A New Coalition Formation Based on Reservation Prices and Locations of Buyers

Santit Narabin

Software Systems Engineering Laboratory
Department of Computer Science, King Mongkut’s Institute
of Technology Ladkrabang, Bangkok, Thailand
S9062901@kmitl.ac.th

Veera Boonjing

National Centre of Excellence in Mathematics
PERDO, Bangkok, Thailand
Kbveera@kmitl.ac.th

Laor Boongasame

Department of Computer Engineering
Bangkok University, Bangkok, Thailand
Laor.b@bu.ac.th

Abstract

This paper defines a coalition as a cluster of buyers in dimensional space of reservation price and location. It defines a new buyer utility accounting for discount and traveling costs. A new coalition with nonnegative utility is proposed to maximize a number of successful buyers. This nonnegative utility coalition also assures that a winner does not have to pay more to subsidize others. Simulation results confirm that performance of the new scheme is significantly better than of a random one and is as good as a reservation price based coalition.

Keywords: GroupBuyMultiple, electronic commerce, group-buying, coalition formation, buyer coalition
Table 1: An example of price schedule

<table>
<thead>
<tr>
<th>Volume (Unit)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>≥5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price($)</td>
<td>100</td>
<td>96</td>
<td>92</td>
<td>88</td>
<td>84</td>
</tr>
</tbody>
</table>

1 Introduction

A Buyer coalition is a group of buyers who join together to purchase a deep discount item. It has become increasing attention because it gives benefits to both buyers and sellers. Buyers are able to improve their bargaining power and negotiation, and sellers can benefit from selling their items in large quantity. Therefore, there are a number of buyer coalition schemes in the literature (e.g., [1], [2], [3], [4], [6], [7], [8], [9], [10], [11], [12], [13], [14]) aiming to maximizing total discount of coalitions. However, these schemes form coalitions using only price information such as seller offered prices and buyer reservation prices. They exclude additional cost customers have to pay to get items such as delivery cost and travelling cost. In a real situation, these costs could make buyers give up coalitions. In addition, a coalition aims at maximizing number of successful buyers would attract buyers more than the ones maximizing their total discounts. This paper defines a new buyer utility accounting for both reservation price and location of buyer. It proposes a new coalition to maximize number of successful buyers by giving a coalition with nonnegative utility. This assures that successful buyers of this coalition do not have to pay more to subsidize other winners.

The article is organized as follows: Section 2 describes a new coalition scheme. Section 3 outlines a simulation study to evaluate the proposed scheme. The conclusion is drawn in Section 4.

2 A New Coalition Scheme

2.1 Scenario

Suppose there are many theatres with a number of available seats shortly before screening of a movie. They are members of an agent responsible for forming buyer coalition. A manager of each theatre will send a price schedule, as shown in Table 1, to the agent. Assume that all theatres share the common price schedule. The agent will distribute this schedule to members who are interested in joining together to purchase discount tickets.

The price schedule of Table 1 shows price per unit based on number of sold tickets. For example, buyers pay at $92 for a ticket when they buy 3 tickets together. Suppose there are five members (buyers) interested in joining
Table 2: Reservation prices of buyers

<table>
<thead>
<tr>
<th>Buyers</th>
<th>Reservation price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theatre 1</td>
</tr>
<tr>
<td>b₁</td>
<td>88</td>
</tr>
<tr>
<td>b₂</td>
<td>84</td>
</tr>
<tr>
<td>b₃</td>
<td>97</td>
</tr>
<tr>
<td>b₄</td>
<td>81</td>
</tr>
<tr>
<td>b₅</td>
<td>85</td>
</tr>
</tbody>
</table>

a coalition, each buyer will send the agent his/her reservation price, the price he/she is willing to pay for an item in different theatres. Table 2 shows, for example, that buyer b1 is willing to spend $88, $94, and $82 for a ticket of theatre 1, theatre 2, and theatre 3, respectively.

The agent detects positions of all buyers from locations of their mobile devices. It then forms a buyer coalition based on buyers’ reservation prices and their locations. This is under a reasonable assumption that buyers’ reservation prices and their locations equally influence buyer decision making.

2.2 Model

Let $T = \{ t_1, t_2, \ldots , t_l \}$ be a set of theatres showing a movie. A descending function $P_{t_k} : a_{t_k} \rightarrow \text{real number}$ is a price schedule function of the theatre $t_k$. Then $P_{t_k}(a_{t_k})$ is a unit price that the theatre $t_k$ would expect from selling a bundle of size ‘$a_{t_k}$’ of the movie tickets. Upon receiving the price schedule, each buyer $b_i$ who wants to purchase a ticket will send the reservation price $r_{i k}$ to the agent system and the system will detect his/her location. A utility of buyer $b_i$ of theatre $k$, $(u^i_k)$ is defined as

$$u^i_k = r^i_k - P|(C_k)| - costD^i_k$$  \hspace{1cm} (1)

where $P|(C_k)|$ denotes the unit price when $|C_k|$ of tickets sold together and $costD^i_k$ denotes the travelling cost of buyer $b_i$. This cost is defined as

$$costD^i_k = \begin{cases} |d^i_k - F(C_k)|/\beta, & \text{if } d^i_k > F(C_k) \\ 0, & \text{otherwise} \end{cases}$$  \hspace{1cm} (2)

where $d^i_k$ is the distance between the $i^{th}$ buyer and the $k^{th}$ theatre, $F(C_k)$ is the mean of distance of the coalition’s members, and $\beta$ is the cost per travelling unit. The utility of coalition is defined as

$$v(C_k) = \Sigma_{b_i \in C_k} u^i_k$$  \hspace{1cm} (3)
The coalitions are formed using k-mean clustering. The winner coalition of the theatre $t_k$, called $C^*_k = W_k = \{w^1_k, w^2_k, \ldots, w^p_k\}$, is the coalition with total utilities at least zero. The surplus of coalition $X_k = \{x^1_k, x^2_k, \ldots, x^p_k\}$ is allocated to the winners based on their reservation prices and their locations. Finally, the agent system sends the final prices $F_k = \{f^1_k, f^2_k, \ldots, f^p_k\}$ back to the winners where $f^i_k = r^i_k - x^i_k$. Moreover, the reject messages will be sent to buyers without the coalition.

### 2.3 Method

The mechanism of the proposed coalition agent consists of the following three major steps.

**Step 1: Preparation of data**

The reservation price data and the location data of all buyers are transformed into a standard normal distribution, called z-score (with mean = 0 and standard deviation = 1), in order to adjust value on different scales to a common scale.

**Step 2: Determination of a buyer coalition structure**

The k-mean clustering method is used for selecting a buyer coalition structure which each buyer belongs to the any coalition or cluster with nearest properties. The number of cluster is assigned by k and we use similarity function to determine nearest between buyer and any coalition. The process of k-mean algorithm in this paper is adopted from Jain and Dubes [5] as shown in Figure 1.

**Algorithm 1: k-mean clustering algorithm**

| Input: | $B = \{b_1, b_2, \ldots, b_n\}$ be the set of buyers, $r^i_k$ is the reservation price of the $i^{th}$ buyer expected to the $k^{th}$ theatre, and $l_i(x_i, y_i)$ is the location of buyer $i^{th}$. |
| Output: | $C_k$ be the coalition of buyer of theatre $k^{th}$ |

1) Select initial centroids of each coalition  
2) Assign each buyer to closet cluster based on similarity.  
3) Compute new cluster centroids.  
4) Repeat steps 2 and 3 until cluster membership stabilizes.

**Figure 1:** The k-mean clustering algorithm
Step 3: Calculating Coalition Values

After the second step, a coalition of buyers is generated equal to the number of theatres. The utility of coalition is less than zero; the members who have the least utility of buyer are eliminated and then will re-calculate the utility of coalition once again. This process will be repeated until the utility of coalition is greater than or equal to zero. The remaining members are the winners. The processes of this step are described in Figure 2.

**Algorithm 2: Coalition’s Value Calculation**

**Input:** $B = \{b_1, b_2, ..., b_n\}$ be the set of buyers, $r_k^i$ is the reservation price of the $i^{th}$ buyer expected to the $k^{th}$ theatre, and $C_k$ be the coalition of buyer of theatre $k^{th}$.

**Output:** $G_k = \{b_k^1, b_k^2, ..., b_k^p\}$ be the set of winners for the $k^{th}$ theatre.

1. While $1 \leq k \leq K$
2. Calculate $F(C_k)$ from $F(C_k) = \frac{\text{MAX}(d_k^i) + \text{MIN}(d_k^i)}{2}$
3. Calculate $u_k^i$ from $u_k^i = r_k^i - P(|C_k|) - \text{costD}_k^i$ where
   \[
   \text{costD}_k^i = \begin{cases} 
   |d_k^i - F(C_k)|\beta, & \text{if } d_k^i > F(C_k) \\
   0, & \text{otherwise}
   \end{cases}
   \]
4. Calculate $v(C_k)$ from $v(C_k) = \sum_{b_i \in C_k} u_k^i$
5. If $v(C_k) < 0$, eliminate the lowest buyer utility $u_k^i$. Otherwise GOTO 4
6. $C_k \leftarrow G_k$
7. GOTO step 1
8. End

Figure 2: The coalition’s value calculation algorithm

The goal of the algorithm in Figure 2 is to maximize the number of buyers who got items instead of maximizing coalition total utility. This would attract buyers to join a coalition because of their high opportunities to get desired items. Moreover, sellers are able to sell a large amount of items as well.

3 Evaluation

This section is a simulation study to evaluate performance of the proposed mechanism (named *GroupBuyMultiple*) compared with a reservation price based mechanism and a reservation price and location based mechanism with random method.

**A Reservation Price Based Mechanism:** The buyer coalition is formed by using k-mean clustering algorithm based on only reservation price information.
Table 3: Simulation parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td>The number of buyers</td>
<td>100</td>
</tr>
<tr>
<td>The reservation price</td>
<td>[80, 99]</td>
</tr>
<tr>
<td>The location of buyers (X, Y)</td>
<td>[1, 20]</td>
</tr>
<tr>
<td>Discount Rates (DR)</td>
<td>0.2, 0.4, 0.6, 0.8, 1.0</td>
</tr>
<tr>
<td>Discount Rates (β)</td>
<td>1, 2, 4, 6, 8, 10</td>
</tr>
</tbody>
</table>

A Reservation Price and Location Based Mechanism with Random Method: The buyer coalition is formed by using random method based on both reservation price and location information.

This simulation study measures the utility of buyer coalition and the number of successful buyers. These two performance measures are averaged from 100 experiments for various simulation parameters. Table 3 shows the set of simulation parameters in our evaluation.

This simulation study measures the utility of buyer coalition and the number of successful buyers. These two performance measures are averaged from 100 experiments for various simulation parameters. Table 3 shows the set of simulation parameters in our evaluation.

The first study is to compare performance of the proposed scheme and the Random scheme. Figure 3 and 4 are results of the study. Figure 5 and 6 show results of the second study to compare performance of the proposed scheme and the reservation price based scheme.

![Figure 3](image1.png)

**Figure 3:**

a): The number of successful buyers at different discount rates
b): The total utility of buyer coalition at different discount rates

From figure 3 a) and 3 b), it is found that the proposed scheme and the Random schemes will increase a number of successful buyers as discount rate...
New coalition formation

increases. Moreover, a number of successful buyers of our scheme is more than those of the random scheme at most of discount rates.

From figure 3 b), it is found that the proposed scheme and the Random schemes will increase coalition total utility as discount rate increases. Moreover, coalition total utility of our scheme is more than those of the random scheme at all of discount rates.

Figure 4: a): The number of successful buyers at different costs per walk
b): The total utility of buyer coalition at different costs per walk

Figure 4 a) shows a number of successful buyers of the proposed scheme and the random scheme at different costs per walk. Figure 4 b) shows coalition total utility of the proposed scheme and the random scheme at different costs per walk.

It is observed that both schemes decrease in a number of successful buyers as cost per walk increases. Additionally, it is shown that coalition total utility of both schemes decreases as cost per walk increases. Hence, increasing traveling cost causes decreasing worth of products because buyers have to pay more to get the products. However, a number of successful buyers and coalition total utility of our scheme is more than those of the random scheme at all costs per walk.

Figure 5 a) shows a number of successful buyers at different discount rates of the proposed scheme and the reservation price based scheme. Figure 5 b) shows coalition total utility at different discount rates of the proposed scheme and the reservation price based scheme.

From figure 5 a), it is found that both schemes give a number of successful buyers higher as discount rate increases. And figure 5 b), it is found that coalition total utility of both schemes increases as discount rate increases.

Although the proposed scheme gives a number of successful buyers less than those of the reservation price based scheme at all discount rates, coalition total utility of the proposed scheme is close to of the reservation price based
scheme. The reasons of such results was setting group by reservation price based scheme makes high reservation price buyers get high opportunity to be successful buyers with high discounts.

Figure 6 shows a number of successful buyers and coalition total utility of the proposed scheme and the reservation price based scheme at different costs per walk.

From figure 6 a), it is found that a number of successful buyers of our proposed scheme lower than of the reservation price based scheme because of
coalition of high reservation price buyers. Therefore, of the reservation price based scheme gives a number of successful buyers higher than of the proposed scheme. Figure 6 b) shows that coalition total utility of our scheme agrees very well with the reservation price based scheme.

In addition, it is found that the more cost per walk increases, the more both of a number of successful buyers and coalition total utility decreases. This is consistent with increment of discount rate.

Although figures 5 and 6 are found that both a number of successful buyers and coalition total utility of the GroupBuyMultiple scheme are less than the Reservation price based scheme, but in practice we could not consider only buyer reservation price information to avoid coalition cancelation of successful buyers. Hence, the simulation results of the propose scheme agrees very well with the real-market.

4 Conclusions

A new coalition scheme for forming buyer coalition based on buyers’ reservation prices and locations is proposed in this article to maximize a number of successful buyers as well as to assure no winners’ subsidy. It achieves its goals by defining a new buyer utility accounting for both discount and traveling costs and by forming a coalition as a cluster with nonnegative utility. Our simulation results confirm that the proposed scheme outperforms a random scheme in terms of a number of successful buyers and total utility of coalitions. Compared with a reservation price based scheme, it is found that performance of the proposed scheme is not significantly different from of a reservation price based scheme.

References


Received: May 18, 2013