Research on Portfolio Investment of Different Risk Attitude with Utility Function

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Abstract

In this paper, firstly we research on portfolio investment of different risk attitude of investors with their classic utility functions, then analyse their difference and connection, at last several numerical examples are given.

Keywords: risk attitude, utility function, portfolio investment, indifference curve

1 Introduction

In the process of securities portfolio investment, every investor has own utility function to express his preference to the returns and risk degree, and use the expected utility maximization principle to choose the optimal portfolio. Investors can be divided into three types according to their attitude towards the risk: (1) risk aversion, (2) risk neutral and (3) risk like. And the properties of the investor's

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utility function are dependent on its attitude toward the risk\textsuperscript{[1,2]},
(1) If the utility function is a convex function, then investors are risk averse type,
(2) If the utility function is a linear function, then investors are risk neutral type,
(3) If the utility function is a concave function, then investors are risk love type.
As shown in Figure 1.

In real life, most investors are risk averse type, and the main present research focus on risk averse investors’ investment behavior under uncertainty environment. But there are some risk neutral and risk love type investors, and they really exist in specific situations, for example, under some pressure, or out of luck and or seeking to stimulate, investors are likely to pursue a higher returns regardless of the risk or willing to bear more risk. These behavior is related to the wealth level and the mental of the investor. In this paper, we mainly research on portfolio investment of different risk attitude, and analyse their difference and connection.

2 Main Results

2.1 Research on risk averse investors’ portfolio investment

In Markowitz portfolio theory, investors are assumed as risk averse type, they pursue maximum profit as the same as pursuing minimization risk, their utility function is convex. As shown in Figure 2. Here we take the classic negative exponential utility function \( U = -e^{-k_rR} \) as the example\textsuperscript{[3,4]}, and \( k_1 > 0 \) (risk averse factor), \( R \) is the investment return and has Gauss distribution \( N(r, \sigma^2) \), and we can solve its expected utility function:

\[
EU = \int_{-\infty}^{\infty} (-e^{-k_r x}) \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-r)^2}{2\sigma^2}\right)dx = -\exp[k_1\left(\frac{1}{2}k_1\sigma^2 - r\right)]
\]

We want \( EU = -\exp[k_1\left(\frac{1}{2}k_1\sigma^2 - r\right)] \) maximum, that is to make \( \frac{1}{2}k_1\sigma^2 - r \)
minimum, so it can be turned into Optimization problem:

\[
\begin{cases}
\min \left( \frac{1}{2} k_i \omega^T \Sigma \omega - \omega^T \mu \right) \\
\text{s.t.} \quad \omega^T e = 1
\end{cases}
\]

We can solve its optimal portfolio investment proportion with Lagrange Multiplier Method: \[
\omega = \frac{\Sigma^{-1}}{k_i \Lambda} [A \mu + (k_i - B)e]
\]

Here \( A = e^T \Sigma^{-1} e \), \( B = e^T \Sigma^{-1} \mu = \mu^T \Sigma^{-1} e \), \( C = \mu^T \Sigma^{-1} \mu \), \( \Delta = AC - B^2 \).

We give one illustrative example, there are 5 security, and the investment return expectation is \( \mu = (0.208 \ 0.353 \ 0.262 \ 0.167 \ 0.318)^T \) and the Covariance matrix is

\[
\Sigma = \begin{bmatrix}
2.10 & 2.21 & -2.16 & 1.62 & -2.15 \\
2.21 & 2.75 & -2.46 & 1.89 & -2.47 \\
-2.16 & -2.46 & 2.56 & -1.85 & 2.54 \\
1.62 & 1.89 & -1.85 & 1.42 & -1.88 \\
-2.15 & -2.47 & 2.54 & -1.88 & 2.66
\end{bmatrix}
\]

Using formula (1) we can get the optimal portfolio, and the proportion

\( \omega = (0.0401, 0.7453, 0.0876, -0.3304, 0.4574)^T \)

\[2.2 \text{ Research on risk neutral investors' portfolio investment}\]

In real life, there are some risk neutral type investors, they pursue profit as the same as ignoring risk, their utility function is line function. As shown in Figure 3.

Here we adopt function \( U = k_2 R \), and \( k_2 > 0 \) (risk neutral factor).

For \( k_2 > 0 \), \( U' = k_2 > 0 \), \( U'' = 0 \), and we can solve its expected utility

\[ EU = \int_{-\infty}^{\infty} k_2 xf(x) dx = k_2 r, \]

so \( EU \) is only depended on return \( r \) and has nothing to do with risk \( \sigma^2 \).

Fig.3 Indifference curves of risk neutral

Fig.4 Indifference curves of risk love

(in Fig.1-3, \( I_i (i = 1, 2, 3) \) express three different indifference curves of investors)
2.3 Research on risk love investors' portfolio investment

Risk love type investors have a positive attitude to risk, and they would think the more risk, the more profit. Usually they will speculate. Their utility function is concave function. As shown in Figure 4. Here we take exponential utility function $U = e^{x^k}$ as the example, and $k > 0$ (risk love factor).

For $k > 0$, $U' = k^xe^{xk} > 0$, $U'' = k^xe^{xk} > 0$, and we can solve its expected utility

$$EU = \int_{-\infty}^{\infty} f(x)dx = \exp[k_3(1/2)k_3\sigma^2 + r)]$$

We can see that $EU \to +\infty$, it has no maximum, and the investors would take more risk to pursue more return. At the same time, $EU$ has a minimum value, namely the utility underscore, which can be taken as reference to the theoretical analysis, that is $\min EU = \min \exp[k_3(1/2)k_3\sigma^2 + r)]$ exists,

Using Lagrange Multiplier Method we can get

$$\omega = \frac{1}{k_3} \Sigma^{-1} [(k_3 + B)e - A\mu]$$

For the above example in 2.1, we can solve $\min EU$, the utility underscore porpotion of the risk love type investors. Here

$$\omega = \frac{1}{A} \Sigma^{-1} [(1 + B)e - A\mu] = (0.0674, -0.8846, 0.3632, 1.5235, -0.0996)^T$$

And the investment return $r = 0.0276$, the risk $\sigma^2 = 0.2000$.

3 Conclusions

From above research, we can see that investors who are risk averse have optimal portfolio and solve it with Lagrange Multiplier Method, and investors who are risk neutral and love type are not. And some investors will show different attitude toward risk with their wealth and changes in different environment.

References


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