Estimation of Parameters

on the BS-BHM Updated Model

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Abstract

This paper will introduce BS-BHM Updated model and show the lognormal distribution for BS-BHM Updated Model. Estimation of two parameters on the BS-BHM Updated model has been done in this paper. It used Monte Carlo estimation method and the method of moments to estimate two parameters on the BS-BHM Updated model. The Empirical results of the two method of estimation obtained similar results for the volatility parameter. While the results
of parameter estimation for the parameter of information flow rate obtained almost the same results.

**Keywords:** BS-BHM Updated Model, Lognormal Distribution, Monte Carlo Estimation, The method of moments

### 1. Introduction

The main purpose of this paper is to introduce and study about BS-BHM Updated model. The BS-BHM Updated model is developed based on the Black Scholes model from information-based perspective by Brody Hughston Macrina that it is updated in the results of Gaussian Integrals, more specifically on the analysis of algebra trick of completing square.

This paper studies about BS-BHM Updated model as the underlying asset pricing model in finance. It is started from the information-based approach asset pricing model constructed by Brody Hughston Macrina, so that it is called BHM model or BHM approach.

Brody Hughston Macrina built the asset pricing model for case cash flow is the payout of the associated dividend with equity. Explicitly, the asset pricing model is presented as follows,

$$S_t = P_{rt} E^Q [D_T | \mathcal{F}_t]$$  \hspace{1cm} (1.1)

$S_t$ is the value of cash flows at time $t$, $0 \leq t < T$ from asset that payout single dividend $D_T$ at time $T$. In equation (1.1), $P_{rt}$ represents the discount factors that it is to be equal to $e^{-r(T-t)}$ with $r$ is the interest rates. Then $Q$ is the risk neutral probability, and $\mathcal{F}_t$ is the market information filtration.

Modeling the information flows is based on an assumption that the information about dividends which is available in market is contained by the process $\{\xi_t\}_{0 \leq t \leq T}$ defined by:

$$\xi_t = \sigma t D_T + \beta t^T$$  \hspace{1cm} (1.2)
{ξ_t} is a market information process. The market information process is composed from two parts, they are σD_T which refers to the true information about dividends and {β_t}_{0≤t≤T} which refers to a standard Brownian Bridge on interval [0, T]. In the formula of asset pricing model by Brody Hughston Macrina in equation (1.1) above, if random variable D_T is equal to x having continuous distribution then,

\[ E^Q[D_T|F_t] = E^Q[D_T|\xi_t] = \int_0^\infty x \pi_t(x) \, dx \]  (1.3)

where

\[ \pi_t(x) = \frac{d}{dx} Q(D_T \leq x|\xi_t) \]  (1.4)

By using Bayes formula [2], \( \pi_t(x) \) is presented in [3, 4, 5, 8] as follows

\[ \pi_t(x) = \frac{p(x)p(\xi_t|D_T=x)}{p(\xi_t)} \]  (1.5)

and the final result of the BHM model or the BHM approach,

\[ S_t = P_{\xi_t} \frac{\int_0^\infty p(x) \exp\left(\frac{T-t}{\sigma^2} n(x)\right) \, dx}{\int_0^\infty p(x) \exp\left(\frac{T-t}{\sigma^2} n(x)\right) \, dx} \]  (1.6)

Brody Hughston Macrina also built the other concept for the asset pricing model that is derived from the formula of equation (1.1) for a specific condition where it is a limited-liability asset which pays no interim dividends and at time T it is sold off for the value S_T. S_T is log-normally distributed and has the form of

\[ S_T = S_0 \exp\left(\sigma \sqrt{T} X_T + \nu \sqrt{T} X_T\right) \]  (1.7)

where \( S_0, \sigma, \nu \) are given constants and \( X_T \) is a standard normally distributed random variable. The corresponding information process is given by

\[ \xi_t = \sigma_t X_T + \beta_t \]  (1.8)

The price process \{S_t\}_{0≤t≤T} is obtained from:

\[ S_t = P_{\xi_t} E^Q(\Delta_t(X_T)|\xi_t) \]  (1.9)

Then for \( t < T \), the equation \( S_t \) results:

\[ S_t = P_{\xi_t} \int_{-\infty}^{\infty} \Delta_t(x) \pi_t(x) \, dx \]  (1.10)

And by the Bayes formula, it is obtained \( \pi_t(x) \) as follows
\[
\pi_{t,T}(x) = \frac{p(x) \exp \left[ \frac{T}{T-t} \left( \sigma x \xi_t - \frac{1}{2} \sigma^2 x^2 t \right) \right]}{\int_{-\infty}^{\infty} p(x) \exp \left[ \frac{T}{T-t} \left( \sigma x \xi_t - \frac{1}{2} \sigma^2 x^2 t \right) \right] dx} \quad (1.11)
\]

In this case, \( S_T \) plays the role of single cash flow \( \Delta_T(x) \) for \( X_T = x \).

So, it is obtained the equation \( S_t \) as follows

\[
S_t = P_{T-t} \int_{-\infty}^{\infty} S_0 \exp \left( rT - \frac{1}{2} \sigma^2 T + \sqrt{T} x \right) \frac{p(x) \exp \left[ \frac{T}{T-t} \left( \sigma x \xi_t - \frac{1}{2} \sigma^2 x^2 t \right) \right]}{\int_{-\infty}^{\infty} p(x) \exp \left[ \frac{T}{T-t} \left( \sigma x \xi_t - \frac{1}{2} \sigma^2 x^2 t \right) \right] dx} dx \quad (1.12)
\]

Because \( X_T \) is assumed to be standard normally distributed then

\[
p(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2} x^2) \quad (1.13)
\]

To follow the Gaussian Integrals [8, 12] then \( S_t \) becomes

\[
S_t = P_{T-t} S_0 \exp \left( rT - \frac{1}{2} \sigma^2 T + \sqrt{T} x \right) \frac{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-\frac{1}{2} x^2) \exp \left[ \frac{T}{T-t} \sigma x \xi_t + \sqrt{T} x - \frac{1}{2} \frac{T}{T-t} \sigma^2 x^2 t \right] dx}{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-\frac{1}{2} x^2) \exp \left[ \frac{T}{T-t} \sigma x \xi_t - \frac{1}{2} \frac{T}{T-t} \sigma^2 x^2 t \right] dx} \quad (1.14)
\]

By using Gaussian integrals, the equation of asset pricing model \( S_t \) is given below

\[
S_t = S_0 \exp \left( rT - \frac{1}{2} \sigma^2 T + \frac{\sigma \sqrt{T} x}{\tau} + \frac{\sigma T x}{\sigma^2 T + 1} \xi_t \right) \quad (1.15)
\]

where \( \tau = \frac{rT}{T-t} \). Successive steps to obtain the model in equation (1.15) can be seen [9]. Furthermore, the model in equation (1.15) is called the BS-BHM Updated model.

### 2. Lognormal Distribution of The BS-BHM Updated Model

In this section, the paper try to show that the BS-BHM Updated model in equation (1.15) follows the lognormal distribution. The initial step is to determine the distribution of random variable \( \xi_t \). Random variable \( \xi_t \) represents the sum of two parts that is \( \sigma t X_T + \beta_{iT} \). The distribution of random variable \( \sigma t X_T \) can be determined based on variable transformation theorem [1]. \( X_T \) is the standard normally distributed random variable i.e. \( X_T \sim N(0, 1) \) and it has the probability density function as follows

\[
f(x_T) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad (2.1)
\]
Suppose the random variable \( Y = \sigma t X_T \), then density function \( g(y) \) is

\[
g(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{y}{\sigma t} \right)^2}
\]

(2.2)

It means that \( Y = \sigma t X_T \) is normally distributed with mean = 0 and variance \( (\sigma t)^2 \) or it can be denoted as \( Y = \sigma t X_T \sim N(0, \sigma^2 t^2) \). \( \beta_{it} \) in the information flow model assumed is the standard Brownian Bridge process on the interval of time \([0, T]\) and in fact a Gaussian process has the mean = 0 and variance \( \frac{t(T-t)}{T} \) or it can be denoted as \( Y = \sigma t X_T \sim N(0, \frac{t(T-t)}{T}) \). The statement about variance \( \beta_{it} \) can be determined through the definition of Brownian motion [11]. The definition of variance \( \beta_{it} \) is a standard Brownian Bridge over the time interval \([0, T]\), so its value is zero at time 0 and \( T \), then \( \beta_{it} \) can be presented as

\[
\beta_{it} = W(t) - \frac{t}{T} W(t)
\]

(2.3)

From equation (2.3), it can be determined the variance \( \beta_{it} \) over the risk neutral probability density. Successive steps to obtain the variance \( \beta_{it} \) can be determined as follows

\[
E^Q(\beta_{it}) = E\left\{ W(t) - \frac{t}{T} W(T) \right\}
\]

\[
= E[W(t)] - \frac{t}{T} E[W(T)]
\]

\[
= 0 - \frac{t}{T} \cdot 0 = 0
\]

(2.4)

\[
Var^Q(\beta_{it}) = E^Q(\beta_{it}^2) - \{E^Q(\beta_{it})\}^2
\]

\[
= E^Q\left\{ \left[ W(t) - \frac{t}{T} W(T) \right]^2 \right\}
\]

\[
= \frac{t}{T} + \frac{t^2}{T^2}
\]

(2.5)

To determine the distribution of the random variable \( \xi_t \), it used the definition of moment generating function (MGF) [1]. It can be referred as belows

Suppose the equation (1.8) is
Y = X_1 + X_2, where X_1 \sim N(0, a) with a = \sigma^2 t^2

X_2 \sim N(0, b) with b = \frac{t(T-t)}{T}

MGF of the random variable Y is

\[ M_Y(t) = E(e^{tY}) = e^{\frac{(a+b)t^2}{2}} \]

Thus it can be concluded that Y \sim N(0, a+b). It means that \( \xi_t \) is normally distributed with mean = 0 and variance = \( \sigma^2 t^2 + \frac{t(T-t)}{T} \) or \( \xi_t \sim N\left(0, \sigma^2 t^2 + \frac{t(T-t)}{T}\right) \).

Finally, lognormal distribution of random variable \( \frac{S_t}{S_0} \) in model (1.15) is determined from the form of

\[ \frac{S_t}{S_0} = \exp(A + b\xi_t) \quad \text{and} \quad \log \frac{S_t}{S_0} = A + b\xi_t \]  \hspace{2cm} (2.7)

where \( A = rt - \frac{1}{2} \frac{\sigma^2 t}{\sigma^2 t + 1} \sqrt{2} T \) and \( b = \frac{\sigma T \sqrt{T}}{t(\sigma^2 t + 1)} \).

From the distribution of \( \xi_t \) above then the probability density of the random variable \( \xi_t \) is

\[ f(\xi_t) = \frac{1}{\sqrt{2\pi} \sigma^2 t^2 + \frac{t(T-t)}{T}} \exp \left\{ -\frac{1}{2} \frac{(\log \frac{S_t}{S_0} - A)^2}{\sigma^2 t^2 + \frac{t(T-t)}{T}} \right\} \]  \hspace{2cm} (2.8)

and by using the definition of random variable transformation [1] it can be obtained the new probability density for random variable \( \frac{S_t}{S_0} \), i.e. the multiplication of the probability density of random variable \( \xi_t \) and Jacobian \( \xi_t \). The probability density of random variable \( \frac{S_t}{S_0} \) is

\[ g\left(\frac{S_t}{S_0}\right) = \frac{1}{\sqrt{2\pi} B} \exp \left\{ -\frac{1}{2} \frac{(\log \frac{S_t}{S_0} - A)^2}{B^2} \right\} \]  \hspace{2cm} (2.9)

where \( A = rt - \frac{1}{2} \frac{\sigma^2 t}{\sigma^2 t + 1} \sqrt{2} T \) and

\[ B^2 = b^2 \left( \frac{\sigma^2 t^2 + \frac{t(T-t)}{T}}{t(\sigma^2 t + 1)} \right) = \left( \frac{\sigma T \sqrt{T}}{t(\sigma^2 t + 1)} \right)^2 \left( \frac{\sigma^2 t^2 + \frac{t(T-t)}{T}}{t(\sigma^2 t + 1)} \right) \]
3. The Method of Estimation on The BS-BHM Updated Model

The BS-BHM Updated model in equation (1.15) has lognormal distribution with the density function of random variable \( \frac{S_t}{S_0} \) is

\[
g \left( \frac{S_t}{S_0} \right) = \frac{1}{\sqrt{2\pi} B} \exp \left\{ -\frac{1}{2} \left( \frac{\log \frac{S_t}{S_0} - A}{B} \right)^2 \right\}
\]

It means that \( \log \frac{S_t}{S_0} \) has normal distribution with mean = \( A = \rho t - \frac{1}{\sigma^2 t^2 + 1} \) \( \sigma^2 \) \( t \) \( \frac{T(t+t)}{T} \) and variance = \( \sigma^2 \frac{T(t+t)}{T} \) or \( \log \frac{S_t}{S_0} \sim N(\mu, \sigma^2) \).

In BS-BHM Updated model, there are volatility parameter \( \nu \) and true information flow rate parameter \( \sigma \) can not be observed directly. This paper will discuss these two parameter estimation for the previous behaviour of the asset price. The estimation value of parameter \( \nu \) and \( \sigma \) arisen from this general procedure is called a historical volatility and information flow rate estimation.

3.1 Monte Carlo Estimation

Suppose that historical asset price data is available at equally spaced time values \( t_i = i \Delta t \), so \( S_{t_i} \) is the asset price at time \( t_i \). Defined \( U_i = \log \frac{S_{t_i}}{S_{t_{i-1}}} \) and \( \{U_i\} \) are independent [7]. To estimate the asset price volatility \( \nu \) and the information flow rate \( \sigma \) on the BS-BHM Updated model, it uses Monte Carlo approach as follows :

Suppose that \( t = t_n \) is the current time and that the \( M+1 \) is most current asset prices. \( \{S_{t_n-M'}, S_{t_n-M'+1}, \ldots, S_{t_n-1}, S_{t_n}\} \) is also available and by using the
corresponding log ratio data which is \( \left\{ U_{n+1-i} \right\}_{i=1}^{M} \) [7], then the sample mean and variance estimation are

\[
a_M = \frac{1}{M} \sum_{i=1}^{M} U_{n+1-i} \tag{3.2}
\]

and

\[
b_M^2 = \frac{1}{M-1} \sum_{i=1}^{M} \left( U_{n+1-i} - a_M \right)^2 \tag{3.3}
\]

Monte Carlo estimation method is done by comparing the sample mean with the mean of BS-BHM Updated model or by comparing the sample variance with the variance of BS-BHM Updated model [7] is

\[
a_M = \tau T - \frac{\sigma_t^2}{2} \frac{T+1}{\sigma^2 T+1} v^2 T \tag{3.4}
\]

then

\[
v^2 = \frac{2(rt-a_M)}{T} + \frac{2(rt-a_M)}{\sigma^2 T} \tag{3.5}
\]

and

\[
b_M^2 = \left( \frac{\sigma_t \sqrt{T}}{t(\sigma^2 T+1)} \right)^2 \left( \sigma^2 T^2 + \frac{T(T-t)}{T} \right) \tag{3.6}
\]

then

\[
v^2 = \frac{T(\sigma^2 T+1)^2 b_M^2}{\sigma^4 T^2 t + \sigma^2 T^2 (T-t)} \tag{3.7}
\]

For equation (3.5) and (3.7), it can written as

\[
\frac{T(\sigma^2 T+1)^2 b_M^2}{\sigma^4 T^2 t + \sigma^2 T^2 (T-t)} = \frac{2(rt-a_M)}{T} + \frac{2(rt-a_M)}{\sigma^2 T} \tag{3.8}
\]

By successive steps using algebra trick, it is obtained the final equation

\[
A_1 \sigma^4 + A_2 \sigma^2 + A_3 = 0 \tag{3.9}
\]

where \( A_1 = 1 - \frac{b_M^2}{2(rt-a_M)^2} \), \( A_2 = \frac{2(T-t)}{t^2} - \frac{b_M^2}{(rt-a_M)^2} \), and \( A_3 = \frac{T-t}{t^2} - \frac{b_M^2}{2(rt-a_M)^2} \).

Suppose \( x = \sigma^2 \) then it is obtained the quadratic equation

\[
A_1 x^2 + A_2 x + A_3 = 0 \tag{3.10}
\]

The solution of equation (3.10) is
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\[ x_1 = \frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1} \]  
\[ (3.11) \]

and

\[ x_2 = \frac{-A_2 - \sqrt{A_2^2 - 4A_1A_3}}{2A_1} \]  
\[ (3.12) \]

Because \( x = \sigma^2 \) then

\[ \sigma_1 = \sqrt{\frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1}} \]  
\[ (3.13) \]

and

\[ \sigma_2 = \sqrt{\frac{-A_2 - \sqrt{A_2^2 - 4A_1A_3}}{2A_1}} \]  
\[ (3.14) \]

Substitution \( \sigma_1^2 \) and \( \sigma_2^2 \) to \( \nu^2 \) in equation (3.5) results in

\[ \nu_1 = \sqrt{\frac{2(\tau - a_M)}{T} + \frac{4(\tau - a_M) - 2b^2_i}{\tau T \left(-A_2 + \sqrt{A_2^2 - 4A_1A_3}\right)}} \]  
\[ (3.15) \]

and

\[ \nu_2 = \sqrt{\frac{2(\tau - a_M)}{T} + \frac{4(\tau - a_M) - 2b^2_i}{\tau T \left(-A_2 - \sqrt{A_2^2 - 4A_1A_3}\right)}} \]  
\[ (3.16) \]

It means the estimators of \( \sigma \) are

\[ \hat{\sigma}_1 = \sqrt{\frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1}} \]  
\[ (3.17) \]

and

\[ \hat{\sigma}_2 = \sqrt{\frac{-A_2 - \sqrt{A_2^2 - 4A_1A_3}}{2A_1}} \]  
\[ (3.18) \]

And the estimators of \( \nu \) are

\[ \hat{\nu}_1 = \sqrt{\frac{2(\tau - a_M)}{T} + \frac{4(\tau - a_M) - 2b^2_i}{\tau T \left(-A_2 + \sqrt{A_2^2 - 4A_1A_3}\right)}} \]  
\[ (3.19) \]

And
\[ \hat{\nu}_2 = \sqrt{\frac{2(\tau - a_M)}{T} + \frac{4(\tau - a_M) - 2 b_M^2}{4T(\sqrt{A_2^2 - 4 A_1 A_3})}} \] (3.20)

3.2. The Method of Moments

Analogous to Monte Carlo estimation method, suppose that historical asset price data is available at equally spaced time values \( t_i = i \Delta t \), so \( S_{t_i} \) is the asset price at time \( t_i \). Defined \( U_i = \log \frac{S_{t_i}}{S_{t_{i-1}}} \) and \( \{U_i\} \) are independent [7]. Estimating parameters of asset price volatility \( \nu \) and the information flow rate \( \sigma \) of BS-BHM Updated model using the method of moments as follows

Suppose that \( t = t_n \) is the current time and that the M+1 is most current asset prices. \( \{S_{t_{n-M}}, S_{t_{n-M+1}}, \ldots, S_{t_{n-1}}, S_{t_n}\} \) is also available and by using the corresponding log rasio data which is \( \{U_{n+1}\}_{i=1}^M \) then the first sample moment \( (m_1 = \text{mean}) \) and the second sample moment \( (m_2) \) [7, 10] are

\[ m_1 = \frac{1}{M} \sum_{i=1}^{M} U_{n+1-i} \] (3.21)

and

\[ m_2 = \frac{1}{M} \sum_{i=1}^{M} (U_{n+1-i})^2 \] (3.22)

Parameter of estimator in method of moments can be obtained by making the k-th moment of the sample to be equal to the k-th moment of the model.

Suppose \( \mu_1 \) dan \( \mu_2 \) are the first moment and the second moment for BS-BHM Updated model, then

\[ \mu_1 = E(U_i) = rt - \frac{1}{2} \frac{\sigma^2}{\sigma^2 + 1} \nu^2 T \] (3.23)

and

\[ \mu_2 = E(U_i^2) = \text{Var}(U_i) + \left( E(U_i) \right)^2 \]
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\[
(\frac{\sigma t \sqrt{T}}{t(\sigma^2 + 1)})^2 \left( \sigma^2 t^2 + \frac{t(T-t)}{T} \right) - \left( rt - \frac{1}{2} \frac{\sigma^2 t^2}{\sigma^2 + 1} \right)^2 = \left( \frac{\sigma t \sqrt{T}}{t(\sigma^2 + 1)} \right) \left( \sigma^2 t^2 + \frac{t(T-t)}{T} \right) - \left( rt - \frac{1}{2} \frac{\sigma^2 t^2}{\sigma^2 + 1} \right)^2
\]  

(3.24)

It can be seen that there are two equation i.e.

\[
m_1 = rt - \frac{1}{2} \frac{\sigma^2 t^2}{\sigma^2 + 1} \sqrt{T}
\]

(3.25)

then

\[
v^2 = \frac{2(rt - m_1)}{T} + \frac{2(rt - m_1)}{\sigma^2 t T}
\]

(3.26)

and

\[
m_2 = \left( \frac{\sigma t \sqrt{T}}{t(\sigma^2 + 1)} \right)^2 \left( \sigma^2 t^2 + \frac{t(T-t)}{T} \right) v^2 - (m_1)^2
\]

(3.27)

then

\[
v^2 = \frac{t(\sigma^2 + 1)^2 (m_2 + (m_1)^2)}{\sigma^4 t^2 T + \sigma^2 t^2 (T-t)}
\]

(3.28)

For equation (3.26) and (3.28), it can be written as

\[
\frac{t(\sigma^2 + 1)^2 (m_2 + (m_1)^2)}{\sigma^4 t^2 T + \sigma^2 t^2 (T-t)} = \frac{2(rt - m_1)}{T} + \frac{2(rt - m_1)}{\sigma^2 t T}
\]

(3.29)

By successive steps algebra trick, it is obtained the final equation below

\[
B_1 \sigma^4 + B_2 \sigma^2 + B_3 = 0
\]

(3.30)

where \( B_1 = 1 - \frac{(m_2 + (m_1)^2)}{2(rt - m_1)} \), \( B_2 = \frac{2(T-t)}{tT} - \frac{(m_2 + (m_1)^2)}{(rt - m_1)^2} \), and \( B_3 = \frac{T-t}{tT} - \frac{(m_2 + (m_1)^2)}{2(rt - m_1)^2} \).

Suppose \( y = \sigma^2 \) then it is obtained the quadratic equation

\[
B_1 y^2 + B_2 y + B_3 = 0
\]

(3.31)

The solution of equation (3.31) is

\[
y_1 = \frac{-B_2 + \sqrt{B_2^2 - 4B_1B_3}}{2B_1}
\]

(3.32)

and

\[
y_2 = \frac{-B_2 - \sqrt{B_2^2 - 4B_1B_3}}{2B_1}
\]

(3.33)

Because \( y = \sigma^2 \) then
\[ \sigma_1 = \sqrt{-B_2 + \frac{B^2_2 - 4B_1B_3}{2B_1}} \]  
(3.34)

and

\[ \sigma_2 = \sqrt{-B_2 - \frac{B^2_2 - 4B_1B_3}{2B_1}} \]  
(3.35)

Substitution \( \sigma_1^2 \) and \( \sigma_2^2 \) to \( \nu^2 \) equation (3.26), results in

\[ \nu_1 = \sqrt{\frac{2(\nu \cdot m_1)}{T} + \frac{4(\nu \cdot m_1) - 2(m_2 + (m_1)^2)}{rT(-B_2 + \sqrt{B^2_2 - 4B_1B_3})}} \]  
(3.36)

and

\[ \nu_2 = \sqrt{\frac{2(\nu \cdot m_1)}{T} + \frac{4(\nu \cdot m_1) - 2(m_2 + (m_1)^2)}{rT(-B_2 - \sqrt{B^2_2 - 4B_1B_3})}} \]  
(3.37)

It means the estimators of \( \sigma \) are

\[ \hat{\sigma}_1 = \sqrt{-B_2 + \frac{B^2_2 - 4B_1B_3}{2B_1}} \]  
(3.38)

and

\[ \hat{\sigma}_2 = \sqrt{-B_2 - \frac{B^2_2 - 4B_1B_3}{2B_1}} \]  
(3.39)

And the estimators of \( \nu \) are

\[ \hat{\nu}_1 = \sqrt{\frac{2(\nu \cdot m_1)}{T} + \frac{4(\nu \cdot m_1) - 2(m_2 + (m_1)^2)}{rT(-B_2 + \sqrt{B^2_2 - 4B_1B_3})}} \]  
(3.40)

and

\[ \hat{\nu}_2 = \sqrt{\frac{2(\nu \cdot m_1)}{T} + \frac{4(\nu \cdot m_1) - 2(m_2 + (m_1)^2)}{rT(-B_2 - \sqrt{B^2_2 - 4B_1B_3})}} \]  
(3.41)
4. Numerical Results

Estimation of historical volatility and the information flow rate of Microsoft (MSFT) shares for Monthly data in Indonesia are done using Monte Carlo estimation method and the method of moments. For the two parameter and the two methods of estimation, it is assumed that the data corresponds to equally spaced points in time [7].

In Monte Carlo estimation, the monthly data runs over 5 years (T = 5 years) and has 60 asset prices (M = 59), so it has \( dt = T/M = 5/59 \approx 0.084746 \).

For the monthly data result in \( a_M = -1.47 \times 10^{-3} \) and \( b_M^2 = 1.056 \times 10^{-3} \).

Estimation based on Monte Carlo produces two estimators of \( \nu \) and \( \sigma \), they are \( \hat{\nu}_1 = 0.0309, \hat{\sigma}_1 = 0.0766 \) and \( \hat{\nu}_2 = 0.0300, \hat{\sigma}_2 = 21.0660i \). Because the second estimator has imaginary number then the first estimator is chosen i.e. \( \hat{\nu}_1 = 0.0309, \hat{\sigma}_1 = 0.0766 \).

In the method of moments, the monthly data runs over 5 years (T = 5 years) and has 60 asset prices (M = 59), so it has \( dt = T/M = 5/59 \approx 0.084746 \).

For the monthly data result in \( m_1 = -1.47 \times 10^{-3} \) and \( m_2 = 1.04 \times 10^{-3} \).

Estimation based on the method of moments produces two estimators of \( \nu \) and \( \sigma \), they are \( \hat{\nu}_1 = 0.0309, \hat{\sigma}_1 = 0.0741 \) and \( \hat{\nu}_2 = 0.0300, \hat{\sigma}_2 = 21.1080i \). Because the second estimator has imaginary number then the first estimator is chosen i.e. \( \hat{\nu}_1 = 0.0309, \hat{\sigma}_1 = 0.0741 \).

5. Conclusion

The BS-BHM Updated model is developed based on the Black Scholes model from information-based perspective by Brody Hughston Macrina in which it is updated Gaussian integrals’s result, more precisely in the analysis of the algebra trick of completing square. The BS-BHM Updated model has lognormal distribution. It means that the log ratio is normally distributed. Estimation of the
volatility parameter and the information flow rate parameter use both Monte Carlo estimation and the method of moments, the results of estimation applied to real data have the same value for the volatility parameter, both using Monte Carlo estimation method and the method of moments. While the results of estimation for the information flow rate has almost the same value, both using Monte Carlo estimation method and the method of moments.

References


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