Assignation Problems’ Solutions by Parallel and Distributed Genetic Algorithms

Roberto Poveda Chaves¹
Facultad de Ingeniería
Universidad Distrital “Francisco José de Caldas”, Bogotá Colombia

Orlando García Hurtado
Facultad de Ingeniería
Universidad Distrital “Francisco José de Caldas”, Bogotá Colombia

Eduardo Cárdenas Gómez
Departamento de Matemáticas.
Universidad Nacional de Colombia, Bogotá Colombia

Copyright © 2014 Roberto Poveda Chaves et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract

The Grisland model is a Parallel Genetic Algorithm designed to find optimal solutions close to the Travelling Salesman Problem. This algorithm combines the classical fine-grained parallel models (grid models) and the coarse-models (island models) with the 2-opt local search heuristics. This article implements on Grisland some assignment problems, which result in particular cases of the Symmetric Euclidian Problem of the Travelling Salesman.

Keywords: Parallel Genetic Algorithm, Travelling Salesman Problem, Assignment Problem

¹Corresponding author
1 Introduction

This paper deals with assignation problems’ solutions with parallel genetic algorithms; assignation problems have the purpose to minimize the total cost to assign \( m \) agents to \( n \) tasks in the most effective way, costs are represented by \( c_{ij} \) and interpret the value to assign the agent \( i, 1 \leq i \leq m \) to the task \( j, 1 \leq j \leq n \). [14].

Assignation problems have been solved in different ways: the usual and most outstanding solving techniques are Dynamic Programming and the Munkres Assignation Algorithm, better known as the Hungarian method, [13].

The solution posited in this paper is made using Parallel and Distributed Genetic Algorithms; this technique has been used by the authors with excellent results to solve routes optimization problems, specially to solve one of the most important combinatorial problems in computer science, the problem of Traveling salesman [2]; the assignation problem is resolved as a particular case of the Traveling salesman problem.

Genetic algorithms is one of the most outstanding approaches in the field of Evolutionary Algorithms and are defined as iterative procedures of general purpose adaptive search with the virtue of describing in an abstract and rigorous way the collective adaptation of the individuals of a population to a particular environment, based on a behavior similar to a natural system [3].

John Holland and a group of students at the university of Michigan invented Genetic Algorithms in 1975, their inspiration were the processes that occur in biologic evolution and the facility to implement those adaptive processes computationally.

In many cases Genetic Algorithms result more appropriate and efficient to solve specific problems of optimization when compared to the traditional Calculus procedures (maximum descent direction, enumerative methods, etc.) since they do not require a specific behavior of the objective function [7].

The implementation of Genetic Algorithms through a parallel approach and in a distributed system provides better possibilities to find better solutions to the problem, given the especially distributed nature of the problems search domain; another obvious reason is the speed to find solutions [10].

This document uses Parallel Genetic Algorithms (PGA) considering the Grisland model implemented by the authors to find solutions to different assignation problems. The Grisland model is a combination of the Islas and Grillas parallel genetic algorithms model and the Lin-Kernighan 2-opt heuristic optimization model [2, [6], [10].

Assignation problems are a particular case of the Traveling salesman problem that has been seen as one of the most important problems of the NP-Complete type.

The Traveling salesman problem consist in finding a Hamiltonian cycle of minimal longitude in a complete graph directed and taged in \( n \) ways. Traditional optimization methods, heuristic procedures and more recently evolutive computing techniques in different flavors has been used for the solution of this problem, [5].


2 Preliminaries

The Grisland Parallel Genetic Algorithm that solves these assignation problems is based in the combination of the Islas and Grillas Genetic models.

The Islas Parallelized Genetic Algorithm is implemented in the individual level and is known also as the Bast Granulated Parallel Genetic Algorithm.

This model is inspired in the spacially distributed structure of natural populations. Demes are independent individuals semi-groups or sub-populations that has a small connection to neighboring demes. The connection to which the definition refers is a migration or slow diffusion of some individuals from one deme to another [1].

The Islas model characterizes geographically separated sub-populations of a relatively large size. The individuals interchange between sub-populations is made according to some predetermined patterns and with a determined frequency. The intention of this approach is to re-inject diversity in a periodical way to sub-populations that tend to converge to sub-populations on which a dedicated exploration in a determined search space is made. Among each sub-population a standard sequential genetic algorithm is executed between migration phases. Usually the K better individuals of each sub-population migrate and replace the K worst individuals of the neighboring sub-population [10].

The Grillas parallel model or fine granulation model proceeds to placing in each cell of a bidimensional grid each individual of the population. The following graph (Figure 1) shows a neighborhood (in black) for a particular individual (in circle).

![Figure 1: Distribution of individuals in the fine granulation model.](image)

The evaluation of the aptitude for each individual is made simultaneously and the selection and crossing take place locally within each neighborhood according to a specific topology. The most used topologies are the following:

1. $4 - n$: Horizontal and vertical neighbors.
2. $5 - n$: Horizontal and vertical neighbors plus the center.
3. $8 - n$: All neighbors of distance 1.
4. $9 - n$: All neighbors of distance 1 plus the center.

5. $16 - n$: In all 8 directions, neighbors with distance 2.

6. $17 - n$: In all 8 directions, neighbors with distance 2 plus the center.

7. $(m)16n$: All neighbors of distance 1, plus vertical and horizontal neighbors with distance 2.

Figure 2: Topologies $4 - n$ and $8 - n$ respectively.

The previous figure considers the 8-n topology (All neighbors of distance 1). The selection of an individual in the vicinity to apply the crossing operator with the central individual is usually made by deterministic or stochastic tournament with 2 individuals. The selection by tournament is appropriate for the Grillas model because it is not necessary to apply it globally on all the population but on sub-populations established spatially.

Then, the crossing between the chosen individual and the central individual is made. The replacement is usually made by changing the central individual by the best of the children or the same individual is preserved if he has an aptitude level superior to the resulting children [10].

Local optimization heuristics 2-opt, is a local search algorithm that is used to solve the Traveling salesman problem, this method betters the trajectory, edge by edge, through the elimination of crossings. This heuristics compares a pair of trajectory edges $(a, b)$ and $(c, d)$ that are revised to connect their 4 nodes in a different way with the intention of obtaining a shorter trajectory:

$$
\overline{ab} + \overline{cd} > \overline{ac} + \overline{bd}
$$

$\overline{ab}$ represents the distance from node $a$ to node $b$, the algorithm makes the comparison of the distance between edges; if $\overline{ac} + \overline{bd}$ is less than the distance $\overline{ac} + \overline{bd}$, it replaces the edges.
Assignation problems’ solutions by PGA

(a, b) and (c, d) by the edges (a, c) and (b, d). The figure 3 to the left shows the original graph and the figure 3 to the right shows the graph after applying the heuristic, that is, the sub-trajectory \((a, b, t, c, d, s, a)\) is optimized with the sub-trajectory \((a, d, c, t, b, s, a)\).

![Figure 3: Heuristic procedure 2-opt.](image)

This procedure is repeated until there is additional betterment \([6], [15]\).

The authors propose to combine the Islas and Grillas parallel models with heuristics 2-opt. They called this model Grisland model. This model has provided important results to the Traveling salesman problem, there are various articles on the model that show the efficiency of the technique, \([2], [9]\).

3 Methodology

An assignation problem of \(m\) agents and \(n\) tasks is a special case of the Traveling salesman problem with a total of nodes (cities) of \(\max\{m, n\}\).

An assignation problem is a particular case of Lineal Optimization whose mathematical model is:

Minimize \(Z = c_{11}x_{11} + c_{12}x_{12} + \cdots + c_{mn}x_{mn}\)

subject to \(x_{11} + x_{12} + \cdots + x_{1n} = 1\)

\(:\)

\(x_{m1} + x_{m2} + \cdots + x_{mn} = 1\)

\(x_{11} + x_{21} + \cdots + x_{m1} \leq 1\)

\(:\)

\(x_{1n} + x_{2n} + \cdots + x_{mn} \leq 1\)

where:

\[x_{ij} = \begin{cases} 
1, & \text{if agent } i \text{ is assigned to task } j \\
0, & \text{otherwise}
\end{cases}\]
The assignation problems can be considered as transportation problems taking agents as sources and tasks as destinies, with all offers and demands equal to one (Figure 4).

In general, assignation problems are resolved considering the number of agents equal the number of tasks (this condition can be guaranteed creating fake agents or tasks, according the needs).

The assignation problem is resolved by Grisland considering the chromosomes as the genes chains of entire value:

$$A_1 \ A_2 \ A_3 \ A_4 \ \cdots \ A_n$$

The $A_i$ gene represents the benefit (cost) to assign the $A_i$ task to the $i$ agent. Of course, $A_i \neq A_j$, if $i \neq j$, $1 \leq A_i \leq n$, $1 \leq i \leq n$.

The aptitude function is $Z = \sum c_{ij} = \sum c_{iA_i}$. The assignation problem $c_{ij}$ costs interpret the value of assigning the agent $i$, $1 \leq i \leq n$ the task $j = A_i$, $1 \leq j \leq n$.

The selection, crossing and mutation are applied in a conventional way, Grisland solves the typical Traveling salesman problems, for example, it solved the TSPLIB ch130 problem, (Figure 5).

![Transportation grid with m offers and n demands.](image)

**Figure 4:** Transportation grid with $m$ offers and $n$ demands.

![ch130 problem.](image)

**Figure 5:** ch130 problem.

Grisland finds the solution (Figure 6):
4 Experimentation and Results

The following assignation problems were solved:

- **Problem 1**
  A 400m relay race includes 4 different swimmers who swim successively 100m backstroke, breaststroke, butterfly stroke and freestyle. A trainer has 6 very fast swimmers whose expected times (in seconds) in the individual events are given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Event 1</th>
<th>Event 2</th>
<th>Event 3</th>
<th>Event 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Backstroke</td>
<td>Breaststroke</td>
<td>Butterfly stroke</td>
<td>Free style</td>
</tr>
<tr>
<td>Swimmer 1</td>
<td>65</td>
<td>73</td>
<td>63</td>
<td>57</td>
</tr>
<tr>
<td>Swimmer 2</td>
<td>67</td>
<td>70</td>
<td>65</td>
<td>58</td>
</tr>
<tr>
<td>Swimmer 3</td>
<td>68</td>
<td>72</td>
<td>69</td>
<td>55</td>
</tr>
<tr>
<td>Swimmer 4</td>
<td>67</td>
<td>75</td>
<td>70</td>
<td>59</td>
</tr>
<tr>
<td>Swimmer 5</td>
<td>71</td>
<td>69</td>
<td>75</td>
<td>57</td>
</tr>
<tr>
<td>Swimmer 6</td>
<td>69</td>
<td>71</td>
<td>66</td>
<td>59</td>
</tr>
</tbody>
</table>

How should the trainer assign the swimmers the relays so as to minimize the sum of the times? The optimal assignation by Grisland is:

Swimmer 1 $\leftarrow - \rightarrow$ Event 1,
Swimmer 2 $\leftarrow - \rightarrow$ Event 3,
Swimmer 3 $\leftarrow - \rightarrow$ Event 4,
Swimmer 5 $\leftarrow - \rightarrow$ Event 2.

- **Problem 2**
  District University of Colombia hires 4 professors that should be assigned to teach four sections of a doctoral program. The following table shows the estimated times
<table>
<thead>
<tr>
<th>Professor</th>
<th>Module A</th>
<th>Module B</th>
<th>Module C</th>
<th>Module D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Backstroke</td>
<td>Breaststroke</td>
<td>Butterfly stroke</td>
<td>Free style</td>
</tr>
<tr>
<td>Professor 1</td>
<td>6</td>
<td>24</td>
<td>36</td>
<td>18</td>
</tr>
<tr>
<td>Professor 2</td>
<td>54</td>
<td>42</td>
<td>60</td>
<td>54</td>
</tr>
<tr>
<td>Professor 3</td>
<td>24</td>
<td>30</td>
<td>66</td>
<td>42</td>
</tr>
<tr>
<td>Professor 4</td>
<td>48</td>
<td>42</td>
<td>48</td>
<td>30</td>
</tr>
</tbody>
</table>

(in days) for each professor in each module according to their experience and expertise.

The faculty administration wants to know what the optimal assignation is so that the professors finish all modules in the least time.

Professor 1 $\prec$ Module 1
Professor 2 $\prec$ Module 3
Professor 3 $\prec$ Module 2
Professor 4 $\prec$ Module 4

The results of the assignation problems, as well as the problems solved by Grisland in their earlier versions were executed on a distributed system of 5 processors under the scheme of passing messages using the software PVM (Parallel Virtual Machine), [16].

The Grisland model code was written in C-Language and was run on the Linux Fedora Release 12 (Constantine) Operative System and the tests were made on Sony Vaio Processors Intel Core 2 Duo 2.26 Ghz, 4 Gb in Memory.

5 Conclusions

Grisland solves in a efficient way the previous assignation problems, each one solved in less than seven iterations on average (7 executions by problem were made, each one with a different topology and in all of them the same solution as the one obtained by Dynamic Programming or the Munkres method was obtained).

Thus, it is perceived that these problems are very easy to solve using the Grisland technique.

It is convenient to solve bigger problems and see the efficiency of the method and, if necessary, to use more processors, change the values of certain genetic operators or implement more sophisticated heuristics like $K$–opt with $K > 2$.

Grisland, with appropriate variants, is seen as a potentially useful technique in the solution of other problems that in principle can be solved by traditional optimization techniques or by any evolutionary technique.
Assignation problems’ solutions by PGA

References


Received: June 8, 2014