_1 + \lambda_2 + \ldots + \lambda_5 \) and by comparing eq.(7) and eq.(9) it can also be seen that for these values \( \text{det}(J_1) \neq \lambda_1 \lambda_2 \ldots \lambda_5 \). Therefore, values given in eqs.(4) and (5) are not the eigenvalues for the matrix \( J_1 \).

**The matrix \( J_0 \) of Theorem 3.1:** The eigenvalues given in reference [1] for the matrix

\[
J_0 = \begin{bmatrix}
-\mu_h & 0 & \lambda_h & 0 & 0 \\
0 & -Q_2 & 0 & 0 & 0 \\
0 & \gamma_h & -Q_3 & 0 & 0 \\
0 & 0 & 0 & -\gamma_\nu & 0 \\
0 & 0 & 0 & 0 & -Q_1
\end{bmatrix}
\]

(10)
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are

\[ \lambda_1 = -\mu_h, \quad \lambda_2 = -Q_2, \quad \lambda_3 = -Q_2Q_3, \quad \lambda_4 = -\gamma_v, \quad \lambda_5 = -Q_1. \]  \hspace{1cm} (11)

These values give

\[ \lambda_1 + \lambda_2 + \ldots + \lambda_5 = -\mu_h - Q_2 - Q_2Q_3 - \gamma_v - Q_1, \]  \hspace{1cm} (12)

\[ \lambda_1\lambda_2\ldots\lambda_5 = -\mu_h\gamma_v Q_1 Q_2^2 Q_3. \]  \hspace{1cm} (13)

For \( J_0 \), we get

\[ \text{tr} (J_0) = -\mu_h - Q_2 - Q_3 - Q_1 - \gamma_v, \]  \hspace{1cm} (14)

\[ \text{det}(J_0) = -Q_1 Q_2 Q_3 \mu_h \gamma_v. \]  \hspace{1cm} (15)

A comparison of eqs. (12) and (14) and eqs. (13) and (15) shows that the values given in eq. (11) do not satisfy properties of eigenvalues given in eq. (1) and eq. (2).

**The matrix \( J_1 \) of Theorem 3.2:** Consider the matrix \( J_1 \) as given in reference [2]

\[ J_1 = \begin{bmatrix} -\mu_h & -\beta_1 & -\lambda_h & 0 & -\beta_2 \\ 0 & -Q_2 + \beta_1 & 0 & 0 & \beta_2 \\ 0 & \gamma_h & -Q_3 & 0 & 0 \\ 0 & 0 & 0 & -\gamma_v & 0 \\ 0 & 0 & 0 & 0 & -Q_1 \end{bmatrix}. \]  \hspace{1cm} (16)

For this matrix the given eigenvalue are

\[ \lambda_1 = -\mu_h, \quad \lambda_2 = -Q_2 + \beta_1, \quad \lambda_3 = Q_2Q_3 - \beta_1Q_3, \quad \lambda_4 = -\gamma_v, \quad \lambda_5 = -Q_1. \]  \hspace{1cm} (17)

These values yield

\[ \lambda_1 + \lambda_2 + \ldots + \lambda_5 = -\mu_h + (-Q_2 + \beta_1) + Q_2Q_3 - \beta_1Q_3 - \gamma_v - Q_1, \]  \hspace{1cm} (18)

\[ \lambda_1\lambda_2\ldots\lambda_5 = \mu_h\gamma_v Q_1 (Q_2 - \beta_1) (Q_2 - \beta_1) Q_3. \]  \hspace{1cm} (19)

But the values of trace and the determinant are

\[ \text{tr} (J_1) = \beta_1 - Q_2 - Q_3 - Q_1 - \mu_h - \gamma_v, \]  \hspace{1cm} (20)

\[ \text{det}(J_1) = Q_1 Q_3 \beta_1 \mu_h \gamma_v - Q_1 Q_2 Q_3 \mu_h \gamma_v. \]  \hspace{1cm} (21)

Therefore, for this matrix also properties given in eqs. (1) and (2) are not satisfied.
Correct Values: We find the correct eigenvalues for matrix $J_1$ of Theorem 3.1 as

$$
\lambda_1 = \frac{1}{2} S_h^o \beta_1 - \frac{1}{2} Q_2 - \frac{1}{2} Q_1 - \frac{1}{2} F_1,
$$

where

$$
F_1 = \sqrt{2 S_h^o Q_1 \beta_1 - 2 Q_1 Q_2 - 2 S_h^o Q_2 \beta_1 + 4 S_h^o S_h^o S_h^o \beta_2 \beta_3 + Q_1^2 + Q_2^2 + S_h^2 \beta_1^2},
$$

$$
\lambda_2 = \frac{1}{2} S_h^o \beta_1 - \frac{1}{2} Q_2 - \frac{1}{2} Q_1 + \frac{1}{2} F_2,
$$

where

$$
F_2 = \sqrt{2 S_h^o Q_1 \beta_1 - 2 Q_1 Q_2 - 2 Q_2 \beta_1 + 4 S_h^o S_h^o \beta_2 \beta_3 + Q_1^2 + Q_2^2 + S_h^2 \beta_1^2},
$$

$$
\lambda_3 = -\gamma_v, \quad \lambda_4 = -Q_3, \quad \lambda_5 = -\mu_h,
$$

and for matrix $J_0$ of Theorem 3.1 as

$$
\lambda_1 = -\gamma_v, \quad \lambda_2 = -\mu_h, \quad \lambda_3 = -Q_3, \quad \lambda_4 = -Q_1, \quad \lambda_5 = -Q_2.
$$

Similarly, for the matrix $J_1$ of Theorem 3.2, the correct eigenvalues are

$$
\lambda_1 = \beta_1 - Q_2, \quad \lambda_2 = -\gamma_v, \quad \lambda_3 = -\mu_h, \quad \lambda_4 = -Q_3, \quad \lambda_5 = -Q_1.
$$

These values satisfy properties of eigenvalues given in eqs.(1) and (2).

Conclusion

In this paper it has been pointed out that the eigenvalues given in Theorems 3.1 and 3.2 of [2] are incorrect. The correct eigenvalues are also provided. Theorems 3.1 and 3.2 can now be proved and analyzed for these values.

References


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