Interval Estimation for a Linear Function of

Variances of Nonnormal Distributions

that Utilize the Kurtosis

Sirima Suwan
Department of Applied Statistics, Faculty of Applied Science
King Mongkut’s University of Technology
North Bangkok, Thailand, 10520
sirimasuwan@hotmail.com

Sa-aat Niwitpong
Department of Applied Statistics, Faculty of Applied Science
King Mongkut’s University of Technology
North Bangkok, Thailand, 10520
snw@kmutnb.ac.th

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Abstract

Confidence interval for a linear combination of variances from \( k \geq 2 \) independent non-normal populations that utilize kurtosis via the method of variance estimates recovery (the MOVER) and its application to a general linear function of parameters presented by Zou et al.[6] is proposed. Our method will compare to the existing confidence interval via Monte Carlo simulation. The coverage probability and the average interval width are used to assess the confidence intervals.

Mathematics Subject Classification: 62F25

Keywords: Minimum mean–squared error, The MOVER, Kurtosis, Interval estimation, Linear combination
1 Introduction

Let $X_{i1}, \ldots, X_{in}, X_{21}, \ldots, X_{2n}, \ldots, X_{kn},$ be $m$ continuous independent samples, each sample being identical independent with any distribution function $G_i(x)$, mean $\mu_i$, variance $\sigma^2_i$, and finite fourth moments $\gamma_4_i$ for $i = 1, 2, \ldots, k$. The sample means and variances are $\bar{X}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} X_{ij}, i = 1, 2, \ldots, k$ and $S^2_i = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2$, respectively. In the sections that follow, we present a new hybrid method for making inference about a confidence interval for linear function of population variances $\sum_{i=1}^{k} c_i \sigma^2_i$, where $k \geq 2$ and the $c_i$ are coefficient in the linear function.

2 The proposed interval estimation procedures

2.1 Asymptotic variance estimators (General approach)

2.1.1 An unbiased population variance estimator

It is well known that the usual unbiased estimate of variance is $S^2_i$, $i = 1, 2, \ldots, k$ and its variance, available in statistical literature, are given by $\text{Var}(S^2_i) = \frac{1}{n_i} \left( \gamma_4_i - (n_i - 3)/(n_i - 1) \right) \sigma^4_i / n_i$, where $\gamma_4_i = \frac{\mu_4}{\sigma^4}$ and $\mu_4$ is the population fourth central moment. For samples sufficiently large provided the population fourth moment is finite, the sample variance is asymptotically normally distributed with mean $E(S^2_i)$ and variance $V(S^2_i)$. A simple large-sample procedure for constructing a 100 $(1 - \alpha)$ % confidence interval for variance can be obtained as

$$\frac{1}{1 + z_{\alpha/2} \sqrt{\gamma_4_i n_i - (n_i - 3)/(n_i - 1)}} \leq \sigma^2_i \leq \frac{1 - z_{\alpha/2} \sqrt{\gamma_4_i n_i - (n_i - 3)/(n_i - 1)}}{n_i},$$

where $\hat{\gamma}_4_i = \frac{1}{n_i} \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^4 / n_i S^4_i$ and $z_{\alpha/2}$ be a critical $z$-value. Another approach is to make use of the MBBE of variance in the similar pattern of (1).

2.1.2 The MBBE of variance

An improved estimator of the variance that utilizes the kurtosis was initially derived by Seals and Intarapanich [2] and later generalized by Wencheko and Chipoyera [5]. The estimator has the form $S_w^2 = w(n-1)S^2$ where the weight, $w = \left[ (n+1) + \gamma_4 - 3 \right] n(n-1)^{-1}$ is an optimal value that minimizes the MSE $(S_w^2)$ and $\gamma_4$ is the kurtosis. Wencheko et al., [5] defined this estimator of variance as
the “minimum mean-squared error best biased estimator” (MBBE). Since the relative efficiency (RE) of the MBBE is larger than 1, thus, implying that the MBBE is always more efficient than the usual unbiased estimator $S^2$ of variance. This statistic is of interested, in the present paper, we intended to deal with the MBBE of variance by adjusting a kurtosis estimation procedure using trimmed mean (and later let’s call the adjusted MBBE of variance), then making use of the two biased estimators, the adjusted MBBE and the MBBE and the usual unbiased estimator $S^2$ to establish the asymptotic confidence intervals for a linear function of independent variances of nonnormal distributions that utilizes the kurtosis via the method of variance estimates recovery, the MOVER and its simple application to a general linear function of parameters(Zou et al.[6]). The basic idea is construction of confidence intervals for a linear function of variances that involves using the readily available method of Donne and Zou [3] and Zou et al.[6] to combine confidence intervals based on separate samples.

The MBBE of variance is of the form $S_{w}^2 = w_i (n_i - 1) S_i^2 = S_i^2 / \left( \{ (n_i + 1) / (n_i - 1) \} + (\gamma_i - 3) / n_i \right)$ and $E(S_{w}^2) = w(n_i - 1) \sigma_i^2$, $i = 1, 2, ..., k$, where $w_i = 1 / (\{ (n_i + 1) + (\gamma_i - 3)(n_i - 1) / n_i \} )$, $0 < w_i < 1$

$MSE(S_{w}^2) = E(S_{w}^2 - \sigma_i^2)^2 = w_i^2 (n_i - 1)^2 \text{Var}(S_i^2) + \left[ (n_i - 1) w_i - 1 \right] \sigma_i^4$, where $\gamma_i$ is the kurtosis. For large $n_i$, when randomly sampling from any distribution with a finite fourth moment, and By the central limit theorem, The MBBE of variance is approximately standard normal with $E(S_{w}^2)$ and $MSE(S_{w}^2)$. Consequently, an approximate two–sided 100 $(1-\alpha)\%$ confidence interval for the variance may be given as

$L = S_i^2 / [1 + z_{\alpha/2} \sqrt{\left\{ \{ \hat{\gamma}_i - (n_i - 3) / (n_i - 1) \} / n_i \right\} + [1 - 1 / \hat{w}_i (n_i - 1)]^2}$, $U = S_i^2 / [1 - z_{\alpha/2} \sqrt{\left\{ \{ \hat{\gamma}_i - (n_i - 3) / (n_i - 1) \} / n_i \right\} + [1 - 1 / \hat{w}_i (n_i - 1)]^2}$, (2)

where $\hat{\gamma}_i = \sum_{j=1}^{n_i} (X_{ji} - \bar{X}_i)^4 / n_i S_i^4$, $z_{\alpha/2}$ be a critical z-value and $\hat{w}_i = [(n_i + 1) + (\hat{\gamma}_i - 3)(n_i - 1) / n_i]^{-1}$.

2.1.3 The adjusted MBBE of variance

Since an estimate of $MSE(S_{w}^2)$ will require an estimate of kurtosis, and it is well known that a usual kurtosis estimate $\hat{\gamma}_i = \sum_{j=1}^{n_i} (X_{ji} - \bar{X}_i)^4 / n_i S_i^4$, $i = 1,2,..,k$, was badly biased in sampling from nonnormal populations, an alternative adjusted kurtosis estimate then has been used, and is of the form: $\hat{\gamma}_i = \sum_{j=1}^{n_i} (X_{pj} - m_i)^4 / n_i S_i^4$, where $m_i$ is a trimmed mean with trim–proportion equal to $1/2\sqrt{n_i - 4}$. Note that we used the trimmed mean in place of mean as suggested by Bonett [1] because the trimmed mean not only tends to provide a
better kurtosis estimate but also tends to improve the accuracy of the interval estimation for leptokurtic (heavy-tailed) or skewed distributions. This adjusted MBBE estimator of variance (adjusted MBBE) yields the two sided 100(1-\(\alpha\))% confidence interval for variance:

\[
L = S_i^2 / \{1 + z_{1-\alpha/2} \sqrt{\left\{\sum_{i=1}^n \left(\hat{\gamma}_{4i} - (n_i - 3)/(n_i - 1)\right) / n_i\right\} + [1 - \hat{w}_i (n_i - 1)]} \}
\]

\[
U = S_i^2 / \{1 - z_{1-\alpha/2} \sqrt{\left\{\sum_{i=1}^n \left(\hat{\gamma}_{4i} - (n_i - 3)/(n_i - 1)\right) / n_i\right\} + [1 - \hat{w}_i (n_i - 1)]} \}, \tag{3}
\]

where \(\hat{\gamma}_{4i} = \sum_{j=1}^{n_i} (X_{ji} - m_i)^4 / n_i S_i^4\), \(z_{1-\alpha/2}\) be a critical z-value, and \(\hat{w}_i = [(n_i + 1) - (\hat{\gamma}_{4i} - 3)(n_i - 1) / n_i]^{-1}\).

2.2 Approximate Intervals for a linear function of variances

2.2.1 The MOVER and its applications [6]
Suppose we would like to construct two sided 100 (1-\(\alpha\))% confidence interval, denoted by (L, U), for \(\theta_1 + \theta_2\) where \(\theta_1, \theta_2\) denote any two interested parameters and \(\hat{\theta}_i, i = 1, 2\) are estimators of \(\hat{\theta}_i\).

Zou et al.,[6] have extended the argument of Donner and Zou [3] to a linear function of parameters by regarding \(\theta_1 + \theta_2\) and \(\theta_1 - \theta_2\) as \(c_1 \hat{\theta}_1 + c_2 \hat{\theta}_2\), where \(c_1\) and \(c_2\) are constants, hence, the interval can be written as

\[
L = c_1 \hat{\theta}_1 + c_2 \hat{\theta}_2 - \sqrt{\left[c_1 \hat{\theta}_1 - \min(c_1 l_1, c_1 u_1)\right]^2 + \left[c_2 \hat{\theta}_2 - \min(c_2 l_2, c_2 u_2)\right]^2}
\]

and

\[
U = c_1 \hat{\theta}_1 + c_2 \hat{\theta}_2 + \sqrt{\left[c_1 \hat{\theta}_1 - \max(c_1 l_1, c_1 u_1)\right]^2 + \left[c_2 \hat{\theta}_2 - \max(c_2 l_2, c_2 u_2)\right]^2} \tag{4}
\]

Their further extension is to use a mathematical induction application in order to derived a generally 100(1-\(\alpha\))% confidence interval for linear functions of parameters \(\sum_{i=1}^k c_i \theta_i, i = 1, 2, \ldots, k\), where \(k \geq 2\), \(\theta_i\) denote any interested parameters and \(c_i\) are coefficient in the linear function, as defined

\[
L = \sum_{i=1}^k c_i \hat{\theta}_i - \sqrt{\sum_{i=1}^k \left[c_i \hat{\theta}_i - \min(c_i l_i, c_i u_i)\right]^2}
\]

\[
U = \sum_{i=1}^k c_i \hat{\theta}_i + \sqrt{\sum_{i=1}^k \left[c_i \hat{\theta}_i - \max(c_i l_i, c_i u_i)\right]^2}. \tag{5}
\]

2.2.2 The intervals estimation for a linear function of variances
Let’s defined a linear function of variances as \(\sum_{i=1}^k c_i \sigma_i^2, k \geq 2\) where \(c_i\) are known constants. Since there are at least three intervals for a single variance as in
Interval estimation for a linear function of variances

To obtain a confidence interval for linear functions of variances via equation (5) we should have k separate confidence limits for the asymptotic variance estimates $\sigma_i^2$, $i = 1, 2, \ldots, k$ (i.e., $(l_1, u_1), \ldots, (l_k, u_k)$), thus the three distinct hybrid confidence intervals for a linear function of variances $\sum_{i=1}^{k} c_i \sigma_i^2$ that arise from each equation (1), (2) and (3) are respectively, as follows,

**i. Namely U1:**

$L = \sum_{i=1}^{k} c_i \sigma_i^2 - \sqrt{\sum_{i=1}^{k} [c_i \sigma_i^2 - \min(c_i, l_i, u_i)]^2}$,  
$U = \sum_{i=1}^{k} c_i \sigma_i^2 + \sqrt{\sum_{i=1}^{k} [c_i \sigma_i^2 - \max(c_i, l_i, u_i)]^2}$,

where $(l_i, u_i), i = 1, 2, \ldots, k$ denote an available (1-$\alpha$)100% confidence intervals for $\sigma_i^2$, $i = 1, 2, \ldots k$ given by equation (1).

**ii. Namely M1:**

$L = \sum_{i=1}^{k} c_i \sigma_i^2 - \sqrt{\sum_{i=1}^{k} [c_i \sigma_i^2 - \min(c_i, l_i, u_i)]^2}$,  
$U = \sum_{i=1}^{k} c_i \sigma_i^2 + \sqrt{\sum_{i=1}^{k} [c_i \sigma_i^2 - \max(c_i, l_i, u_i)]^2}$,

where $(l_i, u_i), i = 1, 2, \ldots, k$ denote an available (1-$\alpha$)100% confidence intervals for $\sigma_i^2$, $i = 1, 2, \ldots k$ given by equation (2), where

$\hat{\sigma}_i^2 = \text{MBBE} = S_{wi}^2 = \hat{w}_i (n_i - 1) S_i^2$,

$\hat{w}_i = [(n_i + 1) + (\hat{\gamma}_{4i} - 3)(n_i - 1)/n_i]^{-1}$ and $\hat{\gamma}_{4i} = \sum_{j=1}^{n_i} (X_j - \bar{X}_i)^4 / n_i S_i^4$.

**iii. Namely M2:**

$L = \sum_{i=1}^{k} c_i \sigma_i^2 - \sqrt{\sum_{i=1}^{k} [c_i \sigma_i^2 - \min(c_i, l_i, u_i)]^2}$,  
$U = \sum_{i=1}^{k} c_i \sigma_i^2 + \sqrt{\sum_{i=1}^{k} [c_i \sigma_i^2 - \max(c_i, l_i, u_i)]^2}$,

where $(l_i, u_i), i = 1, 2, \ldots, k$ denote an available (1-$\alpha$)100% confidence intervals for $\sigma_i^2$, $i = 1, 2, \ldots k$ given by equation (3), where

$\hat{\sigma}_i^2 = \text{adjustedMBBE} = \hat{w}_i (n_i - 1) S_i^2$,

$\hat{w}_i = [(n_i + 1) + (\hat{\gamma}_{4i} - 3)(n_i - 1)/n_i]^{-1}$,  
$\hat{\gamma}_{4i} = \sum_{j=1}^{n_i} (X_{ji} - m_i)^4 / n_i S_i^4$ and $m_i$ is a trimmed mean with trim–proportion equal to $\sqrt{1/2} \sqrt{n_i - 4}$. 
3 Simulation Results

3.1 Method

A simulation study was carried out to investigate the performance of the 95% confidence limits for a linear function of variances. The 10,000 sets of variance values were randomly sampled from a variety of distributions. The U1, M1 and M2 were used to compute the coverage probabilities (Cps) and the average interval widths (Aws) for each of 10,000 sets of variance values and for various balanced and unbalanced sample sizes. The simulation programs were written in R and executed on an Intel computer.

3.2 Results

In constructing confidence intervals for linear functions of variances $\sum_{i=1}^{k} c_i \sigma_i^2$, $k \geq 2$, the performance of those with no difference among variances for a variety of non-normal distributions are first investigated in terms of coverage probabilities and the average intervals widths for U1, M1 and M2, when various group sizes are balanced and unbalanced designs and are summarized in the table. Only the results at the nominal five-percent level of significant are presented.

For a linear function of 2 variances, regardless of balanced or unbalanced designs, the results in Table 1 show that as groups of observations are moderate or large, for asymmetric (skewed) distributions, the M2 performs substantially better than both the M1 and U1 in terms of holding the mean coverage closest to the nominal level, with narrowest average width, for symmetric non-normal distributions, M2 and M1 are identically and both have coverage close to nominal. The U1 is clearly the poorest performer for all cases of skewed distributions, as the coverage never reaches 95% and in many cases of symmetric non-normal distributions its coverage does not maintain its nominal level except when pair of samples is tends to infinity that its coverage converges to the nominal level.
Table 1 compares the coverage probabilities (Cps) and average widths (Aws) of the U1, M1 and M2 for a single linear function of variances, $\sum_{i=1}^{2} c_i \sigma_i^2$ for several different sets of coefficients from 2 nonnormal distributions.

<table>
<thead>
<tr>
<th>n1</th>
<th>n2</th>
<th>$c_1(1,3)$ Cps (U1) Aws (U1) Cps (M1) Aws (M1) Cps (M2) Aws (M2)</th>
<th>$c_1(1,-1)$ Cps (U1) Aws (U1) Cps (M1) Aws (M1) Cps (M2) Aws (M2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>0.7965 0.26 0.8398 0.30 0.9024 0.40</td>
<td>0.8373 0.559 0.8754 0.817</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>0.8702 0.17 0.9032 0.17 0.9331 0.24</td>
<td>0.8844 0.283 0.9111 0.219</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>0.9011 0.04 0.9234 0.18 0.9440 0.09</td>
<td>0.9125 0.74 0.9388 0.74</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>0.9220 0.01 0.9363 0.01 0.9525 0.01</td>
<td>0.9253 0.22 0.9349 0.23</td>
</tr>
<tr>
<td>125</td>
<td>125</td>
<td>0.9284 0.01 0.9389 0.01 0.9527 0.01</td>
<td>0.9297 0.19 0.9376 0.20</td>
</tr>
<tr>
<td>150</td>
<td>150</td>
<td>0.9313 0.01 0.9400 0.01 0.9546 0.01</td>
<td>0.9347 0.17 0.9414 0.17</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>0.9345 0.01 0.9417 0.01 0.9536 0.01</td>
<td>0.9380 0.15 0.9425 0.15</td>
</tr>
<tr>
<td>250</td>
<td>250</td>
<td>0.9391 0.01 0.9446 0.01 0.9564 0.01</td>
<td>0.9384 0.13 0.9426 0.13</td>
</tr>
<tr>
<td>300</td>
<td>300</td>
<td>0.9388 0.01 0.9442 0.01 0.9535 0.01</td>
<td>0.9423 0.12 0.9462 0.12</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>0.8230 0.32 0.8629 0.30 0.9144 0.27</td>
<td>0.8525 0.442 0.8876 0.491</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>0.8413 0.10 0.8908 0.10 0.9357 0.19</td>
<td>0.8953 0.186 0.9158 0.140</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>0.9117 0.02 0.9278 0.07 0.9476 0.05</td>
<td>0.9178 0.33 0.9302 0.37</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>0.9246 0.01 0.9343 0.01 0.9502 0.01</td>
<td>0.9298 0.18 0.9390 0.23</td>
</tr>
<tr>
<td>125</td>
<td>125</td>
<td>0.9311 0.01 0.9407 0.01 0.9537 0.01</td>
<td>0.9340 0.16 0.9399 0.17</td>
</tr>
<tr>
<td>150</td>
<td>150</td>
<td>0.9330 0.01 0.9393 0.01 0.9520 0.01</td>
<td>0.9363 0.15 0.9411 0.15</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>0.9367 0.01 0.9420 0.01 0.9533 0.01</td>
<td>0.9377 0.13 0.9410 0.13</td>
</tr>
</tbody>
</table>

For linear function of 3 variances, regardless of balance or unbalance designs, results (not showed here) state that, for skewed distributions, as groups of observations are moderate to large, both the M1 and M2 outperform the U1 but the M1 sometime performs slightly better than the M2 in the sense of well control the coverage probabilities to be close enough to the nominal level and a little bit narrower widths, while the U1 can better perform when groups of observations are extremely large, for symmetric non-normal distributions, the M2 and M1 are seem to be identically and maintaining their coverage quite well while the U1 perform poorly ,only with the uniform and beta distributions the U1 tends to be most precise in some value of linear coefficients, regularly, it can better perform when groups of observations are large.

Results from constructing confidence intervals for linear functions of 4 variances in Table 2 show again that the M2 is identical to the M1 and both also hold their level well for symmetric non-normal distributions but for some skewed distributions the M2 provides inaccurate coverage that exceeds the nominal level while the M1 is slightly below the target level. However, it is also shows that the M2 and M1 perform comparably well for moderate to large sizes. Results from the U1 depart from the nominal too often, except in some value of linear coefficients for a number of symmetric non-normal distributions (i.e., beta (3,3) and uniform(0,1)), it performs best, generally, as high sizes, the U1 performs reasonably well.
Table 2 compares the coverage probabilities (Cps) and average widths (Aws) of the U1, M1 and M2 for a single linear function of variances, $\sum_{i=1}^{4} C_i \sigma_i^2$ for several different sets of coefficients from 2 nonnormal distributions.

<table>
<thead>
<tr>
<th>c=(1/4,1/4,1/4,1/4)</th>
<th>c=(1,-1,1,-1)</th>
<th>logit(0,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=10,10,10,10</td>
<td>Cps (U1)</td>
<td>Aws (M1)</td>
</tr>
<tr>
<td>30,50,25,25</td>
<td>0.8217</td>
<td>0.0748</td>
</tr>
<tr>
<td>50,50,50,50</td>
<td>0.9064</td>
<td>0.0216</td>
</tr>
<tr>
<td>100,100,100,100</td>
<td>0.9263</td>
<td>0.0040</td>
</tr>
<tr>
<td>125,125,125,125</td>
<td>0.9281</td>
<td>0.0037</td>
</tr>
<tr>
<td>150,150,150,150</td>
<td>0.9340</td>
<td>0.0035</td>
</tr>
<tr>
<td>200,200,200,200</td>
<td>0.9366</td>
<td>0.0027</td>
</tr>
<tr>
<td>300,300,300,300</td>
<td>0.9422</td>
<td>0.0021</td>
</tr>
<tr>
<td>10,20,30,40</td>
<td>0.8622</td>
<td>0.0788</td>
</tr>
<tr>
<td>25,50,75,100</td>
<td>0.9022</td>
<td>0.0239</td>
</tr>
<tr>
<td>50,100,150,200</td>
<td>0.9210</td>
<td>0.0055</td>
</tr>
<tr>
<td>75,150,225,300</td>
<td>0.9264</td>
<td>0.0044</td>
</tr>
<tr>
<td>100,200,300,400</td>
<td>0.9344</td>
<td>0.0021</td>
</tr>
<tr>
<td>125,350,750,100</td>
<td>0.9391</td>
<td>0.0024</td>
</tr>
<tr>
<td>150,300,450,600</td>
<td>0.9381</td>
<td>0.0020</td>
</tr>
<tr>
<td>200,400,600,800</td>
<td>0.9368</td>
<td>0.0020</td>
</tr>
</tbody>
</table>

In further investigation, as comparison, we determined the performances when samples are collected from normal population distributions, in both cases of equal and unequal variances when various group sizes are balanced and unbalanced designs. Simulation results are also established but not shown here. Findings suggest that, the pattern of results were similar for the equal and unequal variances regardless of balanced or unbalanced designs, so in most cases, when sample sizes are moderate, small to large, the top performing intervals are the M1 and M2, moreover, they both yield almost identical results that give reasonably similar coverage and width and these are maintained as sample sizes increases, with the U1, in many cases, as sample sizes increase, generally there is an increase in coverage probability, while in some linear coefficients value, it is the best performer.

Although not reported here we also found that, for a wide variety of linear functions of variances, from $k \geq 2$ populations that are not identical, the results usually show an extremely large departure from the nominal level that are not attractive.

### 4 Conclusions

Two new intervals for a single linear combination of variances, the M1 and M2 which are generated from the MBBE and the adjusted MBBE of variance respectively, are proposed here and demonstrate that they both have better coverage probabilities than the U1 that is generated from the unbiased variance.
estimator in most cases investigated by both symmetric and asymmetric distributions when various group sizes are balanced or unbalanced designs.

As a whole, two possible explanations for our results are that the three intervals, the M2, M1 and U1 for linear function of variances obviously fail to hold their nominal coverage, often providing intervals which are poor. In most cases small-sample performance we investigated, all the confidence intervals converge to the target level, as groups of observations are large.

Confidence interval construct for a single linear combinations of variances $\sum_{i=1}^{k} c_i \sigma_i^2$, from k=2 independent populations, the M2 performed best in terms of holding the correct coverage probabilities with narrowest average width in most cases, for asymmetric distributions regardless of balanced or unbalanced designs, and identical to the M1 for all symmetric distributions when the group sizes are moderate to large.

For k > 2 independent populations, when samples are drawn from symmetric population distributions, regardless of balanced or unbalanced designs, mostly cases for a variety of coefficient, the M2 is also identical to the M1 but has a little bit wider widths than M1 when group of sizes are moderate to large and they both better perform than the U1, except cases in which at least one negative term occurs in a variety of coefficients then they both seem to be inferior in a few distributions. For skewed distributions, in several cases coverage of the M2 is still at or above nominal while the M1 is at or below nominal, though they are still close to nominal, it is not clear whether the M2 or M1 perform best since these two intervals alternately outperform.

In addition, we found a few instance where the coverage probabilities of U1 was superior to the M2 and M1 when samples come from Normal (regardless of equal or unequal variances) or symmetrically distributed populations (i.e., Beta(3,3) and Uniform(0,1)) as there is at least one negative coefficient occurring in a variety of coefficients. To investigate the rule for variances combinations that give the best performer, that is left for a subsequence paper. Finally, we thank the work of Donner and Zou [3], and the work of Zou et al.[6].

References


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