Discrete Multi-target Linear-quadratic Control Problem and Quadratic Programming\textsuperscript{1}

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Abstract

We consider a discrete multi-target linear-quadratic control problem. We reduce the problem into a quadratic programming over a simplex. Computing the coefficients of the cost function requires knowing the descriptions of the orthogonal project onto the vector space $Z$ and the orthogonal complement $Z^\perp$ of the vector space $Z$.

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1 Introduction

Let $(H, <, >)$ be a Hilbert space, $Z$ be its closed vector subspace, $h_1, \cdots, h_m$ and $c$ be vectors in $H$. Consider the following optimization problem:

\[
\max_{1 \leq i \leq m} \|h - h_i\| \rightarrow \min, \quad h \in c + Z, \tag{1}
\]

where $\| \cdot \|$ is the norm in a hilbert space $H$ induced by the inner product $<, >$. This problem was considered in a classical optimal control theory known as a

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tracking problem or Multi-target linear quadratic problem (MTLQP). According to this notation, there are \( m \) targets and we wish to track these targets using a given linear system with a quadratic cost function. A more general version of this problem where the cost function is a general quadratic function is studied in [3] and it is called multi criteria linear quadratic control problem (MCLQP). A solution to MCLQP is also fully described [3] with the help of infinite dimensional interior-point method. A decent direction is calculated on each iteration. It is shown that the approximated solution approaches to the optimal solution. In [2], Faybusovich and Mouktonglang analyzed (1) using duality theory for infinite-dimensional second-order cone programming. They obtain a reduction of this problem to a finite dimensional second-order cone programming and apply this result to a multi-target linear-quadratic control problem on a finite time interval. Later on, [4] considers a reduction (1) to even simpler optimization problem of minimization of convex quadratic function on the \((m-1)\) dimensional simplex. Then by applying this result to the analysis of a continuous version of multi-target linear-quadratic control problem on semi-infinite time interval, they show that the coefficients of the quadratic function admitted a simple expressions in term of the original data. In this paper, we apply the result of the reduction to a discrete version of multi-target linear-quadratic control problem. We also show that the coefficients of the quadratic function admitted a simple expressions in term of the original data.

2 Reduction to a simple quadratic programming problem

The reduction to a quadratic programming problem over a simplex was completely described in full details in [4]. For the completeness of the paper, we shortly describe the reduction in this section. For more details, see [4].

Let \( f_i(h) = \|h - h_i\|^2, i = 1, 2, \ldots, m \). One can easily see that (1) is equivalent to the following optimization problem

\[
\begin{align*}
    z & \rightarrow \min, \\ \\
    f_i(h) & \leq z, i = 1, 2, \ldots, m, \\ \\
    h & \in c + Z.
\end{align*}
\]
Consider the Lagrange function
\[
\mathcal{L}(\lambda_1, \cdots, \lambda_m, h, z) = z + \sum_{i=1}^{m} \lambda_i (f_i(h) - z)
\]
\[
= z(1 - \sum_{i=1}^{m} \lambda_i) + \sum_{i=1}^{m} \lambda_i f_i(h).
\]

The optimality conditions for (2) - (4) take the form
\[
\lambda_i \geq 0, \quad \lambda_i (f_i(h) - z) = 0, \quad i = 0, 1, 2, \cdots, m,
\]
\[
\frac{\partial \mathcal{L}}{\partial z} = 0, \quad \sum_{i=1}^{m} \lambda_i \nabla f_i(h) \in Z^\perp,
\]
where \( \nabla f_i(h) = 2(h - h_i), \ i = 1, 2, \cdots, m, \ Z^\perp \) is the orthogonal complement of \( Z \) in \( H \). Conditions (5), (6) imply that
\[
\sum_{i=0}^{m} \lambda_i = 1, \quad \lambda_i \geq 0, \quad i = 1, 2, \cdots, m,
\]
\[
\pi_Z(h) = \sum_{i=1}^{m} \lambda_i (\pi_Z h_i),
\]
where \( \pi_Z : H \to Z \) is the orthogonal projection onto \( Z \). Similarly, \( \pi_{Z^\perp} : H \to Z^\perp \) is the orthogonal projection onto \( Z^\perp \) of \( Z \). Then the Lagrange dual of (2), (3),(4) takes the form
\[
\varphi(\lambda_1, \lambda_2, \cdots, \lambda_m) = \min\{\mathcal{L}(\lambda_1, \cdots, \lambda_m, h, z) : h \in c + Z, z \in Z\}.
\]
Using (7), (8), we obtain
\[
\varphi(\lambda_1, \lambda_2, \cdots, \lambda_m) = \sum_{i=1}^{m} \lambda_i f_i(h(\lambda_1, \cdots, \lambda_m))
\]
where
\[
h(\lambda_1, \cdots, \lambda_m) = \pi_{Z^\perp}(c) + \sum_{i=1}^{m} \lambda_i \pi_Z(h_i).
\]
For simplicity, we let
\[ h(\lambda) = \sum_{i=1}^{m} \lambda_i h_i. \]
Then
\[ f_j(h(\lambda_1, \cdots, \lambda_m)) = \|\pi_Z(h(\lambda))\|^2 + \|\pi_Z(h_j)\|^2 - 2 < \pi_Z(h(\lambda)), \pi_Z(h_j) > + \|\pi_{Z\perp}(c - h_j)\|^2. \]
Hence, according to (9)
\[ \varphi(\lambda_1, \cdots, \lambda_m) = \|\pi_Z(h(\lambda))\|^2 + \sum_{j=1}^{m} \lambda_j \|\pi_Z(h_j)\|^2 - 2 < \pi_Z(h(\lambda)), \pi_Z(h(\lambda)) > + \sum_{j=1}^{m} \lambda_j \|\pi_{Z\perp}(c - h_j)\|^2. \]
We, hence, arrive at the following expression of \( \varphi \):
\[ \varphi(\lambda_1, \cdots, \lambda_m) = -\|\pi_Z\left(\sum_{i=1}^{m} \lambda_i h_i\right)\|^2 + \sum_{j=1}^{m} \lambda_j (\|\pi_Z(h_j)\|^2 + \pi_{Z\perp}(c - h_j)\|^2). \]
(11)
We can simplify (11) further. Notice that
\[ \|\pi_{Z\perp}(c - h_j)\|^2 = \|\pi_{Z\perp}(c)\|^2 + \|\pi_{Z\perp}(h_j)\|^2 - 2 < \pi_{Z\perp}(c), \pi_{Z\perp}(h_j) >. \]
Consequently,
\[ \varphi(\lambda_1, \cdots, \lambda_m) = -\|\pi_Z(h(\lambda))\|^2 + \sum_{j=1}^{m} \lambda_j \|h_j\|^2 - 2 < \pi_{Z\perp}(c), \pi_{Z\perp}(h(\lambda)) > + \|\pi_{Z\perp}(c)\|^2 \]
\[ = -\|h(\lambda)\|^2 + \|\pi_{Z\perp}(h(\lambda) - c)\|^2 + \sum_{j=1}^{m} \lambda_j \|h_j\|^2 \] (12)
Here,
\[ h(\lambda) = \sum_{i=1}^{m} \lambda_i h_i. \]
Hence, the Lagrange dual to (2), (3), (4) takes the form:
\[ \varphi(\lambda_1, \cdots, \lambda_m) \rightarrow \max, \quad (13) \]

\[ \sum_{i=1}^{m} \lambda_i = 1, \quad \lambda_i \geq 0, \quad i = 1, 2, \cdots, m. \quad (14) \]

The problem (13)-(14) is known as a quadratic programming over a simplex.

3 Discrete linear-quadratic control problem

In this section, we consider the discrete time formulation for the problem (1). For simplicity, let us introduce some useful notations. Let \( x \) denote a sequence \( x = \{x_k\} \subset \mathbb{R}^n \) for \( k = 0, 1, 2, \ldots, \infty \). We say that \( x \in l^2_n(N) \) if \( \sum_{i=1}^{\infty} \|x_i\|^2 < \infty \), where \( \|\cdot\| \) is a norm induced by an inner product \( \langle \cdot, \cdot \rangle \) in \( \mathbb{R}^n \). Let \( (x, u) \in l^2_n(N) \times l^2_m(N) \). We let the pair \( (x, u) \in Z \), where \( Z \) is a vector subspace of the Hilbert space \( l^2_n(N) \times l^2_m(N) \). Observe now the inner product in \( H \) has the following form:

\[ \langle (x, y), (u, \nu) \rangle_H = \sum_{k=0}^{\infty} \left\{ \langle x_k, u_k \rangle + \langle y_k, \nu_k \rangle \right\} \]

The vector subspace \( Z \) now takes the form:

\[ Z = \{(x, u) \in H : x_{k+1} = Ax_k + Bu_k, k = 0, 1, 2, \ldots, x_0 = 0\} \]

Here \( A \) is an \( n \) by \( n \) matrix, and \( B \) is an \( n \) by \( m \) matrix.

The solution to this particular class of optimization problems can be completely described by solving system of recurrence relations and the following well known discrete algebraic Riccati equation (DARE):

\[ K = A^T KA - A^T KB (I + B^T KB)^{-1} (A^T KB)^T + I \]

We assume that this DARE has a positive definite stabilizing solution \( K_{st} \). For sufficient conditions, see [4]. For simplicity, we let

\[ \overline{T} = (I + B^T KB) \]

and we also let

\[ L = B^T \overline{T}^{-1} B^T. \]

Then the next theorem describes the orthogonal complement \( Z^\perp \) of \( Z \).
Theorem 3.1. The orthogonal complement $\mathcal{Z}^\perp$ of $\mathcal{Z}$ is described as follows:

$$\mathcal{Z}^\perp = \{(A^T p_k - p_{k-1}, B^T p_k); \{p_k\} \in l_2^0(\mathbb{N}), \text{for } k = 0, 1, 2, \ldots\}.$$ 

i.e. given $(\psi_k, \varphi_k) \in H$, we have

$$\psi_k = x_k - (A^T p_k - p_{k-1}), \quad (15)$$

$$\varphi_k = u_k - B^T p_k, \quad (16)$$

where $x_k$ is the solution

$$x_{k+1} = (A^T - A^T KL)^T x_k + L\rho_{k+1} + B\mathcal{T}^{-1} \varphi_k, \quad (17)$$

$$u_k = -\mathcal{T}^{-1} BKA x_k + \mathcal{T}^{-1} B T \rho_{k+1} + \mathcal{T}^{-1} \varphi_k, \quad (18)$$

$$p_k = -K x_{k+1} + \rho_{k+1}, \quad (19)$$

and $\rho_k$ is a unique solution

$$\rho_k = (A^T + A^T KL)\rho_{k+1} - A^T KB\mathcal{T}^{-1} \varphi_k + \psi_k \quad (20)$$

belonging to $l_2^0(\mathbb{N})$.

In particular, $(\{x_k\}, \{u_k\}) \in \mathcal{Z}$, $-(\{A^T p_k - p_{k-1}\}, \{B^T p_k\}) \in \mathcal{Z}^\perp$ and consequently $\mathcal{Z}$ is a closed subspace in $H$ with

$$\pi_{\mathcal{Z}}(\{\psi_k\}, \{\varphi_k\}) = (\{x_k\}, \{u_k\}), \quad \pi_{\mathcal{Z}^\perp}(\{\psi_k\}, \{\varphi_k\}) = -(\{A^T p_k - p_{k-1}\}, \{B^T p_k\}).$$

Proof: Let $(\{x_k\}, \{u_k\}) \in \mathcal{Z}$ and $(\{A^T p_k - p_{k-1}\}, \{B^T p_k\}) \in \mathcal{Z}^\perp$
Consider the algebraic Riccati equation [DARE]

\[ K = A^T K A - A^T K L K A + I. \] (21)

We are now looking for \( p_k \) in the form

\[ p_{k-1} = -K x_k + \rho_k. \]

Hence by (20),

\[ p_{k-1} = -K x_k + (A^T - A^T K L) \rho_{k+1} - A^T K B T^{-1} \varphi_k + \psi_k \]

\[ = -K x_k + A^T \rho_{k+1} - A^T K L \rho_{k+1} - A^T K B T^{-1} \varphi_k + \psi_k. \]
Then it follows that
\[
A^T p_k - p_{k-1} = A^T [ -Kx_{k+1} + \rho_{k+1} ] - p_{k-1}
\]
\[
= -A^T Kx_{k+1} + A^T \rho_{k+1} + Kx_k - A^T \rho_{k+1} + A^T KL\rho_{k+1}
\]
\[
+ A^T KB^T \varphi_k - \psi_k
\]
\[
= -A^T K[(A^T - A^T KL)^T x_k + L\rho_{k+1} + B^T \varphi_k] + Kx_k + A^T KL\rho_{k+1}
\]
\[
+ A^T KB^T \varphi_k - \psi_k
\]
\[
= -A^T KA x_k + A^T KL Ax_k - A^T KL\rho_{k+1} - A^T KB^{-1} \varphi_k + Kx_k
\]
\[
+ A^T KL\rho_{k+1} + A^T KB^{-1} \varphi_k - \psi_k
\]
\[
= [-A^T kA + A^T KLKA + K] x_k - \psi_k
\]
By using now the fact that \( K_{st} \) satisfies (21), we obtain:
\[
A^T p_k - p_{k-1} = x_k - \psi_k
\]
which is (15). By using (18), we obtain
\[
u_k - B^T p_k = -B^T KA x_k - B^T KBu_k + B^T \rho_{k+1} + \varphi_k - B^T [ -Kx_{k+1} + \rho_{k+1} ]
\]
\[
= -B^T KA x_k - B^T KBu_k + B^T \rho_{k+1} + \varphi_k + B^T Kx_{k+1} - B^T \rho_{k+1}
\]
\[
= -B^T [Ax_k + Bu_k - x_{k+1}] + \varphi_k
\]
which is (16). Finally, by simple calculation, for \( x_k \) and \( u_k \) defined by (17), (18), we have \( x_{k+1} = Ax_k + Bu_k \). Then the proof is completed.

**Theorem 3.2.** Let \( h = (\psi, \varphi) = (\{\psi_k\}, \{\varphi_k\}) \) \( \in H \) and \( \rho \in L^2_0[0, \infty) \) is the function entering the decomposition (15) and (16) and described in (20). Then

\[
\| \pi_Z(h) \|^2 = \| C(B^T \rho + \varphi) \|^2,
\]
\[
\| \pi_{Z^\perp}(h) \|^2 = \| h \|^2 - \| C(B^T \rho + \varphi) \|^2.
\]

**Proof:** Let \( (\{y_k\}, \{\nu_k\}) \) \( \in Z \) and

\[
\Delta(y_k, \nu_k) = [\nu_k + B^T(KAy_k - \rho_{k+1}) - T^{-1} \varphi_k]^T T[\nu_k + B^T(KAy_k - \rho_{k+1}) - T^{-1} \varphi_k]
\]
\[
= \Delta_1 + \Delta_2 + \Delta_3
\]
where
\[
\Delta_1 = (\nu_k - T^{-1} \varphi_k)^T T(\nu_k - T^{-1} \varphi_k),
\]
\[ \Delta_2 = (KAy_k - \rho_{k+1})^T L(KAy_k - \rho_{k+1}), \]
\[ \Delta_3 = 2(KAy_k - \rho_{k+1})^T B(v_k - \varphi_k). \]

Hence,

\[ \Delta_1 = \nu_k^T \nu_k - 2\nu_k^T \varphi_k + \varphi_k^T \varphi_k \]
\[ = \nu_k^T (I + B^T KB)\nu_k - 2\nu_k^T \varphi_k + \varphi_k^T \varphi_k \]
\[ = \nu_k^T \nu_k + \nu_k^T B^T KB\nu_k - 2\nu_k^T \varphi_k + \varphi_k^T \varphi_k \]
\[ = (\nu_k - \varphi_k)^T (\nu_k - \varphi_k) - \varphi_k^T \varphi + (y_{k+1} - Ay_k)^T K(y_{k+1} - Ay_k) + \varphi_k^T \varphi_k \]
\[ = (\nu_k - \varphi_k)^T (\nu_k - \varphi_k) + y_{k+1}^T K y_{k+1} - 2y_{k+1}^T KAy_k + y_k^T A^T KAy_k - \varphi_k^T \varphi_k \]
\[ + \varphi_k^T \varphi_k \]
\[ \Delta_2 = y_k^T A^T KLKAy_k - 2y_k^T A^T KL \rho_{k+1} + \rho_{k+1}^T L \rho_{k+1} \]
\[ \Delta_3 = 2y_k^T A^T KB\nu_k - 2\rho_{k+1}^T B\nu_k - 2y_k^T A^T KB \varphi_k + 2\rho_{k+1}^T B \varphi_k \]
\[ = 2y_k^T A^T K(y_{k+1} - Ay_k) - 2\rho_{k+1}^T (y_{k+1} - Ay_k) - 2y_k^T A^T KB \varphi_k \]
\[ + 2\rho_{k+1}^T B \varphi_k \]
\[ = 2y_k^T A^T K y_{k+1} - 2y_k^T A^T KAy_k - 2\rho_{k+1}^T y_{k+1} + 2\rho_{k+1}^T Ay_k \]
\[ - 2y_k^T A^T KB \varphi_k + 2\rho_{k+1}^T B \varphi_k \]

\[ \Delta(y_k, \nu_k) = (\nu_k - \varphi_k)^T (\nu_k - \varphi_k) + y_{k+1}^T K y_{k+1} - 2y_{k+1}^T K Ay_k + y_k^T A^T K Ay_k \]
\[ - \varphi_k^T \varphi_k + \varphi_k^T \varphi_k + y_k^T A^T KLKAy_k - 2y_k^T A^T KL \rho_{k+1} \]
\[ + \rho_{k+1}^T L \rho_{k+1} + 2y_k^T A^T K y_{k+1} - 2y_k^T A^T K Ay_k - 2\rho_{k+1}^T y_{k+1} \]
\[ + 2\rho_{k+1}^T Ay_k - 2y_k^T A^T KB \varphi_k + 2\rho_{k+1}^T B \varphi_k \]
\[ = (\nu_k - \varphi_k)^T (\nu_k - \varphi_k) - y_k^T [A^T KA - A^T KLKA] y_k \]
\[ + 2y_k^T [A^T - A^T KL] \rho_{k+1} + y_{k+1}^T K y_{k+1} - \varphi_k^T \varphi_k + \varphi_k^T \varphi_k \]
\[ + \rho_{k+1}^T L \rho_{k+1} - 2\rho_{k+1}^T y_{k+1} - 2y_k^T A^T KB \varphi_k \]
\[ + 2\rho_{k+1}^T B \varphi_k \]

Since \( A^T KA - A^T KLKA = K - I \), and

\[ [A^T - A^T KL] \rho_{k+1} = \rho_k + A^T KB \varphi_k - \psi_k, \]

we have
\[ \Delta(y_k, \nu_k) = (\nu_k - \varphi_k)^T(\nu_k - \varphi_k) + y_k^T[K - I]y_k + 2y_k^T[\rho_k + A^TKB\mathbf{T}^{-1}\varphi_k - \psi_k] \\
+ y_{k+1}^TKy_{k+1} - \varphi_k^T\varphi_k + \varphi_k^T\mathbf{T}^{-1}\varphi_k + \rho_{k+1}^T\rho_{k+1} - 2\rho_{k+1}^Ty_{k+1} \]

\[ = (\nu_k - \varphi_k)^T(\nu_k - \varphi_k) + y_k^TKy_k + y_k^T\rho_k - 2y_k^T\psi_k \\
+ y_{k+1}^TKy_{k+1} - \varphi_k^T\varphi_k + \varphi_k^T\mathbf{T}^{-1}\varphi_k + \rho_{k+1}^T\rho_{k+1} - 2\rho_{k+1}^Ty_{k+1} \]

\[ + 2\rho_{k+1}^T\mathbf{T}^{-1}\varphi_k \]

\[ = (\nu_k - \varphi_k)^T(\nu_k - \varphi_k) + (y_k - \psi_k)^T(y_k - \psi_k) - \psi_k^T\psi_k - \varphi_k^T\varphi_k \\
+ \varphi_k^T\mathbf{T}^{-1}\varphi_k + \rho_{k+1}^T\mathbf{T}^{-1}\mathbf{T}^T\rho_{k+1} + 2\rho_{k+1}^T\mathbf{T}^{-1}\varphi_k \\
- y_k^TKy_k + y_k^T\rho_k - 2\rho_k^Ty_k - 2\rho_{k+1}^Ty_{k+1} \]

\[ = (\nu_k - \varphi_k)^T(\nu_k - \varphi_k) + (y_k - \psi_k)^T(y_k - \psi_k) - (\psi_k^T\psi_k + \varphi_k^T\varphi_k) \\
+ (\mathbf{T}^T\rho_{k+1} + \varphi_k)^T(\mathbf{T}^T\rho_{k+1} + \varphi_k) \\
- y_k^TKy_k + y_k^T\rho_k + 2\rho_k^Ty_k - 2\rho_{k+1}^Ty_{k+1} \]

Notice, since we fixed \( x_0 \), we can let \( y_0 = x_0 \) and then take summation of both sides:

\[ \sum_{k=0}^{\infty} \Delta(y_k, \nu_k) = \sum_{k=0}^{\infty}[(\nu_k - \varphi_k)^T(\nu_k - \varphi_k) + (y_k - \psi_k)^T(y_k - \psi_k)] - \sum_{k=0}^{\infty}[\psi_k^T\psi_k + \varphi_k^T\varphi_k] \]

\[ + \sum_{k=0}^{\infty}[(\mathbf{T}^T\rho_{k+1} + \varphi_k)^T(\mathbf{T}^T\rho_{k+1} + \varphi_k)] - x_0^TKx_0 + 2\rho_0^Tx_0 \]

Using the fact that \( x_0 = 0 \)

\[ \sum_{k=0}^{\infty} \Delta(y_k, \nu_k) = \|(y_k - \psi_k, \nu_k - \varphi_k)\|^2 - \|(\psi_k, \varphi_k)\|^2 + \|C(\mathbf{T}^T\rho_{k+1} + \varphi_k)\|^2 \]

By the definition of \( \Delta(y_k, \nu_k), \Delta(x_k, u_k) = 0. \) Therefore

\[ 0 = \|(y_k - \psi_k, \nu_k - \varphi_k)\|^2 - \|(\psi_k, \varphi_k)\|^2 + \|C(\mathbf{T}^T\rho_{k+1} + \varphi_k)\|^2 \]

Since \( \Delta(y_k, \nu_k) = \pi(\psi_k, \varphi_k) \)

\[ \|(\psi_k, \varphi_k)\|^2 = \|C(\mathbf{T}^T\rho_{k+1} + \varphi_k)\|^2 + \|\pi_Z(\psi_k, \varphi_k)\|^2 \]

Hence,

\[ \|\pi_Z(\psi_k, \varphi_k)\|^2 = \|C(\mathbf{T}^T\rho_{k+1} + \varphi_k)\|^2. \]
That completes the proof.

We are now in the position to compute the coefficients for the quadratic cost function $\varphi$. Since

$$\varphi(\lambda_1, \cdots, \lambda_m) = -\|h(\lambda)\|^2 + \|\pi_Z(h(\lambda) - c)\|^2 + \sum_{j=1}^{m} \lambda_j \|h_j\|^2,$$  \hspace{1cm} (24)

and $h(\lambda) = \sum_{i=1}^{m} \lambda_i h_i$, then we have

$$\|h(\lambda)\|^2 = \|\pi_Z h(\lambda)\|^2 + \|\pi_Z h(\lambda)\|^2$$  \hspace{1cm} (25)

$$= \|\pi_Z(\sum_{i=1}^{m} \lambda_i h_i)\|^2 + \|\pi_Z(\sum_{i=1}^{m} \lambda_i h_i)\|^2$$  \hspace{1cm} (26)

$$= \|\sum_{i=1}^{m} \lambda_i \pi_Z h_i\|^2 + \|\sum_{i=1}^{m} \lambda_i \pi_Z h_i\|^2.$$  \hspace{1cm} (27)

Similarly, as for $\|\pi_{Z^\perp} (h(\lambda) - c)\|^2$ we can apply the theorem 2. This would allow us to express the quadratic cost function (13) in terms of given data.

4 Concluding Remarks:

In this paper, we consider the discrete multi target linear quadratic control problem. We reduce the problem into quadratice programming over a simplex. The coefficients of the cost functions can be computed by knowing the descriptions of the orthogonal project onto the vector space $Z$ and the orthogonal complement $Z^\perp$ of the vector space $Z$.

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