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Modeling Blood Flow in the Blood Vessels of the Cardiovascular System Using Fractals

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Abstract

The essentiality of experimental and speculative investigation is on blood vessels of the cardiovascular system. The principle of the study is to evaluate numerically the blood vessels of the cardiovascular system using Inlet section, Reynold's number, rate of flow and area of cross-section. The concept of inlet section depicts us the affected blood vessel in an accurate way through the numerical calculation. A new algorithm has been introduced for the traversal of blood flow in the blood vessels of the human cardiovascular system in a robust manner and provides us the results in an easier manner.

Keywords: Fractals, Cardiovascular system, Inlet section, Reynold's number.

1. Introduction

In this paper fractal models of human blood vessels of the cardiovascular system have been proposed. The present work deals with the blood circulation of the fractal blood vessel trees that have been classified and examined accordingly with the inlet section and Reynold's number.

1.1 Fractals

A fractal is an object or quantity that displays self-similarity, in a somewhat technical sense, on all scales. The object need not exhibit exactly the same structure at all scales, but the same "type" of structures must appear on all scales. Benoit Mandelbrot, a French mathematician says that a fractal is “a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole.”[8] The term “fractal” means “broken” or “fractured” according to the Latin word *fractus.* Fractal objects and processes are said to display ‘Self-invariant’ (Self-similar or Self-affine) properties [6]. A mathematical fractal is based on an equation that undergoes iteration, a form of feedback based on recursion and they are useful in the field of medicine [2]. The Cardiovascular system with regular beating of the human heart is an example of fractal. Fractal structures do not have a single length scale, while fractal processes (time-series) cannot be characterized by a single-time scale [12].

1.2 Cardiovascular system

The purpose of this study is to investigate the defects of the cardiovascular system due to ischemia. Blood vessels play a role in virtually every medical condition. The blood vessel of the cardiovascular system can be sufficiently modeled through fractals. This analysis permits understanding influence of hemodynamic forces and its role in the development of ischemic diseases. In the cardiovascular system, the flow analysis can be based on fractal geometry and fractal dimension of the vascular tree through fractal models.[7,9-11]

The cardiovascular system is responsible for transporting nutrients and removing gaseous waste from the body. This system is comprised of the heart and the circulatory_system. Structures of the cardiovascular system include the heart, blood_vessels, and blood. Blood vessels are intricate networks of hollow tubes that transport blood throughout the entire body. Blood travels from the heart via arteries to smaller arterioles, then to capillaries or sinusoids, to venules, to veins and back to the heart. Through the process of microcirculation, substances such as oxygen, carbon dioxide, nutrients, and wastes are exchanged between the blood and the fluid that surrounds cells. The blood delivers nutrients to cells and removes wastes that are produced during cellular processes, such as cellular respiration. The blood is composed of red blood cells, white blood cells, platelets, and plasma.(Fig-1).

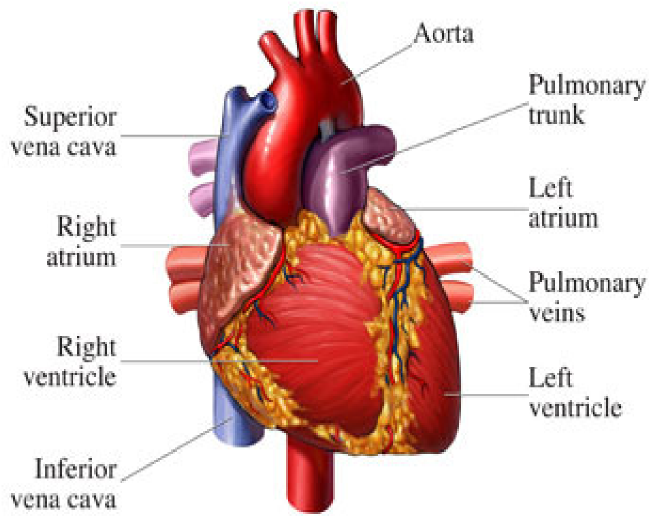

Figure – 1. Human cardiovascular system

The depiction of the blood flow in the blood vessels of the above system has been numerically calculated and also modeled by using fractals.

1.3 Hemodynamics

Hemodynamics means the study of blood flow and related forces in moving the blood through the cardiovascular system. It discusses the physical principles of blood flow through the blood vessels with special reference to the inter-relationships among pressure, flow and resistance. Hemodynamics is defined as the forces affecting the flow of blood throughout the body. Human beings cannot survive without adequate oxygenation, and the primary function of the cardiopulmonary system is to deliver an appropriate amount of oxygen and nutrients to meet the metabolic demands of the body and then to remove metabolic waste products.

1.3 Laminar Flow

Laminar flow is the normal condition for blood flow throughout most of the cardiovascular system. It is characterized by concentric layers of blood moving in parallel down the length of a blood vessel. The highest velocity is found in the center of the vessel. The lowest velocity is found along the vessel wall. This occurs in long, straight blood vessels, under steady flow conditions. In laminar flow the direction of the flow is parallel to the vessel wall and is basically straight.

1.4 Turbulent Flow

Turbulent flow occurs in large arteries at branch points. In turbulent flow the direction of the flow is not parallel to the vessel wall and blood flows in different directions. Turbulent flow can often cause small whirlpools to form in the blood, much like the ones seen in rivers at points of obstruction.

In the section 4, we have modeled an algorithm for the traversal of the blood vessels in a breath first manner. In the section 5, we have briefly described the numerical calculation based on synthetic view for both normal and abnormal patients.

2. Modeling of the blood vessels system through Fractals

In real blood vessels system, vessel walls are elastic and change its diameters. In this way resistance of blood vessel system is regulated. This process is known as auto regulation and corrects nutrition of all cells in human body. Blood flow estimation assumes laminar flow for the entire fractal vascular tree.

The blood vessel of the human cardiovascular system is modeled through the fractal vascular tree like structure is shown below. [3, 4, 12] (Fig.2). The basic idea of fractal is that breaking down an object into smaller parts and each of which resembles the whole has been invoked for the blood vessels.

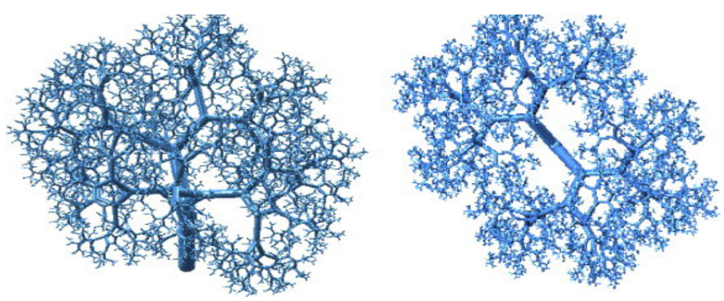

Figure – 2. Structure of the Fractal vascular tree

3. Methods

Blood flow in the cardiovascular system has been precisely calculated by the following methods. These results are very useful to investigate the defects in the above system. The following methods are the most important parameters in transport vessel tree.

3.1. Rate of flow (Q)

It is defined as the quantity of the fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid the rate of flow is expressed as the volume of fluid flowing across the section per second. Mathematically the rate of flow is defined as

$$Q = A \times V \qquad \text{.....(1)}$$

where Q is the Rate of blood flow (mℓ/sec), A is the cross-sectional area (cm^2), V is the velocity of the blood.

3.2 Reynold's number

Using the Reynold's equation we can see that a large diameter, with rapid flow, where the density of the blood is high tends towards turbulence. Rapid changes in vessel diameter may lead to turbulent flow, for instance when a narrower vessel widens to a larger one.

When normal laminar blood flow becomes turbulent flow

- The rate of blood flow (i.e.,) the velocity of flow, is high
- It passes by an obstruction in a vessel (as in case of compression by cuff of Sphygmomanometer).
- It makes a sharp turn.
- It passes over a rough surface

Turbulence is also related to the diameter of the vessel and the viscosity of the blood which can be expressed by ratio of inertia to viscous forces.

$$R_e = \frac{V * L}{\eta} \qquad \text{.....(2)}$$

where R_e is the Reynold's number, L is the length of the blood vessel, V is the Average velocity and η is the Viscosity of the blood. This is the most important dimensionless number; it describes the fluid flow regime. If R_e is higher, there is a greater probability of the turbulence (Table2 and Table 4).

3.3 Inlet Section

The blood flow in the blood vessel is mainly concerned with Inlet section. The length of the section is proportional to vessel diameter, Reynold's number and coefficient

$$l_e = \lambda d R_e \qquad \text{.....(3)}$$

Where l_e is the inlet section, $\lambda = 0.056$ a value from Navier-Stokes equation, d diameter of the blood vessel and R_e Reynold's number

4. Algorithm

The given algorithm circulates the blood in the blood vessels of the cardiovascular system in a recursive manner.

Name: Breath first search traversal of an undirected graph.

Input: A Graph $G = \langle V, E \rangle$ represented by the adjacency list for $v \in V$

Output: *A* breath first search of an undirected graph $G = \langle V, E \rangle$ possess recursive procedure on each vertex marked as visited in the order they were visited.

. **Method:** **BFS** (vertex *v*)

To start, all vertices are unmarked.

- Start at *v*. Visit *v* and mark as visited.
- Visit every unmarked neighbour ui of *v* and mark each ui as visited.
- Mark *v* finished.

We will use p[v] to represent the predecessor of *v* and d[v] to represent the number of edges from *v* (i.e., the distance from *v*).

The procedure is recursively used as follows:

```
Procedure BFS (G = ⟨V, E⟩, v)
Begin
  For all vertices u in V
        color[u] := black
        d[u] := infinity;
        p[u] := NIL;
  End for
  Initialize an empty queue Q;
  color[v] := green;
  d[v] := 0;
  p[v] := NIL;
  ENQUEUE (Q,v);
  While not ISEMPTY (Q) do
        u := DEQUEUE(Q);
        For each edge (u,w) in E do
              If (color[w] == black) then
                    color[w] := green;
                    d[w] := d[u] + 1;
                    p[w] := u;
                    ENQUEUE (Q,w);
              End if
        End for
        color[u] := red;
  End while
END BFS
```

Time complexity: The time complexity can be expressed as $O(|E|+|V|)$ since every vertex and every edge will be explored.
The above algorithm has been programmed and run by C++ [5].

5. Numerical Simulation Results

A synthetic view of human cardiovascular system has been exhibited with a remarkable variety of components is given in the following Table.1 for the normal Human being [1].

Table 1 Data Report of Normal Human Being

Vessel	Radius (r)(cm)	Total Cross Section (A) (cm^2)	Wall thickness (h) (cm)	Average Velocity (V) (cm/sec)	Length (L) (cm)
Aorta	1.25	4.9	0.2	21.25	50
Arteries	0.2	20	0.1	60×10^{-2}	50
Arterioles	1.5×10^{-3}	400	2×10^{-3}	90×10^{-6}	1
Capillaries	3×10^{-4}	4500	1×10^{-4}	160×10^{-8}	0.1
Venules	1×10^{-3}	4000	2×10^{-4}	65×10^{-8}	0.2
Veins	0.25	40	0.05	50×10^{-2}	2.5
Venacava	1.5	18	0.15	2.36	50

The following attributes are calculated using the above table. The viscosity of the blood is 5 Poise. For example, the main blood vessel Aorta calculation has been shown here.

1) Area of the cross section (including wall thickness)

$$C_1 = \pi r^2 = \pi \times (1.45)^2 = 6.6cm^2$$

2) Area of the cross section (excluding wall thickness)

$$C_2 = \pi r^2 = \pi \times (1.25)^2 = 4.9cm^2$$

3) Rate of flow

$$Q = A \times V = 4.9 \times 21.25 = 104.13cm^3/\sec$$

4) Reynolds number

$$R_e = \frac{V * L}{\eta} = \frac{21.25 \times 50}{5} = 212.5$$

5) Inlet Section

$$l_e = \lambda d R_e$$

$$= 0.056 \times 2.5 \times 212.5$$
$$= 29.75 \mu m$$

Table 2. Analysis of the Data for other parts of the cardiovascular system.

Vessel	Area of the cross section (C_1) (cm^2)	Area of the cross section (C_2) (cm^2)	Rate of flow (Q) (cm^3/sec)	Reynolds number (R_e)	Inlet Section (l_e) µm
Aorta	6.6	4.9	104.13	212.5	29.75
Arteries	0.28	0.13	12	6	0.1344
Arterioles	3×10^{-5}	7×10^{-6}	0.036	1.8×10^{-05}	3.204×10^{-9}
Capillaries	5×10^{-7}	50×10^{-6}	7.2×10^{-03}	3.2×10^{-08}	1.075×10^{-11}
Venules	4.5×10^{-6}	3×10^{-6}	2.6×10^{-03}	2.6×10^{-08}	2.912×10^{-12}
Veins	0.28	0.20	20	0.25	0.007
Venacava	8.6	7.0	42.48	23.6	3.9648

The following data belongs to Ischemic heart disease patient for which we can calculate the following attributes.

Table 3 Data Report of Ischemic heart disease

Vessel	Radius (r) (cm)	Total Cross Section (A) (cm^2)	Wall thickness (h) (cm)	Average Velocity (V) (cm/sec)	Length (L) (cm)
Aorta	0.03	0.03	0.95	43.7	43
Arteries	0.29	128	0.76	42.5	43
Arterioles	1.4×10^{-3}	350	2×10^{-3}	90×10^{-6}	1
Capillaries	3×10^{-4}	4500	1×10^{-4}	160×10^{-8}	0.1
Venules	1×10^{-3}	4000	2×10^{-4}	65×10^{-8}	0.2
Veins	0.77	400	0.11	20.4	1.5
Venacava	0.7	3	0.15	11	43

The viscosity of the blood is 5 poise. For example, the main blood vessel Aorta calculation is shown here.

1) Area of the cross section (including wall thickness) $C_1 = \pi r^2$

$$= \pi \times (0.98)^2$$
$$= 3.017\,cm^2$$

2) Area of the cross section (excluding wall thickness) $C_2 = \pi r^2$

$$= \pi \times (0.03)^2$$
$$= 0.003 cm^2$$

3) Rate of flow $Q = A \times V$

$$= 0.03 \times 43.7$$
$$= 1.311 cm^3 / \sec$$

4) Reynolds number

$$R_e = \frac{V * L}{\eta}$$
$$= \frac{43.7 \times 43}{5}$$
$$= 375.82$$

5) Inlet Section $l_e = \lambda d R_e$

$$= 0.056 \times 0.06 \times 375.82$$
$$= 1.263 \mu m$$

Table 4 Analysis of the Data report for Ischemic heart disease patient

Vessel	Area of the cross section (C_1) (cm^2)	Area of the cross section (C_2) (cm^2)	Rate of flow (Q) (cm^3/sec)	Reynolds number (Re)	Inlet Section (l_e) µm
Aorta	3.017	0.003	1.311	375.82	1.263
Arteries	3.464	0.264	5440	365.5	11.871
Arterioles	$3.632 x 10^{-05}$	$6.158 x 10^{-06}$	0.0032	$1.8 x 10^{-06}$	$2.822 x 10^{-10}$
Capillaries	$5.027 x 10^{-7}$	$2.827 x 10^{-6}$	$7.2 x 10^{-04}$	$3.2 x 10^{-08}$	$1.075 x 10^{-11}$
Venules	$4.524 x 10^{-6}$	$3.142 x 10^{-6}$	$2.6 x 10^{-03}$	$2.6 x 10^{-08}$	$2.912 x 10^{-12}$
Veins	2.433	1.863	8160	6.12	0.528
Venacava	2.270	1.539	33	94.6	7.417

6. Conclusion

The blood flow analysis in the blood vessels have been numerically calculated for the parameters such as area of the cross section, rate of flow, Reynold's number and inlet section for a normal and an abnormal patient, in our proposed system where as in the previous journal they have given the solution only for the blood flow in the circulatory system in a graphical manner and in another numerical calculation has been carried based on Poiseuiile's law but we have proved the defects in the blood vessels of the Cardiovascular system by numerical calculation. Inlet section is a new concept which provides us the pressure variation due to inflammation and gives us the accurate result of the affected blood vessels of the cardiovascular system in an effective manner. This numerical simulation could help the surgeon in understanding how different solution may affect blood circulation in the blood vessels and guide him in selection of the most appropriate procedure for a specific patient and is mainly dependent upon the blood vessel parameter. The breath first traversal strategy could be the useful tool to go through the system in an effective manner.

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