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On Fuzzy Soft Γ-Hyperideals over Left Almost Γ-Semihypergroups

***Moin A. Ansari and Ibtisam A. H. Masmali**

Department of Mathematics
Faculty of Science
Jazan University, Jazan
Kingdom of Saudi Arabia
*moinakhtar83@gmail.com

Abstract

In this paper we have introduced the notion of fuzzy soft Γ-hyperideals in left almost Γ-semihypergroups. Also we have studied some properties based on them with t-level set, union and intersection.

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1 Introduction

Hyperstructure theory was born in 1934 when Marty [12], defined hypergroups, began to analyze their properties and applied them to groups, rational algebraic functions. In 1986, Sen and Saha [15], introduced the concept of the Γ-semigroup as a generalization of semigroup and ternary semigroup. Many classical notions and results of the theory of semigroups have been extended and generalized to Γ-semigroups. Davvaz et al. [2, 8], introduced the notion of Γ-semihypergroup. Recently, Yaqoob and Aslam [18], introduced the notion of LA-Γ-semihypergroup as a generalization of semigroup, commutative semihypergroup and of commutative Γ-semigroup. They proved some results in this respect and presented some examples of LA-Γ-semihypergroups. Yaqoob et al. [19, 20], applied rough set theory and soft set theory to LA-Γ-semihypergroups, also see [3, 4, 21, 22, 23].

In 1965, Zadeh [24], introduced the notion of a fuzzy subset of a non-empty set X, as a function from X to $[0, 1]$. Recently, fuzzy set theory has been well developed in the context of hyperalgebraic structure theory. A recent book [6], contains a wealth of applications. Recently in [7], Davvaz and Leoreanu-Fotea studied the structure of fuzzy Γ-hyperideals in Γ-semihypergroups.

The concept of soft set was given by Molodtsov [13] in 1999, which is a completely new approach for modeling and uncertainty. After Molodtsov's work, some different applications of soft sets were studied in [5] and [10]. Also Maji et al. [11], presented the definition of fuzzy soft set. Some corrections were given by Ali et al. [1]. Roy et al. [14], presented some applications of this notion to decision making problems. Many authors studied the properties of fuzzy soft sets in different algebraic structures, for instance, Jun et al. [9], Williams and Saeid [16] and Yang [17].

In this paper, some properties of fuzzy soft Γ-hyperideals in left almost Γ-semihypergroups have been discussed.

2 Left Almost Γ-semihypergroups

In this section, we recall certain definitions and results needed for our purpose. Let S be a non-empty set and $\mathcal{P}^*(S)$ be the set of all non-empty subsets of S. The map $\circ : S \times S \to \mathcal{P}^*(S)$ is called hyperoperation or join operation on the set S. A couple $(S, \circ)$ is called a hypergroupoid. If A and B be two non-empty subsets of S, then we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \qquad a \circ A = \{a\} \circ A \text{ and } a \circ B = \{a\} \circ B.$$

Definition 2.1. *[18] Let S and Γ be two non-empty sets. S is called a left almost Γ-semihypergroup (abbreviated as an LA-Γ-semihypergroup) if every $\gamma \in \Gamma$ is a hyperoperation on S, i.e, $x\gamma y \subseteq S$ for every $x, y \in S$, and for every $\gamma, \beta \in \Gamma$ and $x, y, z \in S$ we have $(x\gamma y)\beta z = (z\gamma y)\beta x$.*

The law $(x\gamma y)\beta z = (z\gamma y)\beta x$ is called left invertive law. Let A and B be two non-empty subsets of an LA-Γ-semihypergroup S. Then we define

$$A\gamma B = \bigcup \{a\gamma b \mid a \in A,\ b \in B \text{ and } \gamma \in \Gamma\},$$

also

$$A\Gamma B = \bigcup_{\gamma \in \Gamma} A\gamma B = \bigcup \{a\gamma b \mid a \in A,\ b \in B \text{ and } \gamma \in \Gamma\}.$$

Example 2.2. *Let $S = \{a, b, c, d\}$ and $\Gamma = \{\beta, \gamma\}$ be the sets of binary hyperoperations defined below:*

β	a	b	c	d
a	$\{a,c\}$	$\{a,b,c\}$	$\{a,c\}$	d
b	$\{a,b,c\}$	$\{a,b\}$	$\{b,c\}$	d
c	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	d
d	d	d	d	S

γ	a	b	c	d
a	$\{a,b\}$	$\{a,b\}$	c	d
b	$\{a,c\}$	$\{a,c\}$	$\{a,b,c\}$	d
c	$\{b,c\}$	c	$\{a,b,c\}$	d
d	d	d	d	S

Clearly S is not a Γ-semihypergroup because $\{a, b, c\} = (a\beta b)\gamma b \neq a\beta(b\gamma b) = \{a, c\}$. Thus S is an LA-Γ-semihypergroup because S satisfies the left invertive law.

Every LA-Γ-semihypergroup satisfies the law $(a\alpha b)\beta(c\gamma d) = (a\alpha c)\beta(b\gamma d)$ for all $a, b, c, d \in S$ and $\alpha, \beta, \gamma \in \Gamma$. This law is known as Γ-hypermedial law. (cf. [18]).

Definition 2.3. *[18] Let K be a non-empty subset of S. Then K is called a sub LA-Γ-semihypergroup of S if $a\gamma b \subseteq K$ for all $a, b \in K$ and $\gamma \in \Gamma$.*

Definition 2.4. *[18] A non-empty subset A of an LA-Γ-semihypergroup S is called a right (left) Γ-hyperideal of S if $A\Gamma S \subseteq A$ ($S\Gamma A \subseteq A$), and is a Γ-hyperideal of S if it is both a right and a left Γ-hyperideal.*

3 Fuzzy Soft Sets

Molodtsov defined the notion of a soft set as follows. Let U be an initial universe and E be the set of parameters. Usually, parameters are attributes, characteristics or properties of an object. Let $P(U)$ denote the power set of U and A is a subset of E.

Definition 3.1. *[13] A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \longrightarrow P(U)$. In other words a soft set over U is a parametrized family of subsets of U.*

Definition 3.2. *[11] A pair $(\widehat{\mathcal{F}}, A)$ is called a fuzzy soft set over U, where $\widehat{\mathcal{F}} : A \longrightarrow \wp(U)$ is a mapping and $\wp(U)$ being the set of all fuzzy subsets of S.*

Definition 3.3. *Let $(\widehat{\mathcal{F}}, A)$ and $(\widehat{\Theta}, B)$ be two fuzzy soft sets over a common universe U, we say $(\widehat{\mathcal{F}}, A)$ is a fuzzy soft subset of $(\widehat{\Theta}, B)$, if (i) $A \subseteq B$, (ii) for all $\delta \in A$, $\widehat{\mathcal{F}}_\delta$ is a fuzzy subset of $\widehat{\Theta}_\delta$ and we denote it by $(\widehat{\mathcal{F}}, A)\widehat{\subseteq}(\widehat{\Theta}, B)$.*

Definition 3.4. *Let $(\widehat{\mathcal{F}}, A)$ and $(\widehat{\Theta}, B)$ be two fuzzy soft sets over a common universe U. Then $(\widehat{\mathcal{F}}, A)\widehat{\wedge}(\widehat{\Theta}, B)$ is defined by $(\widehat{\mathcal{F}}, A)\widehat{\wedge}(\widehat{\Theta}, B) = (\widehat{\Xi}, A \times B)$, where $\widehat{\Xi}_{(\delta,\varepsilon)} = \widehat{\mathcal{F}}_\delta \cap \widehat{\Theta}_\varepsilon$ for all $(\delta, \varepsilon) \in A \times B$.*

Definition 3.5. *Let* $(\widehat{\mathcal{F}}, A)$ *and* $(\widehat{\Theta}, B)$ *be two fuzzy soft sets over a common universe* U*. Then* $(\widehat{\mathcal{F}}, A)\widehat{\vee}(\widehat{\Theta}, B)$ *is defined by* $(\widehat{\mathcal{F}}, A)\widehat{\vee}(\widehat{\Theta}, B) = (\widehat{\Xi}, A \times B)$*, where* $\widehat{\Xi}_{(\delta,\varepsilon)} = \widehat{\mathcal{F}}_\delta \cup \widehat{\Theta}_\varepsilon$ *for all* $(\delta, \varepsilon) \in A \times B$*.*

Definition 3.6. *Let* $(\widehat{\mathcal{F}}, A)$ *and* $(\widehat{\Theta}, B)$ *be two fuzzy soft sets over a common universe* U*. Then,*

(i) the union $\left(\widehat{\Xi}, C\right)$ *of two soft sets* $(\widehat{\mathcal{F}}, A)$ *and* $(\widehat{\Theta}, B)$ *is defined as the soft set* $\left(\widehat{\Xi}, C\right) = (\widehat{\mathcal{F}}, A)\widehat{\cup}(\widehat{\Theta}, B)$*, where* $C = A \cup B$ *and for all* $\delta \in C$

$$\widehat{\Xi}_\delta = \begin{cases} \widehat{\mathcal{F}}_\delta & \text{if } \delta \in A\backslash B \\ \widehat{\Theta}_\delta & \text{if } \delta \in B\backslash A \\ \max\left\{\widehat{\mathcal{F}}_\delta, \widehat{\Theta}_\delta\right\} & \text{if } \delta \in A \cap B \end{cases}$$

(ii) the intersection $\left(\widehat{\Xi}, C\right)$ *of two soft sets* $(\widehat{\mathcal{F}}, A)$ *and* $(\widehat{\Theta}, B)$ *is defined as the soft set* $\left(\widehat{\Xi}, C\right) = (\widehat{\mathcal{F}}, A)\widehat{\cap}(\widehat{\Theta}, B)$*, where* $C = A \cup B$ *and for all* $\delta \in C$

$$\widehat{\Xi}_\delta = \begin{cases} \widehat{\mathcal{F}}_\delta & \text{if } \delta \in A\backslash B \\ \widehat{\Theta}_\delta & \text{if } \delta \in B\backslash A \\ \min\left\{\widehat{\mathcal{F}}_\delta, \widehat{\Theta}_\delta\right\} & \text{if } \delta \in A \cap B. \end{cases}$$

In contrast with the above definitions of fuzzy soft set union and intersection, we may sometimes adopt different definitions of union and intersection given as follows.

Definition 3.7. *Let* $(\widehat{\mathcal{F}}, A)$ *and* $(\widehat{\Theta}, B)$ *be two fuzzy soft sets over a common universe* U *and* $A \cap B \neq \emptyset$*. Then the bi-union of* $(\widehat{\mathcal{F}}, A)$ *and* $(\widehat{\Theta}, B)$ *is defined to be the fuzzy soft set* $\left(\widehat{\Xi}, C\right)$*, where* $C = A \cap B$ *and* $\widehat{\Xi}_\delta = \widehat{\mathcal{F}}_\delta \cup \widehat{\Theta}_\delta$ *for all* $\delta \in C$*. This is denoted by* $\left(\widehat{\Xi}, C\right) = (\widehat{\mathcal{F}}, A)\widehat{\sqcup}(\widehat{\Theta}, B)$*.*

Definition 3.8. *Let* $(\widehat{\mathcal{F}}, A)$ *and* $(\widehat{\Theta}, B)$ *be two fuzzy soft sets over a common universe* U *and* $A \cap B \neq \emptyset$*. Then the bi-intersection of* $(\widehat{\mathcal{F}}, A)$ *and* $(\widehat{\Theta}, B)$ *is defined to be the fuzzy soft set* $\left(\widehat{\Xi}, C\right)$*, where* $C = A \cap B$ *and* $\widehat{\Xi}_\delta = \widehat{\mathcal{F}}_\delta \cap \widehat{\Theta}_\delta$ *for all* $\delta \in C$*. This is denoted by* $\left(\widehat{\Xi}, C\right) = (\widehat{\mathcal{F}}, A)\widehat{\sqcap}(\widehat{\Theta}, B)$*.*

4 Fuzzy Γ-hyperideals in LA-Γ-semihypergroups

In this section, we define the notion of fuzzy left (right) Γ-hyperideals in left almost Γ-semihypergroups. Throughout the paper S will denote an LA-Γ-semihypergroup.

Definition 4.1. *Let μ and λ be two fuzzy subsets of S. Then, we have:*

$$\begin{aligned} \mu \cap \lambda &= \min\{\mu(x), \lambda(x)\}, \\ (\mu\Gamma\lambda)(z) &= \begin{cases} \sup\limits_{z\in x\gamma y}\{\min\{\mu(x),\lambda(y)\}\} & \text{if } z \in x\gamma y, \forall \gamma \in \Gamma \\ 0 & \text{otherwise,} \end{cases} \end{aligned}$$

for all $x, y, z \in S$.

Definition 4.2. *Let μ be a fuzzy subset of S. Then μ is called a sub LA-Γ-semihypergroup of S if for all $x, y \in S$ and $\gamma \in \Gamma$,*

$$\inf_{z\in x\gamma y}\{\mu(z)\} \geq \min\{\mu(x), \mu(y)\}.$$

Lemma 4.3. *Let μ be a fuzzy subset of S. Then μ is a fuzzy sub LA-Γ-semihypergroup of S if and only if $\mu\Gamma\mu \subseteq \mu$.*

Proof. The proof is straightforward. □

Definition 4.4. *Let μ be a fuzzy subset of S. Then, for all $x, y, z \in S$ and $\gamma \in \Gamma$,*

(i) μ is called a fuzzy left Γ-hyperideal of S if $\mu(y) \leq \inf\limits_{z\in x\gamma y}\{\mu(z)\}$,

(ii) μ is called a fuzzy right Γ-hyperideal of S if $\mu(x) \leq \inf\limits_{z\in x\gamma y}\{\mu(z)\}$,

(iii) μ is called a fuzzy Γ-hyperideal of S if it is both a fuzzy left Γ-hyperideal and fuzzy right Γ-hyperideal of S.

Example 4.5. *Let $S = \{a, b, c, d\}$ and $\Gamma = \{\beta, \gamma\}$ be the sets of binary hyperoperations defined below:*

β	a	b	c	d
a	b	b	$\{c,d\}$	d
b	$\{a,b\}$	$\{a,b\}$	$\{c,d\}$	d
c	$\{c,d\}$	$\{c,d\}$	c	d
d	d	d	d	d

γ	a	b	c	d
a	$\{a,b\}$	b	$\{c,d\}$	d
b	b	$\{a,b\}$	$\{c,d\}$	d
c	$\{c,d\}$	$\{c,d\}$	c	d
d	d	d	d	d

Clearly S is not a Γ-semihypergroup because $\{a, b\} = (a\beta a)\gamma b \neq a\beta(a\gamma b) = \{b\}$. Thus S is an LA-Γ-semihypergroup because it satisfies the left invertive law. Let μ be a fuzzy subset of S, which is defined as

$$\mu = \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.5}, \frac{d}{0.7}\right).$$

By routine calculations, it can be seen that μ is a fuzzy Γ-hyperideal of S.

Definition 4.6. *For any $t \in [0,1]$ and fuzzy subset μ of S, the set*

$$U(\mu, t) = \{x \in S : \mu(x) \geq t\},$$

is called an upper t-level cut of μ.

Theorem 4.7. *A fuzzy subset μ of S is a fuzzy left (right) Γ-hyperideal of S if and only if $U(\mu, t)$ is a left (right) Γ-hyperideal of S.*

Proof. The proof is straightforward. □

5 Fuzzy Soft Γ-hyperideals over Left Almost Γ-semihypergroups

In this section, we define the notion of fuzzy soft left (right) Γ-hyperideals over left almost Γ-semihypergroups.

Definition 5.1. *Let S be an LA-Γ-semihypergroup and $(\widehat{\mathcal{F}}, A)$ be a fuzzy soft set over S. Then $(\widehat{\mathcal{F}}, A)$ is called a fuzzy soft LA-Γ-semihypergroup over S if it satisfies*

$$\inf_{z \in x\gamma y}\{\widehat{\mathcal{F}}_\delta(z)\} \geq \min\{\widehat{\mathcal{F}}_\delta(x), \widehat{\mathcal{F}}_\delta(y)\},$$

for all $x, y \in S$, $\gamma \in \Gamma$ and $\delta \in A$.

Definition 5.2. *Let $(\widehat{\mathcal{F}}, A)$ be a fuzzy soft set over S. Then, for all $x, y \in S$, $\gamma \in \Gamma$ and $\delta \in A$,*

(i) $(\widehat{\mathcal{F}}, A)$ is called a fuzzy soft left Γ-hyperideal over S if

$$\widehat{\mathcal{F}}_\delta(y) \leq \inf_{z \in x\gamma y}\left\{\widehat{\mathcal{F}}_\delta(z)\right\};$$

(ii) $(\widehat{\mathcal{F}}, A)$ is called a fuzzy soft right Γ-hyperideal over S if

$$\widehat{\mathcal{F}}_\delta(x) \leq \inf_{z \in x\gamma y}\left\{\widehat{\mathcal{F}}_\delta(z)\right\};$$

(iii) $(\widehat{\mathcal{F}}, A)$ is called a fuzzy soft Γ-hyperideal over S if it is both a fuzzy soft left Γ-hyperideal and fuzzy soft right Γ-hyperideal over S.

Example 5.3. *Let $S = \{a, b, c, d\}$ and $\Gamma = \{\beta, \gamma\}$ be the sets of binary hyperoperations defined below:*

β	a	b	c	d
a	a	a	a	d
b	a	$\{a,c\}$	$\{a,c\}$	d
c	a	$\{b,c\}$	$\{b,c\}$	d
d	d	d	d	d

γ	a	b	c	d
a	a	a	a	d
b	a	$\{a,b,c\}$	$\{a,b,c\}$	d
c	a	$\{b,c\}$	$\{b,c\}$	d
d	d	d	d	d

Clearly S is not a Γ-semihypergroup because $\{a,b,c\} = (c\beta c)\gamma b \neq c\beta(c\gamma b) = \{b,c\}$. Thus S is an LA-Γ-semihypergroup because it satisfies the left invertive law. Let $A = \{\delta, \varepsilon, \kappa, \xi\}$ be the set of parameters. For each parameter $\delta \in A$, $\widehat{\mathcal{F}}_\delta$ is a fuzzy Γ-hyperideal of S, where $\widehat{\mathcal{F}}_\delta$ is a mapping given by $\widehat{\mathcal{F}}_\delta : S \longrightarrow [0,1]$. For each parameter we define

$$\begin{aligned}\widehat{\mathcal{F}}_\delta &= \left(\frac{a}{0.4},\frac{b}{0.1},\frac{c}{0.1},\frac{d}{0.8}\right), \quad \widehat{\mathcal{F}}_\varepsilon = \left(\frac{a}{0.3},\frac{b}{0.3},\frac{c}{0.3},\frac{d}{0.7}\right),\\ \widehat{\mathcal{F}}_\kappa &= \left(\frac{a}{0.5},\frac{b}{0.3},\frac{c}{0.3},\frac{d}{0.5}\right), \quad \widehat{\mathcal{F}}_\xi = \left(\frac{a}{0.6},\frac{b}{0.2},\frac{c}{0.2},\frac{d}{0.9}\right).\end{aligned}$$

Each $\widehat{\mathcal{F}}_\delta$ satisfies the conditions of fuzzy Γ-hyperideal of S. Hence $(\widehat{\mathcal{F}}, A) = \left\{\widehat{\mathcal{F}}_\delta : \delta \in A\right\}$ is a fuzzy soft Γ-hyperideal of S.

Definition 5.4. *Let $(\widehat{\mathcal{F}}, A)$ be a fuzzy soft set over S, for each $t \in [0,1]$, the set $(\widehat{\mathcal{F}}, A)^t = (\widehat{\mathcal{F}}^t, A)$ is called a t-level set of $(\widehat{\mathcal{F}}, A)$, where $\widehat{\mathcal{F}}^t_\delta = \left\{x \in S : \widehat{\mathcal{F}}_\delta(x) \geq t\right\}$ for each $\delta \in A$.*

Theorem 5.5. *Let $(\widehat{\mathcal{F}}, A)$ be a fuzzy soft set over S, $(\widehat{\mathcal{F}}, A)$ is a fuzzy soft LA-Γ-semihypergroup if and only if $(\widehat{\mathcal{F}}, A)^t$ is a soft LA-Γ-semihypergroup over S for each $t \in [0,1]$.*

Proof. Assume that $(\widehat{\mathcal{F}}, A)^t$ is a soft LA-Γ-semihypergroup over S for each $t \in [0,1]$. For each $x_1, x_2 \in S$ and $\delta \in A$, let $t = \min\left\{\widehat{\mathcal{F}}_\delta(x_1), \widehat{\mathcal{F}}_\delta(x_2)\right\}$, then $x_1, x_2 \in \widehat{\mathcal{F}}^t_\delta$. Since $\widehat{\mathcal{F}}^t_\delta$ is an LA-Γ-semihypergroup over S, then for each $\gamma \in \Gamma$, $x_1\gamma x_2 \subseteq \widehat{\mathcal{F}}^t_\delta$. Therefore for all $z \in x_1\gamma x_2$ we have $\widehat{\mathcal{F}}_\delta(z) \geq t$, and so

$$\inf_{z\in x_1\gamma x_2}\left\{\widehat{\mathcal{F}}_\delta(z)\right\} \geq t = \min\left\{\widehat{\mathcal{F}}_\delta(x_1), \widehat{\mathcal{F}}_\delta(x_2)\right\}, \text{ for all } x_1, x_2 \in S,\ \delta \in A \text{ and } \gamma \in \Gamma.$$

This shows that $\widehat{\mathcal{F}}_\delta$ is a fuzzy sub LA-Γ-semihypergroup over S. Thus $(\widehat{\mathcal{F}}, A)$ is a fuzzy soft LA-Γ-semihypergroup over S.

Conversely, let $(\widehat{\mathcal{F}}, A)$ be a fuzzy soft LA-Γ-semihypergroup. For each $\delta \in A$, $t \in [0,1]$ and $x_1, x_2 \in \widehat{\mathcal{F}}^t_\delta$ we have $\widehat{\mathcal{F}}_\delta(x_1) \geq t$ and $\widehat{\mathcal{F}}_\delta(x_2) \geq t$. As $\widehat{\mathcal{F}}_\delta$ is a fuzzy LA-Γ-semihypergroup over S, thus for $\gamma \in \Gamma$ there exist $z \in x_1\gamma x_2$ such that

$$\inf_{z\in x_1\gamma x_2}\left\{\widehat{\mathcal{F}}_\delta(z)\right\} \geq \min\left\{\widehat{\mathcal{F}}_\delta(x_1), \widehat{\mathcal{F}}_\delta(x_2)\right\} \geq t.$$

Therefore for all $z \in x_1\gamma x_2$ we have $z \in \widehat{\mathcal{F}}^t_\delta$, this implies that $x_1\gamma x_2 \subseteq \widehat{\mathcal{F}}^t_\delta$, i.e., $\widehat{\mathcal{F}}^t_\delta$ is a sub LA-Γ-semihypergroup over S. Thus $(\widehat{\mathcal{F}}, A)^t$ is a soft LA-Γ-semihypergroup over S for each $t \in [0,1]$. This completes the proof. □

Theorem 5.6. *Let $(\widehat{\mathcal{F}}, A)$ be a fuzzy soft set over S, $(\widehat{\mathcal{F}}, A)$ is a fuzzy soft left (right) Γ-hyperideal if and only if $(\widehat{\mathcal{F}}, A)^t$ is a soft left (right) Γ-hyperideal over S for each $t \in [0,1]$.*

Proof. Assume that $(\widehat{\mathcal{F}}, A)^t$ is a soft left Γ-hyperideal over S for each $t \in [0,1]$. For each $x \in S$ and $\delta \in A$, let $t = \widehat{\mathcal{F}}_\delta(x)$, then $x \in \widehat{\mathcal{F}}_\delta^t$. Since $\widehat{\mathcal{F}}_\delta^t$ is a left Γ-hyperideal over S, then $y\gamma x \subseteq \widehat{\mathcal{F}}_\delta^t$ for each $y \in S$ and $\gamma \in \Gamma$. Therefore for all $z \in y\gamma x$ we have $\widehat{\mathcal{F}}_\delta(z) \geq t$, and so

$$\inf_{z \in y\gamma x} \left\{ \widehat{\mathcal{F}}_\delta(z) \right\} \geq t = \widehat{\mathcal{F}}_\delta(x), \text{ for all } x, y \in S,\ \delta \in A \text{ and } \gamma \in \Gamma.$$

This shows that $\widehat{\mathcal{F}}_\delta$ is a fuzzy left Γ-hyperideal. Thus $(\widehat{\mathcal{F}}, A)$ is a fuzzy soft left Γ-hyperideal over S.

Conversely, let $(\widehat{\mathcal{F}}, A)$ be a fuzzy soft left Γ-hyperideal. For each $\delta \in A$, $t \in [0,1]$ and $x \in \widehat{\mathcal{F}}_\delta^t$ we have $\widehat{\mathcal{F}}_\delta(x) \geq t$. As $\widehat{\mathcal{F}}_\delta$ is a fuzzy left Γ-hyperideal over S, thus for $y \in S$ there exist $z \in y\gamma x$ such that

$$\inf_{z \in y\gamma x} \left\{ \widehat{\mathcal{F}}_\delta(z) \right\} \geq \widehat{\mathcal{F}}_\delta(x) \geq t.$$

Therefore for all $z \in y\gamma x$ we have $z \in \widehat{\mathcal{F}}_\delta^t$, this implies that $y\gamma x \subseteq \widehat{\mathcal{F}}_\delta^t$, i.e., $\widehat{\mathcal{F}}_\delta^t$ is a left Γ-hyperideal over S. Thus $(\widehat{\mathcal{F}}, A)^t$ is a soft left Γ-hyperideal over S for each $t \in [0,1]$. The case for right Γ-hyperideal can be seen in a similar way. □

Theorem 5.7. *If $(\widehat{\mathcal{F}}, A)$ and $(\widehat{\Theta}, B)$ are two fuzzy soft LA-Γ-semihypergroups over S, then so are $(\widehat{\mathcal{F}}, A)\widehat{\wedge}(\widehat{\Theta}, B)$ and $(\widehat{\mathcal{F}}, A)\widehat{\sqcap}(\widehat{\Theta}, B)$.*

Proof. We know that $(\widehat{\mathcal{F}}, A)\widehat{\wedge}(\widehat{\Theta}, B) = \left(\widehat{\Xi}, C\right)$, where $C = A \times B$ and $\widehat{\Xi}_{(\delta,\varepsilon)} = \widehat{\mathcal{F}}_\delta \cap \widehat{\Theta}_\varepsilon$ for all $(\delta,\varepsilon) \in A \times B$. Now for any $(\delta,\varepsilon) \in A \times B$, since $(\widehat{\mathcal{F}}, A)$ and $(\widehat{\Theta}, B)$ are fuzzy soft LA-Γ-semihypergroups over S, for all $x, y \in S$, $(\delta,\varepsilon) \in A \times B$ and $\gamma \in \Gamma$, we have

$$\begin{aligned}
\inf_{z \in x\gamma y} \left\{ \widehat{\Xi}_{(\delta,\varepsilon)}(z) \right\} &= \inf_{z \in x\gamma y} \left\{ \min\left\{ \widehat{\mathcal{F}}_\delta(z), \widehat{\Theta}_\varepsilon(z) \right\} \right\} \\
&= \min\left\{ \inf_{z \in x\gamma y} \widehat{\mathcal{F}}_\delta(z), \inf_{z \in x\gamma y} \widehat{\Theta}_\varepsilon(z) \right\} \\
&\geq \min\left\{ \min\{\widehat{\mathcal{F}}_\delta(x), \widehat{\mathcal{F}}_\delta(y)\}, \min\{\widehat{\Theta}_\varepsilon(x), \widehat{\Theta}_\varepsilon(y)\} \right\} \\
&= \min\left\{ \min\{\widehat{\mathcal{F}}_\delta(x), \widehat{\Theta}_\varepsilon(x)\}, \min\{\widehat{\mathcal{F}}_\delta(y), \widehat{\Theta}_\varepsilon(y)\} \right\} \\
&= \min\left\{ \widehat{\Xi}_{(\delta,\varepsilon)}(x), \widehat{\Xi}_{(\delta,\varepsilon)}(y) \right\}.
\end{aligned}$$

This shows that $(\widehat{\mathcal{F}}, A)\widehat{\wedge}(\widehat{\Theta}, B)$ is a fuzzy soft LA-Γ-semihypergroup over S. The other case can be seen in a similar way. □

Theorem 5.8. *If* $(\widehat{\mathcal{F}}, A)$ *and* $(\widehat{\Theta}, B)$ *are two fuzzy soft left (right)* Γ*-hyperideals over* S, *then so are* $(\widehat{\mathcal{F}}, A)\widehat{\wedge}(\widehat{\Theta}, B)$ *and* $(\widehat{\mathcal{F}}, A)\widehat{\sqcap}(\widehat{\Theta}, B)$.

Proof. Let $(\widehat{\mathcal{F}}, A)$ and $(\widehat{\Theta}, B)$ be two fuzzy soft left Γ-hyperideals over S. We know that $(\widehat{\mathcal{F}}, A)\widehat{\wedge}(\widehat{\Theta}, B) = \left(\widehat{\Xi}, C\right)$, where $C = A \times B$ and $\widehat{\Xi}_{(\delta,\varepsilon)} = \widehat{\mathcal{F}}_\delta \cap \widehat{\Theta}_\varepsilon$ for all $(\delta, \varepsilon) \in A \times B$. Now for any $(\delta, \varepsilon) \in A \times B$, since $(\widehat{\mathcal{F}}, A)$ and $(\widehat{\Theta}, B)$ are fuzzy soft left Γ-hyperideals over S, for all $x, y \in S$, $(\delta, \varepsilon) \in A \times B$ and $\gamma \in \Gamma$, we have

$$\begin{aligned}
\inf_{z\in x\gamma y}\left\{\widehat{\Xi}_{(\delta,\varepsilon)}(z)\right\} &= \inf_{z\in x\gamma y}\left\{\min\left\{\widehat{\mathcal{F}}_\delta(z), \widehat{\Theta}_\varepsilon(z)\right\}\right\} \\
&= \min\left\{\inf_{z\in x\gamma y}\widehat{\mathcal{F}}_\delta(z), \inf_{z\in x\gamma y}\widehat{\Theta}_\varepsilon(z)\right\} \\
&\geq \min\left\{\widehat{\mathcal{F}}_\delta(y), \widehat{\Theta}_\varepsilon(y)\right\} \\
&= \widehat{\Xi}_{(\delta,\varepsilon)}(y).
\end{aligned}$$

This shows that $(\widehat{\mathcal{F}}, A)\widehat{\wedge}(\widehat{\Theta}, B)$ is a fuzzy soft left Γ-hyperideal over S. The other cases can be seen in a similar way. □

Theorem 5.9. *If* $(\widehat{\mathcal{F}}, A)$ *and* $(\widehat{\Theta}, B)$ *are two fuzzy soft LA-*Γ*-semihypergroups over* S, *then so are* $(\widehat{\mathcal{F}}, A)\widehat{\vee}(\widehat{\Theta}, B)$ *and* $(\widehat{\mathcal{F}}, A)\widehat{\sqcup}(\widehat{\Theta}, B)$.

Proof. We know that $(\widehat{\mathcal{F}}, A)\widehat{\vee}(\widehat{\Theta}, B) = \left(\widehat{\Xi}, C\right)$, where $C = A \times B$ and $\widehat{\Xi}_{(\delta,\varepsilon)} = \widehat{\mathcal{F}}_\delta \cup \widehat{\Theta}_\varepsilon$ for all $(\delta, \varepsilon) \in A \times B$. Now for any $(\delta, \varepsilon) \in A \times B$, since $(\widehat{\mathcal{F}}, A)$ and $(\widehat{\Theta}, B)$ are fuzzy soft LA-Γ-semihypergroups over S, for all $x, y \in S$, $(\delta, \varepsilon) \in A \times B$ and $\gamma \in \Gamma$, we have

$$\begin{aligned}
\inf_{z\in x\gamma y}\left\{\widehat{\Xi}_{(\delta,\varepsilon)}(z)\right\} &= \inf_{z\in x\gamma y}\left\{\widehat{\mathcal{F}}_\delta(z) \cup \widehat{\Theta}_\varepsilon(z)\right\} = \inf_{z\in x\gamma y}\left\{\max\left\{\widehat{\mathcal{F}}_\delta(z), \widehat{\Theta}_\varepsilon(z)\right\}\right\} \\
&= \max\left\{\inf_{z\in x\gamma y}\widehat{\mathcal{F}}_\delta(z), \inf_{z\in x\gamma y}\widehat{\Theta}_\varepsilon(z)\right\} \\
&\geq \max\left\{\min\{\widehat{\mathcal{F}}_\delta(x), \widehat{\mathcal{F}}_\delta(y)\}, \min\{\widehat{\Theta}_\varepsilon(x), \widehat{\Theta}_\varepsilon(y)\}\right\} \\
&= \min\left\{\max\{\widehat{\mathcal{F}}_\delta(x), \widehat{\Theta}_\varepsilon(x)\}, \max\{\widehat{\mathcal{F}}_\delta(y), \widehat{\Theta}_\varepsilon(y)\}\right\} \\
&= \min\left\{\widehat{\Xi}_{(\delta,\varepsilon)}(x), \widehat{\Xi}_{(\delta,\varepsilon)}(y)\right\}.
\end{aligned}$$

This shows that $(\widehat{\mathcal{F}}, A)\widehat{\vee}(\widehat{\Theta}, B)$ is a fuzzy soft LA-Γ-semihypergroup over S. The other case can be seen in a similar way. □

Theorem 5.10. *If* $(\widehat{\mathcal{F}}, A)$ *and* $(\widehat{\Theta}, B)$ *are two fuzzy soft left (right)* Γ*-hyperideals over* S, *then so are* $(\widehat{\mathcal{F}}, A)\widehat{\vee}(\widehat{\Theta}, B)$ *and* $(\widehat{\mathcal{F}}, A)\widehat{\sqcup}(\widehat{\Theta}, B)$.

Proof. Let $(\widehat{\mathcal{F}}, A)$ and $(\widehat{\Theta}, B)$ be two fuzzy soft left Γ-hyperideals over S. We know that $(\widehat{\mathcal{F}}, A)\widehat{\vee}(\widehat{\Theta}, B) = \left(\widehat{\Xi}, C\right)$, where $C = A \times B$ and $\widehat{\Xi}_{(\delta,\varepsilon)} = \widehat{\mathcal{F}}_\delta \cup \widehat{\Theta}_\varepsilon$ for all $(\delta, \varepsilon) \in A \times B$. Now for any $(\delta, \varepsilon) \in A \times B$, since $(\widehat{\mathcal{F}}, A)$ and $(\widehat{\Theta}, B)$ are fuzzy soft left Γ-hyperideals over S, for all $x, y \in S$, $(\delta, \varepsilon) \in A \times B$ and $\gamma \in \Gamma$, we have

$$\begin{aligned}
\inf_{z\in x\gamma y}\left\{\widehat{\Xi}_{(\delta,\varepsilon)}(z)\right\} &= \inf_{z\in x\gamma y}\left\{\max\left\{\widehat{\mathcal{F}}_\delta(z), \widehat{\Theta}_\varepsilon(z)\right\}\right\} \\
&= \max\left\{\inf_{z\in x\gamma y}\widehat{\mathcal{F}}_\delta(z), \inf_{z\in x\gamma y}\widehat{\Theta}_\varepsilon(z)\right\} \\
&\geq \max\left\{\widehat{\mathcal{F}}_\delta(y), \widehat{\Theta}_\varepsilon(y)\right\} \\
&= \widehat{\Xi}_{(\delta,\varepsilon)}(y).
\end{aligned}$$

This shows that $(\widehat{\mathcal{F}}, A)\widehat{\vee}(\widehat{\Theta}, B)$ is a fuzzy soft left Γ-hyperideal over S. The other cases can be seen in a similar way. □

Theorem 5.11. *If $(\widehat{\mathcal{F}}, A)$ and $(\widehat{\Theta}, B)$ are two fuzzy soft LA-Γ-semihypergroups over S, then so is $(\widehat{\mathcal{F}}, A)\widehat{\cap}(\widehat{\Theta}, B)$.*

Proof. We know that $(\widehat{\mathcal{F}}, A)\widehat{\cap}(\widehat{\Theta}, B) = \left(\widehat{\Xi}, C\right)$, where $C = A \cup B$ and for all $\delta \in C$

$$\widehat{\Xi}_\delta = \begin{cases} \widehat{\mathcal{F}}_\delta & \text{if } \delta \in A\backslash B \\ \widehat{\Theta}_\delta & \text{if } \delta \in B\backslash A \\ \min\left\{\widehat{\mathcal{F}}_\delta, \widehat{\Theta}_\delta\right\} & \text{if } \delta \in A \cap B. \end{cases}$$

Now for any $\delta \in C$ and $x, y \in S$, we consider the following cases

Case 1: For any $\delta \in A\backslash B$ and $\gamma \in \Gamma$,

$$\begin{aligned}
\inf_{z\in x\gamma y}\left\{\widehat{\Xi}_\delta(z)\right\} &= \inf_{z\in x\gamma y}\left\{\widehat{\mathcal{F}}_\delta(z)\right\} \geq \min\left\{\widehat{\mathcal{F}}_\delta(x), \widehat{\mathcal{F}}_\delta(y)\right\} \\
&= \min\left\{\widehat{\Xi}_\delta(x), \widehat{\Xi}_\delta(y)\right\}.
\end{aligned}$$

Case 2: For any $\delta \in B\backslash A$ and $\gamma \in \Gamma$,

$$\begin{aligned}
\inf_{z\in x\gamma y}\left\{\widehat{\Xi}_\delta(z)\right\} &= \inf_{z\in x\gamma y}\left\{\widehat{\Theta}_\delta(z)\right\} \geq \min\left\{\widehat{\Theta}_\delta(x), \widehat{\Theta}_\delta(y)\right\} \\
&= \min\left\{\widehat{\Xi}_\delta(x), \widehat{\Xi}_\delta(y)\right\}.
\end{aligned}$$

Case 3: For any $\delta \in A \cap B$ and $\gamma \in \Gamma$, then $\widehat{\Xi}_\delta = \widehat{\mathcal{F}}_\delta \cap \widehat{\Theta}_\delta$. Analogous to the proof of Theorem 5.7, we have

$$\inf_{z\in x\gamma y}\left\{\widehat{\Xi}_\delta(z)\right\} \geq \min\left\{\widehat{\Xi}_\delta(x), \widehat{\Xi}_\delta(y)\right\}.$$

Thus in any case we have $\inf_{z\in x\gamma y}\left\{\widehat{\Xi}_{\delta}\left(z\right)\right\}\geq\min\left\{\widehat{\Xi}_{\delta}(x),\widehat{\Xi}_{\delta}(y)\right\}$, and so $(\widehat{\mathcal{F}},A)\widehat{\cap}(\widehat{\Theta},B)$ is a fuzzy soft LA-Γ-semihypergroup over S. □

Theorem 5.12. *If $(\widehat{\mathcal{F}},A)$ and $(\widehat{\Theta},B)$ are two fuzzy soft left (right) Γ-hyperideals over S, then so is $(\widehat{\mathcal{F}},A)\widehat{\cap}(\widehat{\Theta},B)$.*

Proof. Let $(\widehat{\mathcal{F}},A)$ and $(\widehat{\Theta},B)$ be two fuzzy soft left Γ-hyperideals over S. We know that $(\widehat{\mathcal{F}},A)\widehat{\cap}(\widehat{\Theta},B)=\left(\widehat{\Xi},C\right)$, where $C=A\cup B$ and for all $\delta\in C$

$$\widehat{\Xi}_{\delta}=\begin{cases}\widehat{\mathcal{F}}_{\delta} & \text{if } \delta\in A\backslash B\\ \widehat{\Theta}_{\delta} & \text{if } \delta\in B\backslash A\\ \min\left\{\widehat{\mathcal{F}}_{\delta},\widehat{\Theta}_{\delta}\right\} & \text{if } \delta\in A\cap B.\end{cases}$$

Now for any $\delta\in C$ and $x,y\in S$, we consider the following cases

Case 1: For any $\delta\in A\backslash B$ and $\gamma\in\Gamma$,

$$\inf_{z\in x\gamma y}\left\{\widehat{\Xi}_{\delta}\left(z\right)\right\}=\inf_{z\in x\gamma y}\left\{\widehat{\mathcal{F}}_{\delta}\left(z\right)\right\}\geq\widehat{\mathcal{F}}_{\delta}(y)=\widehat{\Xi}_{\delta}(y).$$

Case 2: For any $\delta\in B\backslash A$ and $\gamma\in\Gamma$,

$$\inf_{z\in x\gamma y}\left\{\widehat{\Xi}_{\delta}\left(z\right)\right\}=\inf_{z\in x\gamma y}\left\{\widehat{\Theta}_{\delta}\left(z\right)\right\}\geq\widehat{\Theta}_{\delta}(y)=\widehat{\Xi}_{\delta}(y).$$

Case 3: For any $\delta\in A\cap B$ and $\gamma\in\Gamma$, then $\widehat{\Xi}_{\delta}=\widehat{\mathcal{F}}_{\delta}\cap\widehat{\Theta}_{\delta}$. Analogous to the proof of Theorem 5.8, we have

$$\inf_{z\in x\gamma y}\left\{\widehat{\Xi}_{\delta}\left(z\right)\right\}\geq\widehat{\Xi}_{\delta}(y).$$

Thus in any case we have $\inf_{z\in x\gamma y}\left\{\widehat{\Xi}_{\delta}\left(z\right)\right\}\geq\widehat{\Xi}_{\delta}(y)$, and so $(\widehat{\mathcal{F}},A)\widehat{\cap}(\widehat{\Theta},B)$ is a fuzzy soft left Γ-hyperideal over S. The other case can be proved in a similar way. □

Theorem 5.13. *If $(\widehat{\mathcal{F}},A)$ and $(\widehat{\Theta},B)$ are two fuzzy soft LA-Γ-semihypergroups over S, then so is $(\widehat{\mathcal{F}},A)\widehat{\cup}(\widehat{\Theta},B)$.*

Proof. The proof of this theorem is similar to the proof of Theorem 5.11. □

Theorem 5.14. *If $(\widehat{\mathcal{F}},A)$ and $(\widehat{\Theta},B)$ are two fuzzy soft left (right) Γ-hyperideals over S, then so is $(\widehat{\mathcal{F}},A)\widehat{\cup}(\widehat{\Theta},B)$.*

Proof. The proof of this theorem is similar to the proof of Theorem 5.12. □

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