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How to Stop a Vampiric Infection?

Using Mathematical Modeling to Fight

Infectious Diseases

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Abstract

We apply models with differential equations to study the possible vampiric disease and to predict its spread across the world. Careful review of all available sources revealed that the mankind might survive the outbreak by laboriously killing all infected and (already) undead.

Keywords: mathematical model, popular culture, predator-prey system, vampires

1. Introduction

The role of infectious disease is devastating for human population growth. Many sources show that most of the infections diseases might put human population in

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great danger (see e.g. Mollison, 1995; or Andersson and Britton, 2000). It is enough to recall the 2011 medical thrilled "Contagion" which portrayed a hypothetical disease (similar to swine flu) to imagine how such a disease would swipe over the world killing millions of people.
Infections spread violently like zombies (or vampires) from the popular horror movies. Just to recall Romero's timeless classic "Night of the Living Dead" from 1968 or most recent Boyle's "28 days later" (or its sequel "28 weeks later"), a British parody "Shawn of the Dead", and, of course, a post-apocalyptic thriller "Resident Evil" with all its sequels.
If scenarios shown in most of zombie films were real, pretty soon the world would have been taken by zombies. Mathematicians from the University of Ottawa claim that zombies would have eventually taken over the world unless quick and aggressive attacks are made (Munz et al., 2009). The progression of zombie infection is fast and unless isolated, quarantined and killed, very soon everyone will become a zombie (and then dies, as far as there is nothing left to feed on). The same would hold true for the infectious diseases. Quarantine, isolation and (if available) vaccination are a must before most of the population is whipped out by a disease.
Since the 1980s, research on vampires found their way into the research literature becoming an inspiration for several academic papers (Hart and Mehlmann, 1982, 1983; Hartl, Mehlmann and Novak, 1992; Neocleus, 2003; Efthimiou and Ghandi, 2007; or most recently Strielkowski et al., 2013).
The first myths and legends about vampires have probably existed since the dawn of human history. In the 19th century, ancient Mesopotamian texts dating back to 4000 B.C. were translated into English revealing some mentioning of "seven spirits" that are very much like the description of vampires as we think of them today (Campbell Thompson, 1904).
Let us look at how mathematical models can be used to fight infectious diseases, just like vampire-like infection. Mathematics can help us to calculate possible scenarios of the outbreak and to come up with the remedy. However, lacking the research literature on the nature of vampiric infections, we would have to rely upon the popular fiction literature on vampires.
In order to do that, we have identified one peculiar type of vampire-human interactions confrontation using the Lotka-Volterra predator-prey model (Volterra, 1931).

2. The Blade model of vampiric infection

The scenario presented below represents a peculiar case of the world inhabited by humans and vampires described in "Blade" comic book series which became an inspiration to the popular film trilogy and TV series (hereinafter referred to as "the Blade model").
The Blade model shows the world where some vampires developed so-called "Reaper virus" making them some sort of "super-vampires". They are stronger

and faster than “normal” vampires and can sustain sunlight and usual weapons (garlic, silver). “Super-vampires” need to feed on other “normal” vampires and are not interested in human beings. The feeding takes place once in about 5-7 days (when “super-vampires” become thirsty). “Super-vampire’s” victim usually becomes another “super-vampire”.
“Normal” vampires still feed on human beings (typically once in 5-7 days) and usually kill them after feeding. Humans can be turned into new “normal” vampires but it takes time and effort.
The initial conditions of the Blade model are the following: five million vampires, 6 159 million people, 1 super-vampire, there are organized groups of both super-vampire slayers and vampire slayers (Figure 1 and Figure 2).

Figure 1: The Blade model: $h > 0$

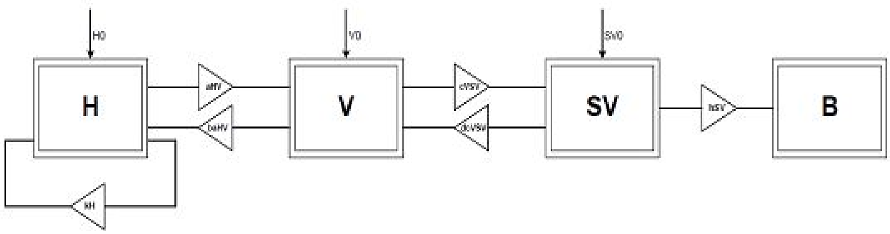
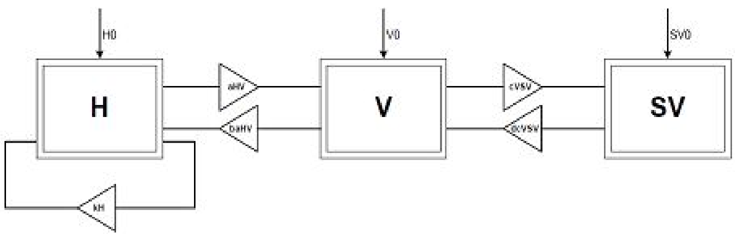


Figure 2: The Blade model: $h = 0$



where H denotes humans, V denotes vampires, VS denotes vampire slayers, SV denotes super-vampires and B denotes a super-vampire slayer (Blade). H0 is the initial state of human population, kH denotes the exponential growth of human population, v0 is the initial state of vampire population, aHV and baHV both describe interactions between a human and a vampire (with a as the coefficient of a lethal outcome for vampire-human interaction for humans and b as the coefficient describing the rate with which humans are turned into vampires), cV denotes the death rate for vampires. SV0 is the initial state of super-vampire population, cVSV and dcVSV both describe interactions between vampires and super-vampires with c being the coefficient of the lethal outcome for vampires and d being the coefficient of turn rate for vampires. hSV denotes the lethal outcome for super-vampires when meeting the super-vampire slayer.
In this specific case, the Blade model is somewhat different from the predator-prey model defined in Volterra (1931) due to the new population represented by super-vampires. The model that describes the interaction of three populations (humans, vampires and super-vampires) looks like the following:

$$\begin{cases} \dfrac{dx}{dt} = x(k - ay) \\ \dfrac{dy}{dt} = y(bax - cz) \\ \dfrac{dz}{dt} = z(dcy - h) \\ x(0) = 6150 \cdot 10^{6} \\ y(0) = 5 \cdot 10^{6} \\ z(0) = 1 \end{cases} \tag{1}$$

3. Model set-up and calibration

Let us calibrate the parameters of the model. The calculation period is set to 500 years with a step of 7 days *(t = 0…26000)*.The coefficient of human population's growth is calculated as
$k = ln(x_1/x_0)/t_1\text{-}t_0$,where $x_1 = 7000$ million people at a moment of time of $t_1 = 2012$, $x_0 = 6150$ million people at a moment of time of $t_0 = 2001$.
Humans almost always die after their encounters with vampires (are turned into new vampires), the coefficient of lethal outcome is high and it is denoted by *a*. The probability of a human being turned into a vampire is rather low and equals to *b=0.001*. There are numerous groups of super-vampire slayers, therefore we use considerably high coefficient of *c*. Almost every vampire becomes a super-vampire after an unfortunate encounter with the latter, therefore *d = 1*
In order to solve the system of ordinary differential equations the fourth-order Runge-Kutta method is employed. This is done by building the following algorithm (2):

$$\mathrm{RK}(x0, y0) := \left| \begin{array}{l} xys \leftarrow \begin{pmatrix} x0 \\ y0 \end{pmatrix} \\ D(t, xys) \leftarrow \begin{bmatrix} xys_0 \cdot (k - a \cdot xys_1) \\ xys_1 \cdot (b \cdot a \cdot xys_0 - c) \end{bmatrix} \\ Res \leftarrow \mathrm{rkfixed}(xys, ts, tf, 100000, D) \\ Res \end{array} \right. \tag{2}$$

4. Estimating the Blade model

Following the scenario described in "Blade 2" film, here are two cases to be considered:

- Blade agrees to help vampires in exterminating super-vampires. The parameter regulating the extermination rate of super-vampires is $h > 0$ (Figure 1).
- Blade refuses to help vampires in killing super-vampires (super-vampires are no threat for humans), $h = 0$ (Figure 2).

In the first case, the stationary solution can be reached only if $y_s = k/a = h/dc$, $x_s = cz_s/ba$ (strict regulation of the vampire population and reaching proportionality between the population of humans and super-vampires). Slight deviations from the stationary solution will lead to cycles in all 3 populations (see Charts 1 and 2).

From Chart 1 it is apparent that the human population will be diminishing from the initial period of time and in 21 years (1092 weeks) will reach its minimum with the critical point of 1.23 billion people. Moreover, the human population will grow and by 2196 (10140 weeks) will reach its maximum of 8.53 billion people. This process will repeat itself with a cyclical periodicity.

Chart 1: Changes in human population in the Blade model (cyclical nature)

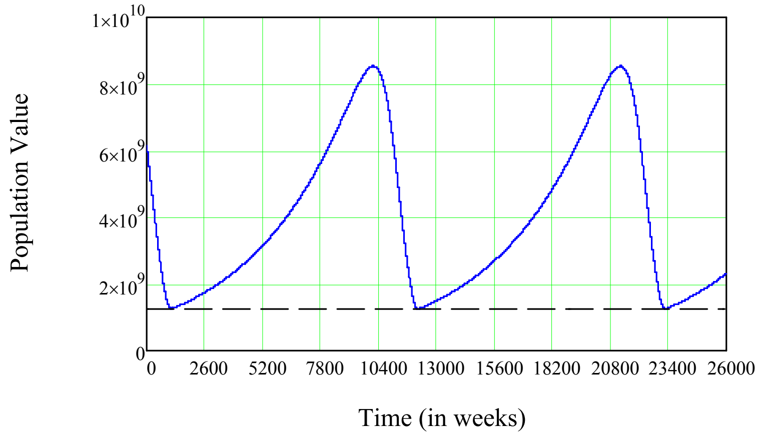
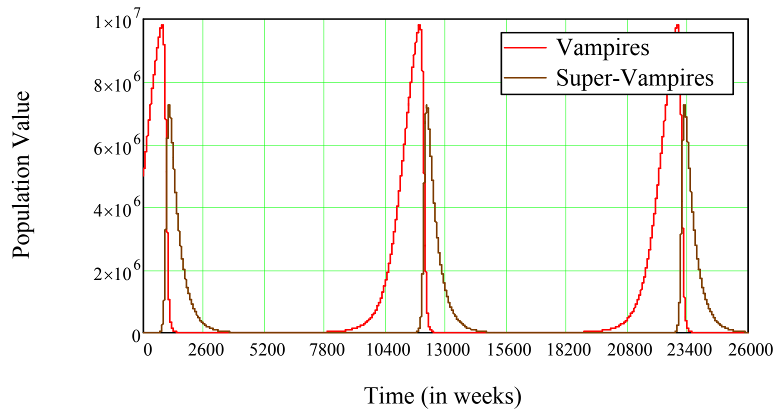


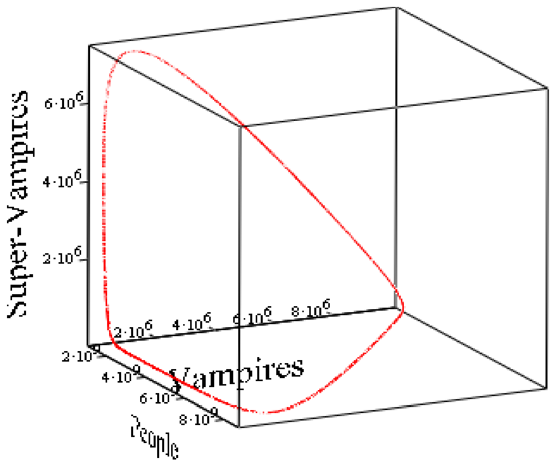
Chart 2: Changes in vampire population and super-vampire population (cyclical nature)



The changes in the populations of vampires and super-vampires are also of a cyclical nature (Chart 2). Starting from the initial point of time, the number of vampires will grow from 5 million to 9.8 million within 15 years. Further, it will decline to the point of mere 554 vampires in 2055 (2808 weeks). The population of super-vampires will grow from just one super-vampire to 7.2 million vampires in 21 years (1092 weeks) and then will decline until the super-vampires will be on a brink of extinction.

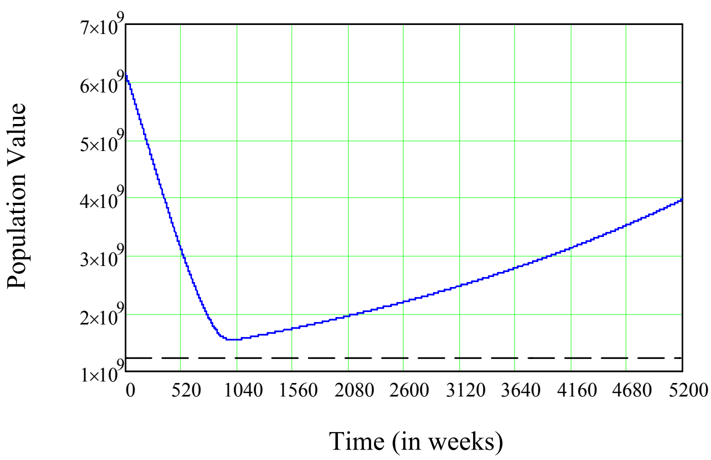
Chart 3 presents the phase diagram of the system encountering vampires, humans and super-vampires, which resembles the connected curve. In case Blade refuses to help vampires in their quest to exterminate their new enemy, super-vampires, the Blade model cannot sustain a stable co-existence of three species. Chart 4 depicts the situation when the human population is diminishing being under continuous attacks by vampires and by 2020 (988 weeks) will reach its minimum of 1.5 billion, whereupon it will start growing steadily again. This is going to happen due to the fact that super-vampires will exterminate all vampires (Chart 5) and there will be no natural enemies for humans.

Chart 3: Phase diagram for the populations of vampires (Y), humans (X) and super-vampires (Z) in the Blade model

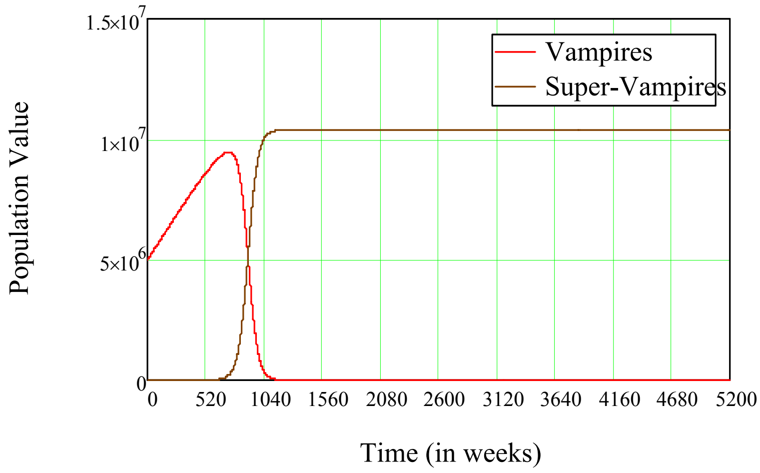


By 2032 (1612 weeks) there will be no vampires left in the world of the Blade model (they all will be killed or turned into super-vampires). The super-vampire population will reach its peak of 10.4 million and will remain at its values until super-vampires die off due to food shortages (since they cannot feed on humans, due to their nature, and require vampires' blood for feeding).

Chart 4: The change in human population (disbalance)



er-vampire population



Therefore, according to the conditions of the Blade model, it would be more beneficial for humans to persuade Blade not to interfere. As a result, the infected will all become the bearers of “Reaper virus” and therefore harmless to humans. Unless some new infectious disease is introduced into this system (or “Reaper virus” mutates to affect the humans), humanity will be saved.

4. Conclusions

The paper demonstrated how mathematical models can be used to predict and fight the spread of infectious diseases. In addition, it analyzed the possibility of co-existence for human and vampire (the metaphor for the infected humans) populations in a scenario described by the conditions narrated in comic books and films.

It appears that although vampire-human interactions would in most cases lead to great imbalances in the ecosystems, there are several cases that might actually convey plausible models of co-existence between humans and vampires.
The Blade model (based on Marvel Comics’ “Blade”) presents an extension to the Lotka-Volterra class of models introducing the super-vampires (a metaphor for a mutated disease) and examines the balance in the new system. According to the Blade model, all three populations are in disbalance and it largely depends on whether vampires and humans would join their forces to fight the super-vampires that vampires remain in existence or are exterminated by the super-vampires.
Overall, it appears that fighting a vampiric infection would require extensive resources and unless the virus mutates and start attacking those already infected with its primal form, the existence of humanity might be endangered.

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