Soret Effect on an Oscillatory MHD Mixed Convective Mass Transfer Flow Past an Infinite Vertical Porous Plate with Variable Suction

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Abstract

A parametric study to investigate the effect of thermal diffusion (Soret effect) on an oscillatory MHD mixed convective mass transfer flow of an incompressible viscous electrically conducting fluid past an infinite vertical porous plate when the normal suction velocity as well as free stream velocity varies periodically with time is presented. The magnetic Reynolds number is assumed to be so small that the induced magnetic field can be neglected in comparison with the applied magnetic field. The resultant set of the non-dimensional governing equations are solved analytically by adopting regular perturbation technique. The profiles of the velocity, skin friction and Sherwood number at the plate are demonstrated graphically for various values of the parameters involved in the problem and the results are physically interpreted. It is found that due to the increase in velocity ratio, the fluid motion is accelerated and the viscous drag on the plate falls significantly.

Keywords: MHD, mass transfer, Nusselt number, Soret effect, Velocity ratio.

INTRODUCTION

MHD is the science of motion of electrically conducting fluid in presence of magnetic field. There are numerous examples of application of MHD principles, including...
MHD generators, MHD pumps and MHD flow meters etc. The dynamo and motor is a classical example of MHD principle. MHD principles also find its application in medicine and biology. The principle of MHD is also used in stabilizing a flow against the transition from laminar to turbulent flow. Convection problems of electrically conducting fluid in presence of transverse magnetic field have got much importance because of its wide application in Geophysics, Astrophysics, Plasma Physics and Missile Technology etc. The present form of MHD is due to the pioneer contribution of several authors like Alfven [1942], Cowling [1957], Crammer and Pai [1978], Shercliff [1965] and Ferraro and Pulmption [1966]. Model studies on MHD free and forced convection with heat and mass transfer problems have been carried out by many of the authors due to their application in many branches of science and technology. Some of them are Ahmed [2010], Elbashbeshy [2003] and Singh and Singh [2000].

Soret effect which is also known as thermal-diffusion effect concerns with the methods of separating heavier gas molecules from lighter ones by maintaining temperature gradient over a volume of a gas containing particles of different masses. When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate nature. It has been found that an energy flux can be generated not only by temperature gradient but by composition gradient as well. The mass flux created by temperature gradient is termed as Soret or thermal–diffusion effect. Soret effect is of a smaller order of magnitude than the effects described by Fourier’s and Fick’s laws and is often neglected in heat and mass transfer processes. The Soret effect is utilized for isotope separation, and in mixtures between gases with very light molecular weight (\(H_2, H_\text{e}\)). In view of the importance of Soret effect, several authors have carried out their research works to investigate the problems related to thermal-diffusion effect. Some of them are Anghel et al. [2000], Postenlnicu [2004], Alam et al. [2004] and Ahmed [2010].

The main objective of the present investigation is to study the Soret effects as well as the MHD effects on the unsteady mixed convective mass transfer flow past an infinite vertical porous plate with variable suction, where the plate temperature oscillates with the same frequency as that of variable suction velocity. It is assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the direction of the flow in presence of a transverse magnetic field.

**MATHEMATICAL FORMULATION**

We now consider an unsteady MHD conducting flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate with variable...
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suction under the influence of a uniform transverse magnetic field. Our investigation is restricted to the following assumptions:

- The polarization effects are assumed to be negligible and hence the electric field is also negligible.
- The variations of all fluid properties other than the variations of density except in so far as they give rise to a body force are ignored completely.
- All the physical variables are functions of $y'$ and $t'$ only as the plate are infinite.
- It is assumed that the variation of expansion co-efficient is negligibly small and the pressure and influence of the pressure on the density are negligible.

We introduce a co-ordinate system $(x', y', z')$ with X- axis vertically upwards along the plate, Y- axis perpendicular to the plate and directed into the fluid region and Z- axis along the width of the plate as shown in figure 1. Let the components of velocity along with X and Y axes should be $u'$ and $v'$. Let these velocity components are chosen in the upward direction along the plate and normal to the plate respectively.

Figure 1: Physical model of the problem
Under these assumptions, the equations that describe the physical situation are given by:

\[ \frac{\partial v'}{\partial y'} = 0 \]  

\[ \frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} + g \beta (T' - T_c' - C' - C_c') - \nu' \frac{u'}{K'} \frac{\sigma B' u'}{\rho} \]  

\[ \frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{K_r}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} \]  

\[ \frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} + D_1 \frac{\partial^2 T'}{\partial y'^2} \]  

All the physical quantities involved in the above equations are defined in the nomenclature.

The boundary conditions are:

\[ u' = 0, T' = T_c' + (1 + \varepsilon e^{i\omega t}) (T_u' - T_c'), C' = C_c' \text{ at } y' = 0 \]  

\[ u' \to U' = U_0 (1 + \varepsilon e^{i\omega t}), T' \to T_u', C' \to C_c' \text{ as } y' \to \infty \]  

The equation (1) yields that the suction velocity at the plate is either a constant or a function of time and we take the suction velocity normal to the plate in the form

\[ v' = -V_0 (1 + \varepsilon A e^{i\omega t}) \]  

where \( A \) is a real positive constant, \( \varepsilon \) is a small value less than unity, \( V_0 \) is a scale of suction velocity which is non-zero positive constant. The negative sign indicates that the suction is towards plate.

Outside the boundary layer, equation (2) gives

\[ \frac{1}{\rho} \frac{\partial p'}{\partial x'} = \frac{dU'}{dt'} + \frac{\nu}{K'} U' + \frac{\sigma B' u'}{\rho} \]  

In order to write the governing equations and the boundary conditions in dimensional form, the following non-dimensional quantities are introduced:

\[ u = \frac{u'}{V_0}, v = \frac{v'}{V_0}, y = \frac{V_0 y'}{\nu}, U = \frac{U'}{V_0}, \lambda = \frac{V_0}{\nu}, t = \frac{t' V_0^2}{4 \nu}, \theta = \frac{T' - T_c'}{T_w' - T_c'}, C = \frac{C' - C_c'}{C_c' - C_c'}, \]  

\[ Sc = \frac{\nu}{D'}, M = \frac{\sigma B^2 \nu}{\rho V_0^2}, Gr = \frac{\nu \rho g (T_u' - T_c')}{V_0^3}, \omega = \frac{4 \nu \omega'}{V_0^2}, K = \frac{KV_0^2}{V_0^2}, Pr = \frac{\nu}{\alpha}, \]  

\[ Gm = \frac{\nu \beta^2 g (C_c' - C_c')}{V_0^3}, Sr = \frac{D_1 (T_w' - T_c')}{(C_c' - C_c')} \]  

In view of the equations (6) – (8), the equations (2) – (4) reduce to the following dimensional form:
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\[
\frac{1}{4} \frac{\partial u}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial u}{\partial y} = \frac{\lambda}{4} \frac{dU}{dt} + \frac{\partial^2 u}{\partial y^2} + Gr \theta + GmC + N(\lambda U - u) \tag{9}
\]

\[
\frac{1}{4} \frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \tag{10}
\]

\[
\frac{1}{4} \frac{\partial C}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2} \tag{11}
\]

The corresponding boundary conditions are:

\[\begin{align*}
    u &= 0, \theta = 1 + \varepsilon e^{i\omega t}, C = 1 & \text{at} & & y = 0 \\
    u &\to U = 1 + \varepsilon e^{i\omega t}, \theta \to 0, C \to 0 & \text{as} & & y \to \infty
\end{align*}\tag{12}\]

**SOLUTION OF THE PROBLEM**

Equations (9) – (11) are coupled non-linear partial differential and these cannot be solved in closed-form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the plate as

\[
\begin{align*}
    u &= u_0(y) + \varepsilon e^{i\omega t} u_1(y) + o(\varepsilon^2) + \ldots \\
    \theta &= \theta_0(y) + \varepsilon e^{i\omega t} \theta_1(y) + o(\varepsilon^2) + \ldots \\
    C &= C_0(y) + \varepsilon e^{i\omega t} C_1(y) + o(\varepsilon^2) + \ldots
\end{align*}
\]

\[U = 1 + \varepsilon e^{i\omega t}\]

Substituting (13) in equations (9) – (11), equating the harmonic terms and neglecting the higher order terms of \(o(\varepsilon^2)\), we obtain

\[
\begin{align*}
    u_0'' + Lu_0' - Nu_0 &= -N\lambda - Gr\theta_0 - GmC_0 \tag{14} \\
    u_1'' + Lu_1' - \left(N + \frac{i\omega}{4}\right) u_1 &= -\lambda \left(N + \frac{i\omega}{4}\right) - Gr\theta_1 - GmC_1 \tag{15} \\
    \theta_0'' + Pr L\theta_0' &= 0 \tag{16} \\
    \theta_1'' + Pr L\theta_1' - \frac{i\omega}{4} Pr \theta_1 &= 0 \tag{17} \\
    C_0'' + LScC_0' &= -ScSr\theta_0'' \tag{18} \\
    C_1'' + LScC_1' - \frac{i\omega}{4} C_1 &= -ScSr\theta_1'' \tag{19}
\end{align*}
\]

where primes denotes ordinary differentiation with respect to \(y\).
The corresponding boundary conditions can be written as
\[ u_0 = 0, u_0 = 0, \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 0 \] at \( y = 0 \)
\[ u_0 \to 1, u_1 \to 1, \theta_0 \to 0, \theta_1 \to 0, C_0 \to 0, C_1 \to 0 \] as \( y \to \infty \) \hspace{1cm} (20)
Solving equations (14) – (19) under the boundary conditions (20), we obtain the velocity, temperature and concentration distribution in the boundary layer. But due to shake of brevity the solutions are not shown here.

**Skin friction :**
The non dimensional form of skin friction at the plate is given by :
\[ Cf = \left( \frac{\partial u}{\partial y} \right)_{y=0} = \left\{ \left[ (-m_1m_6 + m_{10} \Pr L + m_0LSc) + \varepsilon \omega^{i\omega} (-m_{12}m_{17} + m_{16}m_1 - m_{15}m_4) \right] \right\} \]

**Nusselt number :**
The non dimensional form of the rate of heat transfer in terms of Nusselt number at the plate is given by :
\[ Nu = -\left( \frac{\partial \theta}{\partial y} \right)_{y=0} = [- \Pr L + (-m_1 \varepsilon \omega^{i\omega})] \]

**Sherwood number :**
The non-dimensional form of the rate of mass transfer in terms of Sherwood number at the plate is given by :
\[ Sh = -\left( \frac{\partial C}{\partial y} \right)_{y=0} = \left\{ (-m_3LSc - m_2 \Pr L) + \varepsilon \omega^{i\omega} (m_4m_5 - m_1m_5) \right\} \]

**RESULTS AND DISCUSSION**

In order to get the physical insight in to the problem, we have carried out numerical calculations for non-dimensional velocity field, co-efficient of skin friction \( Cf \) at the plate and the rate of mass transfer in terms of Sherwood number \( Sh \) have been carried out by assigning some specific arbitrary values to the different parameters involved in the problem, viz, Hartmann number \( M \), Soret number \( Sr \), Porosity parameter \( K \) and velocity ratio parameter \( \lambda \). Throughout our investigation the values of Prandtl number \( \Pr \), Grashof number \( Gr \), Solutal Grashof number \( Gm \) and Schmdit number \( Sc \) are kept constant. The effects of these values are demonstrated through different graphs and the results are interpreted physically.

The figures (2)-(4) exhibit the variation of the velocity field \( u \) against \( y \) under the influence of Hartmann number \( M \), Porosity number \( K \) and velocity ratio parameter \( \lambda \). Figure (2) indicates that the fluid velocity decreases with the increase in magnetic intensity indicating the fact that the fluid motion is decelerated under the action of transverse magnetic field.
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Figure (3) illustrates the effect of Porosity parameter $K$ on fluid velocity. It is simulated from this figure that an increase in porosity parameter $K$ leads to an increase in fluid velocity, i.e. the fluid motion is accelerated for increasing the permeability of the medium. Figure (4) demonstrates the effect of velocity ratio against $y$ on the velocity field. It is inferred from the figure that under the action of velocity ratio, the fluid motion is accelerated. We recall that the velocity ratio parameter increases means that the free stream velocity increases i.e. the boundary layer velocity is enhanced due to increase in the free stream velocity.

Figure (5) and (6) show the variation of Skin-friction $C_f$ against magnetic field under the influence of Soret number $Sr$ and velocity ratio parameter $\lambda$. In both the figures it is observed that an increase in thermal-diffusion effect and velocity ratio parameter tends the viscous drag to rise and to fall significantly.

The figure (7) presents the variation of the rate of mass transfer from the plate to the fluid. It is inferred from this figure that the Sherwood number decreases under thermal-diffusion effect. i.e. the mass flux from the plate to the fluid is reduced under the influence of diffusion-thermo effect.

**CONCLUSIONS**

Our investigation leads to the following conclusions:

1. The fluid motion is accelerated due to increase in porosity of the medium and decelerated under the action of transverse magnetic field.
2. The velocity ratio parameter accelerates the fluid motion and decelerates the viscous drag.
3. Thermal Diffusion effect raised the magnitude of the viscous drag on the plate and reduced the mass flux from the plate to the fluid.
Figure 4: Effect of Velocity ratio parameter on velocity profile

\( Pr = .71, K = 5, M = 1, \omega = 1, \epsilon = .002, t = 1, A = .5, Sc = .60, Sr = 1, Gr = 5, Gm = 5 \)

Figure 5: Effect of Soret number on skin friction

\( Pr = .71, K = 5, \lambda = 1, \omega = 1, \epsilon = .002, t = 1, A = .5, Sc = .60, Gr = 5, Gm = 5 \)

Figure 6: Effect of velocity ratio parameter on Skin-friction

\( Pr = .71, K = 5, \omega = 1, \epsilon = .002, t = 1, A = .5, Sc = .60, Gr = 5, Gm = 5, Sr = 1 \)

Figure 7: Effect of Soret number on Sherwood number

\( \lambda = 1, K = 5, \omega = 1, \epsilon = .002, t = 1, A = .5, Sc = .60, Gr = 5, Gm = 5, Pr = .71 \)
NOMENCLATURE

$B_0$ = Strength of the applied magnetic field  
$C'$ = Dimensional species concentration  
$C_{w} =$ Dimensional species concentration near the plate  
$C_{w}' =$ Dimensional species concentration in the free stream  
$C =$ Non-dimensional concentration  
$C_p =$ Specific heat at constant pressure  
$D =$ Molecular mass diffusivity  
$D_i =$ Chemical thermal diffusivity  
$Gr =$ Thermal Grashof number  
$Gm =$ Solutal Grashof number  
$K =$ Porosity parameter  
$M =$ Local Hartmann number  
$Nu =$ Nusselt number  
$K' =$ Permiability of the porous medium  

$\rho =$ Density of the fluid  
$\nu =$ Kinematic viscosity  
$\alpha =$ Thermal diffusivity  
$\omega =$ Frequency parameter  
$\theta =$ Dimensionless temperature  

$\beta =$ Co-efficient of volume expansion for heat transfer  
$\beta' =$ Co-efficient of volume expansion for mass transfer  
$\lambda =$ Velocity ratio parameter  
$\sigma =$ Electrical conductivity  

Greek symbols

$\rho =$ Density of the fluid  
$\nu =$ Kinematic viscosity  
$\alpha =$ Thermal diffusivity  
$\omega =$ Frequency parameter  
$\theta =$ Dimensionless temperature

REFERENCES


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