On Fuzzy Soft Γ-Hyperideals over Left Almost Γ-Semihypergroups

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Abstract

In this paper we have introduced the notion of fuzzy soft Γ-hyperideals in left almost Γ-semihypergroups. Also we have studied some properties based on them with t-level set, union and intersection.

Mathematics Subject Classification: 20N20, 20N25, 08A72

Keywords: LA-Γ-semihypergroups; Fuzzy soft sets; Fuzzy soft Γ-hyperideals

1 Introduction

Hyperstructure theory was born in 1934 when Marty [12], defined hypergroups, began to analyze their properties and applied them to groups, rational algebraic functions. In 1986, Sen and Saha [15], introduced the concept of the Γ-semigroup as a generalization of semigroup and ternary semigroup. Many classical notions and results of the theory of semigroups have been extended and generalized to Γ-semigroups. Davvaz et al. [2, 8], introduced the notion of Γ-semihypergroup. Recently, Yaqoob and Aslam [18], introduced the notion of LA-Γ-semihypergroup as a generalization of semigroup, commutative semihypergroup and of commutative Γ-semigroup. They proved some results in this
respect and presented some examples of LA-Γ-semihypergroups. Yaqoob et al. [19, 20], applied rough set theory and soft set theory to LA-Γ-semihypergroups, also see [3, 4, 21, 22, 23].

In 1965, Zadeh [24], introduced the notion of a fuzzy subset of a non-empty set \( X \), as a function from \( X \) to \([0, 1]\). Recently, fuzzy set theory has been well developed in the context of hyperalgebraic structure theory. A recent book [6], contains a wealth of applications. Recently in [7], Davvaz and Leoreanu-Fotea studied the structure of fuzzy Γ-hyperideals in Γ-semihypergroups.

The concept of soft set was given by Molodtsov [13] in 1999, which is a completely new approach for modeling and uncertainty. After Molodtsov’s work, some different applications of soft sets were studied in [5] and [10]. Also Maji et al. [11], presented the definition of fuzzy soft set. Some corrections were given by Ali et al. [1]. Roy et al. [14], presented some applications of this notion to decision making problems. Many authors studied the properties of fuzzy soft sets in different algebraic structures, for instance, Jun et al. [9], Williams and Saeid [16] and Yang [17].

In this paper, some properties of fuzzy soft Γ-hyperideals in left almost Γ-semihypergroups have been discussed.

## 2 Left Almost Γ-semihypergroups

In this section, we recall certain definitions and results needed for our purpose. Let \( S \) be a non-empty set and \( \mathcal{P}^*(S) \) be the set of all non-empty subsets of \( S \). The map \( \circ : S \times S \rightarrow \mathcal{P}^*(S) \) is called hyperoperation or join operation on the set \( S \). A couple \( (S, \circ) \) is called a hypergroupoid. If \( A \) and \( B \) be two non-empty subsets of \( S \), then we denote

\[
A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \quad a \circ A = \{a\} \circ A \quad \text{and} \quad a \circ B = \{a\} \circ B.
\]

**Definition 2.1.** [18] Let \( S \) and \( \Gamma \) be two non-empty sets. \( S \) is called a left almost \( \Gamma \)-semihypergroup (abbreviated as an LA-Γ-semihypergroup) if every \( \gamma \in \Gamma \) is a hyperoperation on \( S \), i.e., \( x\gamma y \subseteq S \) for every \( x, y \in S \), and for every \( \gamma, \beta \in \Gamma \) and \( x, y, z \in S \) we have \( (x\gamma y)\beta z = (z\gamma y)\beta x \).

The law \( (x\gamma y)\beta z = (z\gamma y)\beta x \) is called left invertive law. Let \( A \) and \( B \) be two non-empty subsets of an LA-Γ-semihypergroup \( S \). Then we define

\[
A\gamma B = \bigcup \{a\gamma b \mid a \in A, \ b \in B \text{ and } \gamma \in \Gamma\},
\]

also

\[
A\Gamma B = \bigcup_{\gamma \in \Gamma} A\gamma B = \bigcup \{a\gamma b \mid a \in A, \ b \in B \text{ and } \gamma \in \Gamma\}.
\]
Example 2.2. Let $S = \{a, b, c, d\}$ and $\Gamma = \{\beta, \gamma\}$ be the sets of binary hyperoperations defined below:

<table>
<thead>
<tr>
<th>$\beta$</th>
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</table>

Clearly $S$ is not a $\Gamma$-semihypergroup because $\{a, b, c\} = (a\beta b)\gamma b \neq a\beta (b\gamma b) = \{a, c\}$. Thus $S$ is an LA-$\Gamma$-semihypergroup because $S$ satisfies the left invertive law.

Every LA-$\Gamma$-semihypergroup satisfies the law $(a\alpha b)\beta (c\gamma d) = (a\alpha c)\beta (b\gamma d)$ for all $a, b, c, d \in S$ and $\alpha, \beta, \gamma \in \Gamma$. This law is known as $\Gamma$-hypermedial law. (cf. [18]).

Definition 2.3. [18] Let $K$ be a non-empty subset of $S$. Then $K$ is called a sub LA-$\Gamma$-semihypergroup of $S$ if $a\gamma b \subseteq K$ for all $a, b \in K$ and $\gamma \in \Gamma$.

Definition 2.4. [18] A non-empty subset $A$ of an LA-$\Gamma$-semihypergroup $S$ is called a right (left) $\Gamma$-hyperideal of $S$ if $A\Gamma S \subseteq A$ ($S\Gamma A \subseteq A$), and is a $\Gamma$-hyperideal of $S$ if it is both a right and a left $\Gamma$-hyperideal.

3 Fuzzy Soft Sets

Molodtsov defined the notion of a soft set as follows. Let $U$ be an initial universe and $E$ be the set of parameters. Usually, parameters are attributes, characteristics or properties of an object. Let $P(U)$ denote the power set of $U$ and $A$ is a subset of $E$.

Definition 3.1. [13] A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by $F : A \rightarrow P(U)$. In other words a soft set over $U$ is a parametrized family of subsets of $U$.

Definition 3.2. [11] A pair $(\hat{F}, A)$ is called a fuzzy soft set over $U$, where $\hat{F} : A \rightarrow \wp(U)$ is a mapping and $\wp(U)$ being the set of all fuzzy subsets of $S$.

Definition 3.3. Let $(\hat{F}, A)$ and $(\hat{\Theta}, B)$ be two fuzzy soft sets over a common universe $U$, we say $(\hat{F}, A)$ is a fuzzy soft subset of $(\hat{\Theta}, B)$, if (i) $A \subseteq B$, (ii) for all $\delta \in A$, $\hat{F}_\delta$ is a fuzzy subset of $\hat{\Theta}_\delta$ and we denote it by $(\hat{F}, A) \subseteq (\hat{\Theta}, B)$.

Definition 3.4. Let $(\hat{F}, A)$ and $(\hat{\Theta}, B)$ be two fuzzy soft sets over a common universe $U$. Then $(\hat{F}, A) \hat{\wedge} (\hat{\Theta}, B)$ is defined by $(\hat{F}, A) \hat{\wedge} (\hat{\Theta}, B) = (\hat{\Xi}, A \times B)$, where $\hat{\Xi}_{(\delta, \varepsilon)} = \hat{F}_\delta \cap \hat{\Theta}_\varepsilon$ for all $(\delta, \varepsilon) \in A \times B$. 

Definition 3.5. Let $(\mathcal{F}, A)$ and $(\tilde{\Theta}, B)$ be two fuzzy soft sets over a common universe $U$. Then $(\mathcal{F}, A)\sqcup(\tilde{\Theta}, B)$ is defined by $(\mathcal{F}, A)\sqcup(\tilde{\Theta}, B) = (\tilde{\Xi}, A \times B)$, where $\tilde{\Xi}(\delta, \varepsilon) = \mathcal{F}_\delta \cup \tilde{\Theta}_\delta$ for all $(\delta, \varepsilon) \in A \times B$.

Definition 3.6. Let $(\mathcal{F}, A)$ and $(\tilde{\Theta}, B)$ be two fuzzy soft sets over a common universe $U$. Then,

(i) the union $(\tilde{\Xi}, C)$ of two soft sets $(\mathcal{F}, A)$ and $(\tilde{\Theta}, B)$ is defined as the soft set $(\tilde{\Xi}, C) = (\mathcal{F}, A)\sqcup(\tilde{\Theta}, B)$, where $C = A \cup B$ and for all $\delta \in C$

\[
\tilde{\Xi}_\delta = \begin{cases} 
\mathcal{F}_\delta & \text{if } \delta \in A \setminus B \\
\tilde{\Theta}_\delta & \text{if } \delta \in B \setminus A \\
\max \{\mathcal{F}_\delta, \tilde{\Theta}_\delta\} & \text{if } \delta \in A \cap B
\end{cases}
\]

(ii) the intersection $(\tilde{\Xi}, C)$ of two soft sets $(\mathcal{F}, A)$ and $(\tilde{\Theta}, B)$ is defined as the soft set $(\tilde{\Xi}, C) = (\mathcal{F}, A)\cap(\tilde{\Theta}, B)$, where $C = A \cup B$ and for all $\delta \in C$

\[
\tilde{\Xi}_\delta = \begin{cases} 
\mathcal{F}_\delta & \text{if } \delta \in A \setminus B \\
\tilde{\Theta}_\delta & \text{if } \delta \in B \setminus A \\
\min \{\mathcal{F}_\delta, \tilde{\Theta}_\delta\} & \text{if } \delta \in A \cap B
\end{cases}
\]

In contrast with the above definitions of fuzzy soft set union and intersection, we may sometimes adopt different definitions of union and intersection given as follows.

Definition 3.7. Let $(\mathcal{F}, A)$ and $(\tilde{\Theta}, B)$ be two fuzzy soft sets over a common universe $U$ and $A \cap B \neq \emptyset$. Then the bi-union of $(\mathcal{F}, A)$ and $(\tilde{\Theta}, B)$ is defined to be the fuzzy soft set $(\tilde{\Xi}, C)$, where $C = A \cap B$ and $\tilde{\Xi}_\delta = \mathcal{F}_\delta \cup \tilde{\Theta}_\delta$ for all $\delta \in C$. This is denoted by $(\tilde{\Xi}, C) = (\mathcal{F}, A)\sqcup(\tilde{\Theta}, B)$.

Definition 3.8. Let $(\mathcal{F}, A)$ and $(\tilde{\Theta}, B)$ be two fuzzy soft sets over a common universe $U$ and $A \cap B \neq \emptyset$. Then the bi-intersection of $(\mathcal{F}, A)$ and $(\tilde{\Theta}, B)$ is defined to be the fuzzy soft set $(\tilde{\Xi}, C)$, where $C = A \cap B$ and $\tilde{\Xi}_\delta = \mathcal{F}_\delta \cap \tilde{\Theta}_\delta$ for all $\delta \in C$. This is denoted by $(\tilde{\Xi}, C) = (\mathcal{F}, A)\cap(\tilde{\Theta}, B)$.

4 Fuzzy $\Gamma$-hyperideals in LA-$\Gamma$-semihypergroups

In this section, we define the notion of fuzzy left (right) $\Gamma$-hyperideals in left almost $\Gamma$-semihypergroups. Throughout the paper $S$ will denote an LA-$\Gamma$-semihypergroup.
Definition 4.1. Let $\mu$ and $\lambda$ be two fuzzy subsets of $S$. Then, we have:

\[
\mu \cap \lambda = \min \{\mu(x), \lambda(x)\},
\]

\[
(\mu \Gamma \lambda)(z) = \begin{cases} 
\sup \{\min\{\mu(x), \lambda(y)\}\} & \text{if } z \in x \gamma y, \forall \gamma \in \Gamma \\
0 & \text{otherwise},
\end{cases}
\]

for all $x, y, z \in S$.

Definition 4.2. Let $\mu$ be a fuzzy subset of $S$. Then $\mu$ is called a sub LA-$\Gamma$-semihypergroup of $S$ if for all $x, y \in S$ and $\gamma \in \Gamma$,

\[
\inf_{z \in x \gamma y} \{\mu(z)\} \geq \min\{\mu(x), \mu(y)\}.
\]

Lemma 4.3. Let $\mu$ be a fuzzy subset of $S$. Then $\mu$ is a fuzzy sub LA-$\Gamma$-semihypergroup of $S$ if and only if $\mu \Gamma \mu \subseteq \mu$.

Proof. The proof is straightforward. \qed

Definition 4.4. Let $\mu$ be a fuzzy subset of $S$. Then, for all $x, y, z \in S$ and $\gamma \in \Gamma$,

(i) $\mu$ is called a fuzzy left $\Gamma$-hyperideal of $S$ if $\mu(y) \leq \inf_{z \in x \gamma y} \{\mu(z)\}$,

(ii) $\mu$ is called a fuzzy right $\Gamma$-hyperideal of $S$ if $\mu(x) \leq \inf_{z \in x \gamma y} \{\mu(z)\}$,

(iii) $\mu$ is called a fuzzy $\Gamma$-hyperideal of $S$ if it is both a fuzzy left $\Gamma$-hyperideal and fuzzy right $\Gamma$-hyperideal of $S$.

Example 4.5. Let $S = \{a, b, c, d\}$ and $\Gamma = \{\beta, \gamma\}$ be the sets of binary hyperoperations defined below:

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<th>$\beta$</th>
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Clearly $S$ is not a $\Gamma$-semihypergroup because $\{a, b\} = (a \beta a) \gamma b \neq a \beta (a \gamma b) = \{b\}$. Thus $S$ is an LA-$\Gamma$-semihypergroup because it satisfies the left invertive law. Let $\mu$ be a fuzzy subset of $S$, which is defined as

\[
\mu = \begin{pmatrix} a & b & c & d \\
0.2 & 0.2 & 0.5 & 0.7
\end{pmatrix}.
\]

By routine calculations, it can be seen that $\mu$ is a fuzzy $\Gamma$-hyperideal of $S$. 

Definition 4.6. For any \( t \in [0, 1] \) and fuzzy subset \( \mu \) of \( S \), the set
\[
U(\mu, t) = \{ x \in S : \mu(x) \geq t \}
\]
is called an upper \( t \)-level cut of \( \mu \).

Theorem 4.7. A fuzzy subset \( \mu \) of \( S \) is a fuzzy left (right) \( \Gamma \)-hyperideal of \( S \) if and only if \( U(\mu, t) \) is a left (right) \( \Gamma \)-hyperideal of \( S \).

Proof. The proof is straightforward. \( \square \)

5 Fuzzy Soft \( \Gamma \)-hyperideals over Left Almost \( \Gamma \)-semihypergroups

In this section, we define the notion of fuzzy soft left (right) \( \Gamma \)-hyperideals over left almost \( \Gamma \)-semihypergroups.

Definition 5.1. Let \( S \) be an LA-\( \Gamma \)-semihypergroup and \((\hat{F}, A)\) be a fuzzy soft set over \( S \). Then \((\hat{F}, A)\) is called a fuzzy soft LA-\( \Gamma \)-semihypergroup over \( S \) if it satisfies
\[
\inf_{z \in x \gamma y} \{\hat{F}_\delta(z)\} \geq \min\{\hat{F}_\delta(x), \hat{F}_\delta(y)\},
\]
for all \( x, y \in S, \gamma \in \Gamma \) and \( \delta \in A \).

Definition 5.2. Let \((\hat{F}, A)\) be a fuzzy soft set over \( S \). Then, for all \( x, y \in S, \gamma \in \Gamma \) and \( \delta \in A \),

(i) \((\hat{F}, A)\) is called a fuzzy soft left \( \Gamma \)-hyperideal over \( S \) if
\[
\hat{F}_\delta(y) \leq \inf_{z \in x \gamma y} \{\hat{F}_\delta(z)\};
\]

(ii) \((\hat{F}, A)\) is called a fuzzy soft right \( \Gamma \)-hyperideal over \( S \) if
\[
\hat{F}_\delta(x) \leq \inf_{z \in x \gamma y} \{\hat{F}_\delta(z)\};
\]

(iii) \((\hat{F}, A)\) is called a fuzzy soft \( \Gamma \)-hyperideal over \( S \) if it is both a fuzzy soft left \( \Gamma \)-hyperideal and fuzzy soft right \( \Gamma \)-hyperideal over \( S \).

Example 5.3. Let \( S = \{a, b, c, d\} \) and \( \Gamma = \{\beta, \gamma\} \) be the sets of binary hyperoperations defined below:

<table>
<thead>
<tr>
<th>( \beta )</th>
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Clearly $S$ is not a $\Gamma$-semihypergroup because \( \{a, b, c\} = (c\beta c)\gamma b \neq c\beta(c\gamma b) = \{b, c\} \). Thus $S$ is an LA-$\Gamma$-semihypergroup because it satisfies the left invertive law. Let $A = \{\delta, \varepsilon, \kappa, \xi\}$ be the set of parameters. For each parameter $\delta \in A$, $\tilde{F}_\delta$ is a fuzzy $\Gamma$-hyperideal of $S$, where $\tilde{F}_\delta$ is a mapping given by $\tilde{F}_\delta : S \rightarrow [0, 1]$. For each parameter we define

\[
\tilde{F}_\delta = \left( \frac{a}{0.4}, \frac{b}{0.1}, \frac{c}{0.1}, \frac{d}{0.8} \right), \quad \tilde{F}_\varepsilon = \left( \frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}, \frac{d}{0.7} \right),
\]

\[
\tilde{F}_\kappa = \left( \frac{a}{0.5}, \frac{b}{0.3}, \frac{c}{0.3}, \frac{d}{0.5} \right), \quad \tilde{F}_\xi = \left( \frac{a}{0.6}, \frac{b}{0.2}, \frac{c}{0.2}, \frac{d}{0.9} \right).
\]

Each $\tilde{F}_\delta$ satisfies the conditions of fuzzy $\Gamma$-hyperideal of $S$. Hence $(\tilde{F}, A) = \{\tilde{F}_\delta : \delta \in A\}$ is a fuzzy soft $\Gamma$-hyperideal of $S$.

**Definition 5.4.** Let $(\tilde{F}, A)$ be a fuzzy soft set over $S$, for each $t \in [0, 1]$, the set $(\tilde{F}, A)\gamma = (\tilde{F}\gamma, A)$ is called a $t$-level set of $(\tilde{F}, A)$, where $\tilde{F}\gamma = \{ x \in S : \tilde{F}_\delta (x) \geq t \}$ for each $\delta \in A$.

**Theorem 5.5.** Let $(\tilde{F}, A)$ be a fuzzy soft set over $S$, $(\tilde{F}, A)$ is a fuzzy soft LA-$\Gamma$-semihypergroup if and only if $(\tilde{F}, A)\gamma$ is a soft LA-$\Gamma$-semihypergroup over $S$ for each $t \in [0, 1]$.

**Proof.** Assume that $(\tilde{F}, A)\gamma$ is a soft LA-$\Gamma$-semihypergroup over $S$ for each $t \in [0, 1]$. For each $x_1, x_2 \in S$ and $\delta \in A$, let $t = \min \{ \tilde{F}_\delta (x_1), \tilde{F}_\delta (x_2) \}$, then $x_1, x_2 \in \tilde{F}_\delta\gamma$. Since $\tilde{F}_\delta\gamma$ is an LA-$\Gamma$-semihypergroup over $S$, then for each $\gamma \in \Gamma$, $x_1\gamma x_2 \subseteq \tilde{F}_\delta\gamma$. Therefore for all $z \in x_1\gamma x_2$ we have $\tilde{F}_\delta (z) \geq t$, and so

\[
\inf_{z \in x_1\gamma x_2} \{ \tilde{F}_\delta (z) \} \geq t = \min \{ \tilde{F}_\delta (x_1), \tilde{F}_\delta (x_2)\}, \quad \text{for all } x_1, x_2 \in S, \ \delta \in A \text{ and } \gamma \in \Gamma.
\]

This shows that $\tilde{F}_\delta$ is a fuzzy sub LA-$\Gamma$-semihypergroup over $S$. Thus $(\tilde{F}, A)$ is a fuzzy soft LA-$\Gamma$-semihypergroup over $S$.

Conversely, let $(\tilde{F}, A)$ be a fuzzy soft LA-$\Gamma$-semihypergroup. For each $\delta \in A$, $t \in [0, 1]$ and $x_1, x_2 \in \tilde{F}_\delta\gamma$ we have $\tilde{F}_\delta (x_1) \geq t$ and $\tilde{F}_\delta (x_2) \geq t$. As $\tilde{F}_\delta$ is a fuzzy LA-$\Gamma$-semihypergroup over $S$, thus for $\gamma \in \Gamma$ there exist $z \in x_1\gamma x_2$ such that

\[
\inf_{z \in x_1\gamma x_2} \{ \tilde{F}_\delta (z) \} \geq \min \{ \tilde{F}_\delta (x_1), \tilde{F}_\delta (x_2)\} \geq t.
\]

Therefore for all $z \in x_1\gamma x_2$ we have $z \in \tilde{F}_\delta\gamma$, this implies that $x_1\gamma x_2 \subseteq \tilde{F}_\delta\gamma$, i.e., $\tilde{F}_\delta\gamma$ is a sub LA-$\Gamma$-semihypergroup over $S$. Thus $(\tilde{F}, A)\gamma$ is a soft LA-$\Gamma$-semihypergroup over $S$ for each $t \in [0, 1]$. This completes the proof. \[\square\]
Theorem 5.6. Let \((\widehat{\mathcal{F}}, A)\) be a fuzzy soft set over \(S\), \((\widehat{\mathcal{F}}, A)\) is a fuzzy soft left (right) \(\Gamma\)-hyperideal if and only if \((\widehat{\mathcal{F}}, A)^t\) is a soft left (right) \(\Gamma\)-hyperideal over \(S\) for each \(t \in [0, 1]\).

Proof. Assume that \((\widehat{\mathcal{F}}, A)^t\) is a soft left \(\Gamma\)-hyperideal over \(S\) for each \(t \in [0, 1]\). For each \(x \in S\) and \(\delta \in A\), let \(t = \widehat{\mathcal{F}}_\delta(x)\), then \(x \in \widehat{\mathcal{F}}_\delta^t\). Since \(\widehat{\mathcal{F}}_\delta^t\) is a soft \(\Gamma\)-hyperideal over \(S\), then \(y^t x \subseteq \widehat{\mathcal{F}}_\delta^t\) for each \(y \in S\) and \(\gamma \in \Gamma\). Therefore for all \(z \in y^t x\) we have \(\widehat{\mathcal{F}}_\delta(z) \geq t\), and so

\[
\inf_{z \in y^t x} \{\widehat{\mathcal{F}}_\delta(z)\} \geq t = \widehat{\mathcal{F}}_\delta(x), \text{ for all } x, y \in S, \delta \in A \text{ and } \gamma \in \Gamma.
\]

This shows that \(\widehat{\mathcal{F}}_\delta\) is a fuzzy left \(\Gamma\)-hyperideal. Thus \((\widehat{\mathcal{F}}, A)\) is a fuzzy soft left \(\Gamma\)-hyperideal over \(S\).

Conversely, let \((\widehat{\mathcal{F}}, A)\) be a fuzzy soft left \(\Gamma\)-hyperideal. For each \(\delta \in A\), \(t \in [0, 1]\) and \(x \in \widehat{\mathcal{F}}_\delta\) we have \(\widehat{\mathcal{F}}_\delta(x) \geq t\). As \(\widehat{\mathcal{F}}_\delta\) is a fuzzy left \(\Gamma\)-hyperideal over \(S\), thus for \(y \in S\) there exist \(z \in y^t x\) such that

\[
\inf_{z \in y^t x} \{\widehat{\mathcal{F}}_\delta(z)\} \geq \widehat{\mathcal{F}}_\delta(x) \geq t.
\]

Therefore for all \(z \in y^t x\) we have \(z \in \widehat{\mathcal{F}}_\delta^t\), this implies that \(y^t x \subseteq \widehat{\mathcal{F}}_\delta^t\), i.e., \(\widehat{\mathcal{F}}_\delta^t\) is a left \(\Gamma\)-hyperideal over \(S\). Thus \((\widehat{\mathcal{F}}, A)^t\) is a soft left \(\Gamma\)-hyperideal over \(S\) for each \(t \in [0, 1]\). The case for right \(\Gamma\)-hyperideal can be seen in a similar way.

Theorem 5.7. If \((\widehat{\mathcal{F}}, A)\) and \((\widehat{\Theta}, B)\) are two fuzzy soft LA-\(\Gamma\)-semihypergroups over \(S\), then so are \((\widehat{\mathcal{F}}, A)\wedge(\widehat{\Theta}, B)\) and \((\widehat{\mathcal{F}}, A)\wedge(\widehat{\Theta}, B)\).

Proof. We know that \((\widehat{\mathcal{F}}, A)\wedge(\widehat{\Theta}, B) = \left(\widehat{\Xi}, C\right)\), where \(C = A \times B\) and \(\widehat{\Xi}_{(\delta, \varepsilon)} = \widehat{\mathcal{F}}_\delta \cap \widehat{\Theta}_\varepsilon\) for all \((\delta, \varepsilon) \in A \times B\). Now for any \((\delta, \varepsilon) \in A \times B\), since \((\widehat{\mathcal{F}}, A)\) and \((\widehat{\Theta}, B)\) are fuzzy soft LA-\(\Gamma\)-semihypergroups over \(S\), for all \(x, y \in S\), \((\delta, \varepsilon) \in A \times B\) and \(\gamma \in \Gamma\), we have

\[
\inf_{z \in x^t y} \{\widehat{\Xi}_{(\delta, \varepsilon)}(z)\} = \inf_{z \in x^t y} \left\{\min \{\widehat{\mathcal{F}}_\delta(z), \widehat{\Theta}_\varepsilon(z)\}\right\}
\]

\[
= \min \left\{\inf_{z \in x^t y} \widehat{\mathcal{F}}_\delta(z), \inf_{z \in x^t y} \widehat{\Theta}_\varepsilon(z)\right\}
\]

\[
\geq \min \left\{\min \{\widehat{\mathcal{F}}_\delta(x), \widehat{\mathcal{F}}_\delta(y)\}, \min \{\widehat{\Theta}_\varepsilon(x), \widehat{\Theta}_\varepsilon(y)\}\right\}
\]

\[
= \min \left\{\min \{\widehat{\mathcal{F}}_\delta(x), \widehat{\Theta}_\varepsilon(x)\}, \min \{\widehat{\mathcal{F}}_\delta(y), \widehat{\Theta}_\varepsilon(y)\}\right\}
\]

\[
= \min \left\{\widehat{\Xi}_{(\delta, \varepsilon)}(x), \widehat{\Xi}_{(\delta, \varepsilon)}(y)\right\}.
\]

This shows that \((\widehat{\mathcal{F}}, A)\wedge(\widehat{\Theta}, B)\) is a fuzzy soft LA-\(\Gamma\)-semihypergroup over \(S\). The other case can be seen in a similar way.
Theorem 5.8. If \((\hat{F}, A)\) and \((\hat{\Theta}, B)\) are two fuzzy soft left (right) \(\Gamma\)-hyperideals over \(S\), then so are \((\hat{F}, A)\cap(\hat{\Theta}, B)\) and \((\hat{F}, A)\cap(\hat{\Theta}, B)\).

Proof. Let \((\hat{F}, A)\) and \((\hat{\Theta}, B)\) be two fuzzy soft left \(\Gamma\)-hyperideals over \(S\). We know that \((\hat{F}, A)\cap(\hat{\Theta}, B) = (\hat{\Xi}, C)\), where \(C = A \times B\) and \(\hat{\Xi}_{(\delta, \varepsilon)} = \hat{F}_\delta \cap \hat{\Theta}_\varepsilon\) for all \((\delta, \varepsilon) \in A \times B\). Now for any \((\delta, \varepsilon) \in A \times B\), since \((\hat{F}, A)\) and \((\hat{\Theta}, B)\) are fuzzy soft left \(\Gamma\)-hyperideals over \(S\), for all \(x, y \in S\), \((\delta, \varepsilon) \in A \times B\) and \(\gamma \in \Gamma\), we have

\[
\inf_{z \in x \gamma y} \{\hat{\Xi}_{(\delta, \varepsilon)}(z)\} = \inf_{z \in x \gamma y} \{\min\{\hat{F}_\delta(z), \hat{\Theta}_\varepsilon(z)\}\} = \min \left\{\inf_{z \in x \gamma y} \hat{F}_\delta(z), \inf_{z \in x \gamma y} \hat{\Theta}_\varepsilon(z)\right\} \geq \min \left\{\hat{F}_\delta(y), \hat{\Theta}_\varepsilon(y)\right\} = \hat{\Xi}_{(\delta, \varepsilon)}(y).
\]

This shows that \((\hat{F}, A)\cap(\hat{\Theta}, B)\) is a fuzzy soft left \(\Gamma\)-hyperideal over \(S\). The other cases can be seen in a similar way. \(\Box\)

Theorem 5.9. If \((\hat{F}, A)\) and \((\hat{\Theta}, B)\) are two fuzzy soft LA-\(\Gamma\)-semihypergroups over \(S\), then so are \((\hat{F}, A)\cap(\hat{\Theta}, B)\) and \((\hat{F}, A)\cap(\hat{\Theta}, B)\).

Proof. We know that \((\hat{F}, A)\cap(\hat{\Theta}, B) = (\hat{\Xi}, C)\), where \(C = A \times B\) and \(\hat{\Xi}_{(\delta, \varepsilon)} = \hat{F}_\delta \cup \hat{\Theta}_\varepsilon\) for all \((\delta, \varepsilon) \in A \times B\). Now for any \((\delta, \varepsilon) \in A \times B\), since \((\hat{F}, A)\) and \((\hat{\Theta}, B)\) are fuzzy soft LA-\(\Gamma\)-semihypergroups over \(S\), for all \(x, y \in S\), \((\delta, \varepsilon) \in A \times B\) and \(\gamma \in \Gamma\), we have

\[
\inf_{z \in x \gamma y} \{\hat{\Xi}_{(\delta, \varepsilon)}(z)\} = \inf_{z \in x \gamma y} \{\hat{F}_\delta(z) \cup \hat{\Theta}_\varepsilon(z)\} = \inf_{z \in x \gamma y} \{\max\{\hat{F}_\delta(z), \hat{\Theta}_\varepsilon(z)\}\} = \max \left\{\inf_{z \in x \gamma y} \hat{F}_\delta(z), \inf_{z \in x \gamma y} \hat{\Theta}_\varepsilon(z)\right\} \geq \max \left\{\min\{\hat{F}_\delta(x), \hat{F}_\delta(y)\}, \min\{\hat{\Theta}_\varepsilon(x), \hat{\Theta}_\varepsilon(y)\}\right\} = \min \left\{\max\{\hat{F}_\delta(x), \hat{\Theta}_\varepsilon(x)\}, \max\{\hat{F}_\delta(y), \hat{\Theta}_\varepsilon(y)\}\right\} = \min \left\{\hat{\Xi}_{(\delta, \varepsilon)}(x), \hat{\Xi}_{(\delta, \varepsilon)}(y)\right\}.
\]

This shows that \((\hat{F}, A)\cap(\hat{\Theta}, B)\) is a fuzzy soft LA-\(\Gamma\)-semihypergroup over \(S\). The other case can be seen in a similar way. \(\Box\)

Theorem 5.10. If \((\hat{F}, A)\) and \((\hat{\Theta}, B)\) are two fuzzy soft left (right) \(\Gamma\)-hyperideals over \(S\), then so are \((\hat{F}, A)\cap(\hat{\Theta}, B)\) and \((\hat{F}, A)\cap(\hat{\Theta}, B)\).
Proof. Let \((\mathcal{F}, A)\) and \((\mathcal{\tilde{\Theta}}, B)\) be two fuzzy soft left \(\Gamma\)-hyperideals over \(S\). We know that \((\mathcal{F}, A)\widehat{\vee}(\mathcal{\tilde{\Theta}}, B) = \left(\mathcal{\hat{\Xi}}, C\right)\), where \(C = A \times B\) and \(\mathcal{\hat{\Xi}} = \mathcal{F}_\delta \cup \mathcal{\tilde{\Theta}}\) for all \((\delta, \varepsilon) \in A \times B\). Now for any \((\delta, \varepsilon) \in A \times B\), since \((\mathcal{F}, A)\) and \((\mathcal{\tilde{\Theta}}, B)\) are fuzzy soft left \(\Gamma\)-hyperideals over \(S\), for all \(x, y \in S\), \((\delta, \varepsilon) \in A \times B\) and \(\gamma \in \Gamma\), we have
\[
\inf_{z \in x \gamma y} \left\{ \widehat{\Xi}(\delta, \varepsilon)(z) \right\} = \inf_{z \in x \gamma y} \left\{ \max \left\{ \mathcal{F}_\delta(z), \mathcal{\tilde{\Theta}}(\varepsilon)(z) \right\} \right\} \\
= \max \left\{ \inf_{z \in x \gamma y} \mathcal{F}_\delta(z), \inf_{z \in x \gamma y} \mathcal{\tilde{\Theta}}(\varepsilon)(z) \right\} \\
\geq \max \left\{ \mathcal{F}_\delta(y), \mathcal{\tilde{\Theta}}(\varepsilon)(y) \right\} \\
= \hat{\Xi}(\delta, \varepsilon)(y).
\]
This shows that \((\mathcal{F}, A)\widehat{\vee}(\mathcal{\tilde{\Theta}}, B)\) is a fuzzy soft left \(\Gamma\)-hyperideal over \(S\). The other cases can be seen in a similar way. \(\Box\)

**Theorem 5.11.** If \((\mathcal{F}, A)\) and \((\mathcal{\tilde{\Theta}}, B)\) are two fuzzy soft \(LA\)-\(\Gamma\)-semihypergroups over \(S\), then so is \((\mathcal{F}, A)\bar{\cap}(\mathcal{\tilde{\Theta}}, B)\).

**Proof.** We know that \((\mathcal{F}, A)\bar{\cap}(\mathcal{\tilde{\Theta}}, B) = \left(\mathcal{\hat{\Xi}}, C\right)\), where \(C = A \cup B\) and for all \(\delta \in C\)
\[
\hat{\Xi}_\delta = \begin{cases} \\
\mathcal{F}_\delta & \text{if } \delta \in A \setminus B \\
\mathcal{\tilde{\Theta}}_\delta & \text{if } \delta \in B \setminus A \\
\min \{ \mathcal{F}_\delta, \mathcal{\tilde{\Theta}}_\delta \} & \text{if } \delta \in A \cap B. \\
\end{cases}
\]
Now for any \(\delta \in C\) and \(x, y \in S\), we consider the following cases

**Case 1:** For any \(\delta \in A \setminus B\) and \(\gamma \in \Gamma\),
\[
\inf_{z \in x \gamma y} \left\{ \hat{\Xi}_\delta(z) \right\} = \inf_{z \in x \gamma y} \left\{ \mathcal{F}_\delta(z) \right\} \geq \min \{ \mathcal{F}_\delta(x), \mathcal{F}_\delta(y) \} \\
= \min \left\{ \hat{\Xi}_\delta(x), \hat{\Xi}_\delta(y) \right\}.
\]

**Case 2:** For any \(\delta \in B \setminus A\) and \(\gamma \in \Gamma\),
\[
\inf_{z \in x \gamma y} \left\{ \hat{\Xi}_\delta(z) \right\} = \inf_{z \in x \gamma y} \left\{ \mathcal{\tilde{\Theta}}_\delta(z) \right\} \geq \min \{ \mathcal{\tilde{\Theta}}_\delta(x), \mathcal{\tilde{\Theta}}_\delta(y) \} \\
= \min \left\{ \hat{\Xi}_\delta(x), \hat{\Xi}_\delta(y) \right\}.
\]

**Case 3:** For any \(\delta \in A \cap B\) and \(\gamma \in \Gamma\), then \(\hat{\Xi}_\delta = \mathcal{F}_\delta \cap \mathcal{\tilde{\Theta}}_\delta\). Analogous to the proof of Theorem 5.7, we have
\[
\inf_{z \in x \gamma y} \left\{ \hat{\Xi}_\delta(z) \right\} \geq \min \left\{ \hat{\Xi}_\delta(x), \hat{\Xi}_\delta(y) \right\}.
\]
Thus in any case we have \( \inf_{z \in x \gamma y} \{ \hat{\Xi}_\delta(z) \} \geq \min \{ \hat{\Xi}_\delta(x), \hat{\Xi}_\delta(y) \} \), and so \((\hat{F}, A) \cap (\hat{\Theta}, B)\) is a fuzzy soft LA-\(\Gamma\)-semihypergroup over \(S\).

**Theorem 5.12.** If \((\hat{F}, A)\) and \((\hat{\Theta}, B)\) are two fuzzy soft left (right) \(\Gamma\)-hyperideals over \(S\), then so is \((\hat{F}, A) \cap (\hat{\Theta}, B)\).

**Proof.** Let \((\hat{F}, A)\) and \((\hat{\Theta}, B)\) be two fuzzy soft left \(\Gamma\)-hyperideals over \(S\). We know that \((\hat{F}, A) \cap (\hat{\Theta}, B) = \left(\hat{\Xi}, C\right)\), where \(C = A \cup B\) and for all \(\delta \in C\)

\[
\hat{\Xi}_\delta = \begin{cases} 
\hat{F}_\delta & \text{if } \delta \in A \setminus B \\
\hat{\Theta}_\delta & \text{if } \delta \in B \setminus A \\
\min \{ \hat{F}_\delta, \hat{\Theta}_\delta \} & \text{if } \delta \in A \cap B.
\end{cases}
\]

Now for any \(\delta \in C\) and \(x, y \in S\), we consider the following cases

**Case 1:** For any \(\delta \in A \setminus B\) and \(\gamma \in \Gamma\),

\[
\inf_{z \in x \gamma y} \{ \hat{\Xi}_\delta(z) \} = \inf_{z \in x \gamma y} \{ \hat{F}_\delta(z) \} \geq \hat{F}_\delta(y) = \hat{\Xi}_\delta(y).
\]

**Case 2:** For any \(\delta \in B \setminus A\) and \(\gamma \in \Gamma\),

\[
\inf_{z \in x \gamma y} \{ \hat{\Xi}_\delta(z) \} = \inf_{z \in x \gamma y} \{ \hat{\Theta}_\delta(z) \} \geq \hat{\Theta}_\delta(y) = \hat{\Xi}_\delta(y).
\]

**Case 3:** For any \(\delta \in A \cap B\) and \(\gamma \in \Gamma\), then \(\hat{\Xi}_\delta = \hat{F}_\delta \cap \hat{\Theta}_\delta\). Analogous to the proof of Theorem 5.8, we have

\[
\inf_{z \in x \gamma y} \{ \hat{\Xi}_\delta(z) \} \geq \hat{\Xi}_\delta(y).
\]

Thus in any case we have \(\inf_{z \in x \gamma y} \{ \hat{\Xi}_\delta(z) \} \geq \hat{\Xi}_\delta(y)\), and so \((\hat{F}, A) \cap (\hat{\Theta}, B)\) is a fuzzy soft left \(\Gamma\)-hyperideal over \(S\). The other case can be proved in a similar way.

**Theorem 5.13.** If \((\hat{F}, A)\) and \((\hat{\Theta}, B)\) are two fuzzy soft LA-\(\Gamma\)-semihypergroups over \(S\), then so is \((\hat{F}, A) \cup (\hat{\Theta}, B)\).

**Proof.** The proof of this theorem is similar to the proof of Theorem 5.11.

**Theorem 5.14.** If \((\hat{F}, A)\) and \((\hat{\Theta}, B)\) are two fuzzy soft left (right) \(\Gamma\)-hyperideals over \(S\), then so is \((\hat{F}, A) \cup (\hat{\Theta}, B)\).

**Proof.** The proof of this theorem is similar to the proof of Theorem 5.12.
References


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