Nonlinear Dynamic Analysis of the Human Ear

Structure based on the Natural Element Method

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Abstract

The modes of vibration of middle ear ossicular chain are mainly controlled both by the tympanic membrane, being inhomogeneous material, the elastic parameters of which gradually changes along the radial direction, and by the ligaments that will deform greatly in motion. Due to huge computation, existing distortion elements and difficulty in convergence when numerically simulating the ear structure through the method of finite element model (FEM), therefore, the natural element method (NEM), which is a novel method, is applied to derive the 3D governing equations for the ear structure. Programming the calculation process based on NEM, nonlinear dynamic analysis of the middle ear sound transmission is conducted. The validity of NEM is confirmed by comparing the results with the results of finite element calculation and the published experimental data. It is concluded from the results that NEM is feasible to simulate and calculate the ear structure made of the physiologically inhomogeneous material and it is more effective and precise to simulate the soft tissue of hyperelastically large deformation in live body using the natural element method.
Keyword: Natural element method; Numerical simulation; Amplitude of vibration; Ear structure; Vibration

1. Introduction

Life science is one of the foundation natural science most concerned about in the world. It has been a hot spot that mature mechanics and structural engineering analysis thought have been applied to research and analysis of body structure. The ear structure of human body is a typical conduction vibration structure excited by acoustic waves. Sound conduction is a complicated dynamic transmission process combined with solid dynamics and fluid dynamics.

Use mechanics principle to study the human ear structure began at the end of the 20th century. On one hand, analytic equation has been derived by mechanics theory, such as mathematical physics equation of basement membrane and cochlear wall [1], partial differential equation of ear cavity gas diffusion [2], tympanic membrane vibration equation and analysis method of artificial auditory ossicle detection [3], [4]. On the other hand, finite element simulation and analysis is hot research. In 1992, Wada et al [5] established finite element model including tympanic membrane and auditory ossicle chain. The partial middle ear simulation model was firstly established at that time. After then, Takuji and Gan [6],[7] established three dimension finite element model of middle ear and the whole ear by tissue slice from freezing temporal bone, and acoustic-solid-liquid coupling calculation was made by this model. With the development of imaging technology, people used CT scan and MRI nuclear magnetic resonance imaging to obtain anatomical structure data of middle ear in human physiological state, and constructed entity model in exaggerated scale [8]-[11]. Recently, the authors of this paper established three dimension finite element model of middle ear based on CT scan integrating with advanced synchrotron X-ray radiation imaging technology, and dynamics behaviour of the middle ear was analyzed [12].

Because of the fact that the tympanic membrane, ligaments and tendons are made of inhomogeneous and viscoelastic materials [13], [14], however, in the process of aforementioned finite element analysis, the material parameters of elements were defined as isotropic materials. Therefore, it is difficult to deal with inner parts and boundary of elements for inhomogeneous and viscoelastic materials. The main problems are that the modulus of adjacent elements change obviously and the material properties of inner parts of elements vary with the stress of elements, which will bring errors and reduce accuracy when calculating the results. However, the novel meshless numerical methods can deal with these problems. These meshless methods can greatly improve computing accuracy by assigning material parameters one by one to every node with no need of correlation information. In addition, the methods can make the material properties of elements vary
with stress. In view of this, a three dimensional meshless natural element numerical model is established in this paper. Based on the model, nonlinear dynamics analysis of human middle ear structure is performed. With the above consideration, the paper is organized as follows. In Section 2, we present the basic theory of 3-D natural element method. In Section 3, we derive the 3D governing equations for the ear structure. In Section 4, we establish the middle ear FE model and present corresponding boundary conditions and material properties. In Section 5, we program the calculation process based on NEM, nonlinear dynamic analysis of the middle ear sound transmission is conducted. The validity of NEM is confirmed by comparing the results with the results of finite element calculation and the published experimental data. In Section 6, we conclude the paper.

2. Theory of 3-D Natural Element Method

Establish space eight nodes Voronoi structure according to the theories of two dimensions Voronoi diagram. This paper is based on vertex node of the cube as the division of the node to the space and 8 nodes Voronoi structure is acquired, which is shown in Figure 1. A node X is interpolated in Figure 1 and Figure 1 is divided again, and then gains second-order Voronoi structure of interpolating X. Second-order Voronoi structure 1/8 is in Figure 2. The natural neighbor node of X is found based on hollow sphere rule----if X locates in circumscribing sphere of the tetrahedron, vertex node of the tetrahedron is natural neighbor node of X.

![Figure 1 First-order Voronoi structure](image-url)
Label 1, 2, 3, 4, 5, 6, 7, 8 are the natural neighbor node of X. X and its natural neighbor node constructs second-order Voronoi structure. Because space structure is complex, shape function is derived based on second-order Voronoi structure 1/8. Label 1-8 are space node in Figures 1, 2 and label A-H are vertex node of the Voronoi diagram of node 7. X and A coincide in Figure 1. X and its natural neighbor node construct second-order Voronoi structure. A, b and c are three vertex nodes of the Voronoi diagram of node X. The tetrahedron Aabc which relates node 7 is took out and is overlapping part between second-order Voronoi structure of X and first-order Voronoi structure of 7. Line A7 is vertical plate abc. According to symmetry of figure, overlapping part of other 7 nodes simulates node 7. Second-order Voronoi structure of X is composed of 8 tetrahedrons. The paper applies edge of polyhedron on constructing function. For example node 7, the process of constructing displacement function is explained.

\[ \phi_i(x) = \frac{\alpha_i(x)}{\sum_{j=1}^{\alpha_j(x)}} \]  

(1)

\[ \alpha_i(x) = \frac{S_i(x)}{h_i(x)} \]  

(2)

Where \( S_i(x) = ac+ab+bc \) is the sum of which edge of Voronoi structure relate node 7, \( h_7(x) = A7 \) is the vertical distance between node X and plate abc (shown in Figure 2).

Shape function of node 7 is extended to Shape function of any neighbor node I in the domain:

\[ \phi_i(x) = \frac{\beta_i(x)}{\sum_{j=1}^{\beta_j(x)}} \]  

(3)

\[ \beta_i(x) = \frac{S_i(x)}{h_i(x)} \]  

(4)
Where $S_i(x)$ is the sum of which edge of Voronoi structure relate node $I$, $h_i(x)$ is the vertical distance between node $x$ and the relating plate of Voronoi diagram.

3. Governing Equation

Tympanic membrane functions as the key component to transmit the sound into vibration and thus the variation of its elastic modulus directly influence the sound transmission. In this paper, the constitutive equation proposed by TAO CHENG [14] is employed to investigate the effects of different material properties for tympanic membrane on sound transmission.

The constitutive relationship of tympanic membrane is given as follows [15]:

$$\frac{d\sigma}{d\lambda} = 0.88\lambda^{24.76} + 0.49\lambda^{-15.38} \quad (5)$$

Where $\sigma$ is the stress, $\lambda = 1 + \varepsilon$, and $\varepsilon$ is the strain.

Transform the form of equation (5) into the relation between elastic modulus and strain, we get:

$$E(\varepsilon) = \frac{d\sigma}{d\varepsilon} = 0.88(1 + \varepsilon)^{24.76} + 0.49(1 + \varepsilon)^{-15.38} \quad (6)$$

For simplification, expand $\varepsilon$ of equation (6) based on the power series expansion method, we obtain:

$$E(\varepsilon) = 0.88(1 + \varepsilon^2 + \varepsilon^3) + 0.49/(1 + \varepsilon^2 + \varepsilon^3) \quad (7)$$

Substitute equation (7) into the elastic matrix $D$:

$$D = \frac{E(\varepsilon)(1 - \nu)}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} 1 & \nu & \nu & 0 & 0 & 0 \\ 0 & 1 & \nu & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 - 2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 - 2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 - 2\nu \\ \end{bmatrix} \quad (8)$$

Balance equation

$$\sigma_{i,j}(x,t) + f_i(x,t) - \rho(x)u_{i,t}(x,t) - \mu u_{i,t}(x,t) = 0 \quad \text{in } \Omega \quad (9)$$

Geometric equation
\[ \varepsilon_{ij}(x,t) = \frac{1}{2} (u_{ij}(x,t) + u_{ji}(x,t)) \quad (\text{in } \Omega) \quad (10) \]

**Physical equation**

\[ \sigma_{ij}(x,t) = D_{ijkl} \varepsilon_{kl}(x,t) \quad (\text{in } \Omega) \quad (11) \]

**Boundary condition**

\[ u_i(x,t) = \overline{u}_i(x,t) \quad (\text{on } S_u) \quad (12) \]

\[ \sigma_{ij}(x,t) n_j = \overline{t}_i(x,t) \quad (\text{on } S_{\sigma}) \quad (13) \]

**Initial condition**

\[ u_i(x,t_0) = \overline{u}_i(x) \quad (\text{in } \Omega) \quad (14) \]

\[ u_{i,t}(x,t_0) = \overline{u}_{i,t}(x) \quad (\text{in } \Omega) \quad (15) \]

Where \( \sigma_{ij}(x,t) \) is stress components on time \( t \) and at point \( x \), \( f(x,t) \) is body components on time \( t \) and at point \( x \), \( \rho(x) \) is mass density at point \( x \), \( u_i(x,t) \) is displacement components on time \( t \) and at point \( x \), \( \varepsilon_{ij}(x,t) \) is strain components on time \( t \) and at point \( x \), \( D_{ijkl} \) is elastic coefficient matrix, \( \overline{u}_i(x,t) \) is displacement components on time \( t \) and at point \( x \), \( \overline{t}_i(x,t) \) is surface components on time \( t \) and at point \( x \). 

**Equivalent integral form of balance equation (9) and force boundary condition (13)** is expressed in the Galerkin form as follows:

\[ \int_{\Omega} (\sigma_{ij}(x,t) + f_i(x,t)) \delta u_i(x,t) d\Omega - \int_{\Omega} (\sigma_{ij}(x,t))_{ij} \delta u_i(x,t) d\Omega = 0 \quad (16) \]

\[ \int_{\Omega} \sigma_{ij}(x,t) \delta u_i(x,t) d\Omega \quad \text{is integration by parts to (16), substituted to physical equation (11) and equation (16) becomes:} \]

\[ \int_{\Omega} (\delta \varepsilon_{ij}(x,t) + \delta u_i(x,t)) \overline{u}_j(x,t) d\Omega + \delta u_i(x,t) \overline{u}_j(x,t) + \delta u_i(x,t) \overline{u}_j(x,t) d\Omega = \int_{\Omega} \delta u_i(x,t) f_i d\Omega \int_{\partial \Omega} u_i(x,t) \overline{t}_i(x,t) d\Omega \]

(17)
Nonlinear dynamic analysis of the human ear structure

Where \( \delta u(x,t) \) is displacement increment in any space.

When the domain is discreted by nem, the interpolation of the displacement \( u, v, w \) in any \( x \):

\[
u(x,t) = \sum_{i=0}^{n} \Phi_i(x) v_i(t)
\]

\[
w(x,t) = \sum_{i=0}^{n} \Phi_i(x) w_i(t)
\]

Where \( \Phi(x) \) is interpolation shape of node, \( x \) is space coordinate.

Substitute equation (18) to (20) into equation (17) gives:

\[
 Ku(t) + C \dot{u}(t) + M \ddot{u}(t) = F(t)
\]

Where:

\[
 M = \int_{\Omega} \rho \Phi^T \Phi d\Omega
\]

\[
 C = \int_{\Omega} \mu \Phi^T \Phi d\Omega
\]

\[
 K = \int_{\Omega} \rho B^T DB d\Omega
\]

\[
 F = \int_{\Omega} \Phi^T f(t) d\Omega + \int_{\gamma} \Phi^T T(t) ds
\]

Where \( M, C, K, F \) is respectively mass matrix, damping matrix, stiff matrix, loading vector.
4. Numerical Model

4.1 Establishment of The Middle Ear FE Model

Based on a normal healthy volunteer (a woman at age of 30)’s left ear specimen supplied by Zhongshan Hospital of Fudan University, CT scan was used to capture the relevant data of tympanic membrane. By combining with synchrotron radiation X ray imaging technology in Shanghai Synchrotron Radiation Facility (SSRF) of Chinese Academy of Sciences, further treatment is given to the image. Using the self-programming, the CT-scan data is quantized. And then three-dimensional finite element model of human ear structure (including external auditory canal gas, tympanic membrane, middle ear ossicular chain, ligament tendons of middle ear, and inner ear including scala vestibuli, scala tympani, and basement membrane with three dimensional helical structure) is established by using the finite element software PATRAN. The model was divided into grid, and its boundary conditions and the material parameters were defined, and a three-dimensional finite element model of human ear structure was obtained in NASTRAN (As is shown in Figure 3 and Figure 4).
4.2 Boundary Conditions and Material Properties

The nodes from the finite element model are adopted in the numerical model of the meshless natural element method to conduct calculation and analysis. That is to say, the nodes of the finite element model act as discrete nodes that are needed in the meshless method calculation. The boundary conditions and material properties of the numerical model for the meshless method are defined based on the following methods:

(1) Tympanic membrane
The discrete nodes of the tympanic membrane are divided into three parts: nodes of TM pars tensa; nodes of TM pars flaccida and nodes of tympanic annulus ligament. The material properties are given to the nodes of each part, as listed in Table 1. The nodes of tympanic annulus ligament and the common nodes of between TM pars tensa and TM pars flaccida are all provided with the same elastic modulus of ligaments. And the peripheral nodes of tympanic membrane annular ligament were fixed.

(2) Auditory ossicles
Three auditory ossicles of the ossicular chain possess the same elastic modulus. Therefore, every node has the same elastic modulus and the elements of local elastic matrix are constant. The node mass of the ossicular chain are specified according to the space Voronoi diagram in which the node mass are obtained by multiplying the volume of each node with its density.

(3) Connecting parts
The nodes of contact elements between Malleus and Incus are both firmly bonded with those of Malleus and Incus. The material properties for the nodes are defined the same modulus as the ossicle’s. The connecting elements between Incus and Stapes are all fixed with those of Incus and Stapes. The material properties for the elements are defined the same modulus as the ossicle’s. All the other nodes have the same elastic modulus of the Incudostapedial joint. In addition, the connecting elements mass are equally given to every node.

(4) Ligaments and tendons
The common nodes collecting the ossicles with both the ligaments and tendons are given the same material properties as the ossicles. All the other nodes possess respective material property.

(5) Boundary conditions
The edge nodes of tympanic membrane are fixed in all three directions. Another node, which is given the equivalent elastic modulus of the simplified cochlear, is added into the middle part between the two spring elements collecting with stapes footplate. The other two nodes of each spring element connect with stapes footplate and the parallel direction of the surface for oval window respectively.

Material properties and acoustic properties [6], [8], [13], [14] used in this paper are
shown in Table 1 and Table 2. Poisson’s ratios of every part are all 0.3. Through simulation tests, the damping coefficients of hearing system structure are all set as 0.5.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Density (kg/m³)</th>
<th>Young’s Modulus (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tympanic membrane (Pars tensa)</strong></td>
<td>1.2 × 10³</td>
<td>3.5 × 10⁷</td>
</tr>
<tr>
<td><strong>Tympanic membrane (Pars flaccida)</strong></td>
<td>1.2 × 10³</td>
<td>2.0 × 10⁷</td>
</tr>
<tr>
<td>The tympanic membrane malleus attachment at the umbo</td>
<td>1.2 × 10³</td>
<td>3.5 × 10⁷[11]</td>
</tr>
<tr>
<td>The tympanic membrane malleus attachment at malleus handle</td>
<td>1.2 × 10³</td>
<td>3.5 × 10³[11]</td>
</tr>
<tr>
<td>Malleus head</td>
<td>2.55 × 10³</td>
<td>1.41 × 10¹⁰</td>
</tr>
<tr>
<td>Malleus Neck</td>
<td>4.53 × 10³</td>
<td>1.41 × 10¹⁰</td>
</tr>
<tr>
<td>Malleus Handle</td>
<td>3.7 × 10³</td>
<td>1.41 × 10¹⁰</td>
</tr>
<tr>
<td>Incudomalleolar Joint</td>
<td>3.2 × 10³</td>
<td>1.41 × 10¹⁰</td>
</tr>
<tr>
<td>Incus Body</td>
<td>2.36 × 10³</td>
<td>1.41 × 10¹⁰</td>
</tr>
<tr>
<td>Incus Short process</td>
<td>2.26 × 10³</td>
<td>1.41 × 10¹⁰</td>
</tr>
<tr>
<td>Incus Long process</td>
<td>5.08 × 10³</td>
<td>1.41 × 10¹⁰</td>
</tr>
<tr>
<td>Incudostapedial Joint</td>
<td>1.2 × 10³</td>
<td>6.0 × 10⁵</td>
</tr>
<tr>
<td>Stapes</td>
<td>2.2 × 10³</td>
<td>1.41 × 10¹⁰</td>
</tr>
<tr>
<td>Superior mallear ligament</td>
<td>2.5 × 10³</td>
<td>4.9 × 10⁶</td>
</tr>
<tr>
<td>Lateral mallear ligament</td>
<td>2.5 × 10³</td>
<td>6.7 × 10⁶</td>
</tr>
<tr>
<td>Anterior mallear ligament</td>
<td>2.5 × 10³</td>
<td>2.1 × 10⁷</td>
</tr>
</tbody>
</table>
Table 1 (continued) Material properties of the FE model

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's Modulus (GPa)</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superior incudal ligament</td>
<td>2.5×10³</td>
<td>4.9×10⁴</td>
</tr>
<tr>
<td>Posterior incudal ligament</td>
<td>2.5×10³</td>
<td>6.5×10⁶</td>
</tr>
<tr>
<td>Tensor tympani tendon</td>
<td>2.5×10³</td>
<td>8.7×10⁶</td>
</tr>
<tr>
<td>Stapedial tendon</td>
<td>2.5×10³</td>
<td>5.2×10⁶</td>
</tr>
<tr>
<td>Oval window</td>
<td>1.2×10³</td>
<td>5.5×10⁶</td>
</tr>
<tr>
<td>Round window membrane</td>
<td>1.2×10³</td>
<td>3.5×10⁵</td>
</tr>
<tr>
<td>Basilar membrane</td>
<td>1.0×10³</td>
<td>2.0×10⁵</td>
</tr>
</tbody>
</table>

Table 2 Acoustic properties of ear components

<table>
<thead>
<tr>
<th>Structure</th>
<th>Density (kg/m³)</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.21</td>
<td>340</td>
</tr>
<tr>
<td>Perilymphatic fluid</td>
<td>1000</td>
<td>1400</td>
</tr>
</tbody>
</table>

5. Numerical Simulation and Calculation

In this paper, 90 dB SPL (0.632 Pa) was applied on the surface of tympanic membrane and we analyze frequencies response in order to obtain the model-predicted frequencies response curves of the displacements at umbo and stapes footplate, and further a comparison between the computational curve from NEM and the experimental data from the fresh temporal bone specimen [7] was made, shown in Figure 5 and Figure 6.

![Figure 5. Displacement comparison of Umbo.](image-url)
As shown in Figure 5 and Figure 6, comparing the computational curve from NEM with the experimental curve, there is a good agreement in the trend and distribution. This shows that the method presented in this paper is correct and credible.
As is shown in Figure 5 and Figure 6, after introducing nonlinear constitutive relationship of soft tissue, the frequency response curve of the umbilical region of tympanic membrane and stapes footplate have not noticeable change in the low frequencies, whereas in the high frequencies, the curves vary with the elastic modulus of tympanic membrane and fluctuate. The calculated results are identical to the conclusions of medical analysis [15]. Comparison between the computational curve from classical FEM and the experimental data from the fresh temporal bone specimen [7] was made, shown in Figure 7 and Figure 8. In the result of FEM, the frequency response curve of both Umbo and Stapes differ largely from experimental data in the high frequencies, which imply that nonlinear and large deformed ligaments’ element distortion leading to not reflective actual displacement.

6. Conclusion

The meshless natural element method (NEM ) developed in this paper could compute the frequency response curve which better fitted with experimental data than that obtained by finite element method (FEM). This demonstrates that NEM better simulates inhomogeneous soft tissue of hyperelastically large deformation in live body than FEM does.

According to the analysis of numerical simulation results, if elastic modulus of soft tissue is constant in the process of calculation iteration, the frequency response curve in the high frequencies would decreases linearly. However, when considering the actual material properties of biological material and the elastic modulus of tympanic membrane varying with strain, the amplitudes of Umbo and Stapes in the high frequencies will experience oscillation which reflects the characteristic of wave. This result is well aligned with experimental results and clinical analysis. This further suggests that it is more preferable using natural element method to simulate middle ear structure in order to reflect vibration performance of middle ear structure. Meanwhile, it suggests that the sound wave make tympanic membrane, which is a kind of nonhomogeneous material, produce nonlinear vibration mode.

The NEM presented in this paper can precisely calculate the frequency response of sound transmission for the human ear structure, and effectively simulate dynamic behavior in living organism.

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All the authors in this paper state that they do not possess any of the financial interests and do not have any conflicts of interest.

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