A Validation of Profit Sharing Ratio Determination Mathematical Model for Islamic Hire-Purchase Contract

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Abstract

This paper discusses the mathematics formulation of determining the profit sharing ratio for Islamic hire-purchase contract named Al-Ijarah Thumma Al-Bai or AITAB. This paper presents an approach which maximizes the profits of the hire-purchase participating parties by implementing the fair profit sharing ratio determination mentioned in the profit sharing concept with stochastic optimization technique. Then the validation of the model is also given.

Keywords: Islamic hire-purchase instrument, stochastic optimization, profit sharing ratio determination
1 INTRODUCTION

Banking and financing atmosphere in Malaysia is unique for Malaysia is the only country in the world operating both conventional and Islamic systems side by side (Seif & N.Irwani 2007). Compared to conventional banking system which is well established and have existed for nearly many years, Islamic banking is incomparable for it only plays a significant role in banking and finance system for 30 years since 1980 (Muhammad 2007). Even tough Islamic banking and financing system is young in age, it has played a significant role, with recent data shows that the total assets of Islamic banking and finance exceeding USD200 billion and growing at annual rate of 6% (Amir & Ali 2007). Despite the attention received, Islamic banks need to work harder in order to compete with the conventional banks in attracting customers and investors. It have resulting in the creation of more attractive and innovative products (Zeti 2004), which result with the attractive and innovative products, as well as a flourish reception. But still there are some improvement needed to be done because the lack of Shari‘ah (Islamic law) outline within the products itself (Nurdianawati & Asyraf 2006). This is due to lack of experience, research and scarcity of literature especially in English (Sudin 1996) and also not able to compete with captivate products and interest rate of the conventional banks.

Islamic hire-purchase agreement known as Al-Ijarah Thumma Al-Bai’ or AITAB is not an exception of the problem. Chong and Liu (2009) found that there are no significant difference between conventional and Islamic systems. As an alternative system of commerce, Islamic finance has failed to achieve its articulated and functional goal regarding the notions of fairness and social justice (Haider 2007).

Thus, this study aims to construct mathematical formulations for AITAB contract which is more in line with Shari‘ah rules and regulations. Our purpose in this paper is to propose our mathematical formulation of profit sharing ratio (PSR) using stochastic programming which suits the AITAB agreement. Moreover, we discuss the bound for term charges consider within the PSR formulation framework.

This article is divided into three parts excluding this introduction. In the second section, we briefly introduce Islamic banking and finance system and also Islamic hire-purchase contract namely AITAB. The third section is the construction of PSR mathematical formulation.

2 ISLAMIC HIRE-PURCHASE CONTRACT

Ijara wa-iqtina or hire-purchase is one of the most widely used instrument and popular among customers. Hire-purchase contract is more preferable compared
to other banking instruments because of its tax exemption (Mandell 2002). In addition, it gives an opportunity for those who cannot afford to buy goods on cash basis to use and own goods through periodic monthly payments (Seif & N. Irwani 2007). Hire-purchase is also a key for Islamic banking and financial institutions operating on interest-free principles to avoid interest rate risk as suggested by Radiah and Yap (2009). Thus, Islamic hire-purchase is an important and powerful method which is inline with Shari‘ah and could be used and fully developed for benefits.

In Islam, ijara or lease agreement have two different perspectives which are (i) a hire contract to employ services of a person, or (ii) a lease contract of particular asset (Usmani 2006). In this study, we only concentrate on the second type of lease agreement.

IJara wa-iqtina, is a contract which transfer the ownership of leased asset to lessee in the form of sale (Bank Negara Malaysia 2009). Referring to Bank Negara Malaysia (BNM) (2009), Al- Ijarah Thumma Al-bai’ (AITAB), an Islamic hire-purchase product, is a form of Ijara wa-iqtina. AITAB is the only Islamic hire-purchase contract in Malaysia which has been introduced back in year 1995 by Bank Islam (Nurdianawati & Asyraf 2006). AITAB contract consists of two different agreements; al-ijarah which is a hiring agreement and al-bai’ that is purchasing agreement. The hiring agreement will take place first followed by the purchasing agreement. Therefore, a process of transferring hiring agreement to purchasing agreement without condition is an AITAB contract (Abdul Ghafar 2010).

As usual, a contract must involve at least two parties. We consider two parties participating in the AITAB contract namely the lessor and the lessee. The lessor is the owner of the leased asset while lessee is the one who rent and gains benefit from using the leased asset. Since lessee gains benefit by using the leased asset, as a compensation, lessee pays monthly rent to the lessor for a certain time period agreed before. Normally the lease term used most of the leased asset lifetime span (C. Myres et al. 1976), hence the lessor usually passes ownership of the leased asset to the lessee at the maturity date of the AITAB contract. During the leased period, lessor obtains monthly rent which is equal to the periodic payments of leased asset price (or periodic payments of principal) and a profit. On the other hand, lessee obtains the benefits from using the asset as well as ownership at the end of the contract. But if the lessee in any way defaults on the rent payment, the lessor has the right to repossess the asset. In this sense, AITAB contract has no difference with financial leased from conventional banking (Abdul Sattar 1998).

Thus, there is a need to embed Shari‘ah methods into AITAB agreement in order to make it more appealing as Islamic banking product and considered as fair to all the contracted parties. Thereupon referring to studies done by previ-
ous researchers (Abdul Sattar 1998, Seif & N. Irwani 2007, Mohammad Hashim 2007, Ala’ Eddin 2004, Nurdianawati & Asyraf 2006, Nurfadhlina et al. 2009, Ros Aniza & Abdul Rahim 2003, Usmani 2006), Islamic hire-purchase agreement need to follow specific rules and regulations as stipulated by the Shari’ah in order to make it valid in Islamic perspective (for details refer to (Nurfadhlina et al. 2009)). Thus, in this study the implementation of Musyarakah Mutanaqisah or diminishing ownership into AITAB modus operandi and is explain in the next chapter.

2.0.1 Modus Operandi Of Al-Ijarah Thumma Al-Bai’ With Musyarakah Mutanaqisah (Diminishing Ownership)

In this study, we consider an AITAB contract with two actors namely lessor and lessee. At time $t = 0$, the lessor must make a decision of whether or not to enter into the AITAB contract for $T$ months with the lessee. It is assumed that the lessor will only agree to enter into the contract if and only if he believes the benefit from AITAB transaction is at least equal to risk-free investment return for the same duration and principle. It is because in a dual financial system, Islamic investment return though not based on interest must be as comparative as the conventional products, and at least equal to the risk-free return in order to attract investors/customers. Therefore, this is the first constraint to consider and fulfill in order for lessor to enter the AITAB contract.

Furthermore based on Hire-Purchase Act 1967 (Act 212) (Legal Research Board 2009), the lessee must put in place at least 10% of leased asset price and the balance will be paid by the lessor before the agreement is to be bounded. Considering the regulation, the lessee is now not only renting the leased asset but is also a partner to the lessor. Thus the profit generated from leasing the asset must be shared fairly between the two partners. Therefore, problem of finding the right profit sharing ratio between the lessor and the lessee is the main aim of this paper and will be discussed in the next section.

According to Abdul Aziz (1992), rent consists of periodic principal payments and compensation for benefits gained through the leased asset by lessee. Compensation for gains of the leased asset is captured through the profit rate. Meanwhile periodic principal payments will be sum up to equal to original principal made by lessor in obtaining the leased asset. Thenceforth lessor’s ownership will decrease with time and will be zero at time interval $t = T - 1$, the opposite will happen to lessee’s ownership. The concept applied here is known as Musyarakah Mutanaqisah. Based on Abdul Aziz, we embed the Musyarakah Munataqisah (diminishing ownership) concept into the AITAB modeling.
Profit generated is based on the profit rate. As currently practised, Islamic finance and banking products only consider a fixed profit rate return though in reality the opposite actually happen. Fixed interest rates are not permitted in Islam (Karsten 1982). Realizing this we consider a fluctuating profit rate for AITAB agreement, which is assumed to be random and varies with time interval in monthly basis. Randomness in profit rate is permitted in Shari’ah (Islamic Law) because it gives more justice to the contracted parties compared to the fixed profit rate since it reflect true market forces and gives an opportunity for the lessor and lessee to generate more benefits (Abdul Sattar 1998). But the uncertainty lies within it open to a opportunity of manipulation. Manipulation is not permitted in Islam and to overcome this, the agreement contents must be well defined with no opportunities of manipulation and agreed by contracting parties. Thus the content of agreement must specified all the particular in details with no spaces of manipulation. The discussion of steps in each time intervals of AITAB contract and the benefit equations is given in the next section.

2.1 Mathematical Formulation Of Islamic Hire-Purchase Contract

The construction of mathematical models for AITAB benefit equations follow the process describe by equations below. At each time interval, $t$ there are events taking place as describe in equations follows. First it is necessary to understand situations appear within AITAB contract at each time $t$ with $t = 0, 1, ..., T - 1$. The notations use in this paper are given as follows:

- $P$: principal or lessor’s investment
- $a_{t+1}$: periodic principal payment at time interval $t + 1$
- $X_t$: lessor’s ownership at time interval $t$
- $Y_t$: lessee’s ownership at time interval $t$
- $\theta_{t,pp}$: lessor’s profit sharing ratio at time interval $t$
- $\theta_{t,p}$: lessee’s profit sharing ratio at time interval $t$
- $B_{t,pp}$: lessor’s benefit at time interval $t$
- $B_{t,p}$: lessee’s benefit at time interval $t$
- $u_{t+1}(\omega)$: profit rate (or term charges) at time $t + 1$
- $r_f$: risk-free rate

At the beginning of each time interval $t$, the participating parties must determine profit sharing ratio before the realization of profit rate (or terms charges). The terms charges is a random variable and changing in each month. $\theta_{t,pp}$ and $\theta_{t,p}$ are lessor’s profit ratio and lessee’s profit ratio accordingly. This arguments apply to all time interval throughout the AITAB contract.

At time $t = 0$
The lessor initial contribution is at most $0.9P$ and lessee initial contribution is at least $0.1P$. Thus $X_0 = 0.9P - a_1$ and $Y_0 = 0.1P + a_1$. Then $X_0 + Y_0 = P$.

Lessor’s and lessee’s benefit is given by equation below. With $B_0^{pp}$ and $B_0^p$ lessor’s and lessee’s benefit accordingly.

\[ B_0^{pp} = (0.9P - a_1)u_1(\omega)\theta_0^{pp} + (0.1P + a_1)u_1(\omega)\theta_0^p \]

\[ B_0^p = (0.9P - a_1)u_1(\omega)\theta_0^p + (0.1P + a_1)u_1(\omega)\theta_0^p \]

At time $t = 1$

\[ X_1 = X_0 - a_2 \text{ and } Y_1 = Y_0 + a_2. \] Then $X_1 + Y_1 = X_0 + Y_0 = P$.

Lessor’s and lessee’s benefit given by equation below.

\[ B_1^{pp} = (X_0 - a_2)u_2(\omega)\theta_1^{pp} + (Y_0 + a_2)u_2(\omega)\theta_1^p \]

\[ B_1^p = (X_0 - a_2)u_2(\omega)\theta_1^p + (Y_0 + a_2)u_2(\omega)\theta_1^p \]

At time $t = 2$

\[ X_2 = X_1 - a_3 \text{ and } Y_2 = Y_1 + a_3. \] Then $X_2 + Y_2 = X_1 + Y_1 = P$.

Lessor’s and lessee’s benefit given by equation below.

\[ B_2^{pp} = (X_1 - a_3)u_3(\omega)\theta_2^{pp} + (Y_1 + a_3)u_3(\omega)\theta_2^p \]

\[ B_2^p = (X_1 - a_3)u_3(\omega)\theta_2^p + (Y_1 + a_3)u_3(\omega)\theta_2^p \]

At time $t = T - 1$

\[ X_{T-1} = X_{T-2} - a_T \text{ and } Y_{T-1} = Y_{T-2} + a_T. \] Then $X_{T-1} + Y_{T-1} = P$.

Lessor’s and lessee’s benefit given by equation below.

\[ B_{T-1}^{pp} = (X_{T-2} - a_T)u_T(\omega)\theta_{T-1}^{pp} + (Y_{T-2} + a_T)u_T(\omega)\theta_{T-1}^p \]

\[ B_{T-1}^p = (X_{T-2} - a_T)u_T(\omega)\theta_{T-1}^p + (Y_{T-2} + a_T)u_T(\omega)\theta_{T-1}^p \]

Based on equation of each time interval above, we conclude total benefit for lessor and lessee is given by equation (1) and equation (2) respectively.

\[ B^{pp} = \sum_{t=0}^{T-1} Pu_{t+1}(\omega)\theta_t^{pp} \]
Periodic rent payments also consist of periodic principal payments. Considering this, lessee is also consider as the owner of the leased asset having increasing ownership over time. Therefore profit gain from leased asset must be divided fairly between the two partners. From our result, it shows that the total of the sharing for each parties must be always equal to the principal.

As shown above, it is clear that benefit for lessor and lessee at each time interval depending on the profit rate and the profit sharing ratio. Profit sharing ratio must be decided at the beginning of each time period $t$ even before profit rate $u_{t+1}^{}(\omega)$ is known. To overcome the issue of indeterministic profit, we introduce profit sharing ratio (PSR) determination given by $\vartheta_t$. Using stochastic programming approach, we construct a model for PSR determination.

In this paper we only consider lessor’s PSR, given that $\vartheta_{t}^{pp} + \vartheta_{t}^{p} = 1$, given $\vartheta_{t}^{pp}$ and $\vartheta_{t}^{p}$ is lessor’s PSR and lessee’s PSR accordingly. The assumptions considered in the modeling are listed below:

(A1) Lessor’s profit from AITAB contract must at least equal to the risk-free return.

(A2) Malaysia government bond is the risk-free securities considered here with fixed interest rate and $T$ maturity duration.

(A3) Terms charges (or profit rate) considered is changing within bounded set given by Base Financing Rate (BFR) provided by BNM which the maximum BFR is $\omega^+$ and the minimum is $\omega^-$. 

(A4) There is no depreciation considered.

(A5) Lessee is indifferent to the profit.

PSR for both contracting parties must be decided at the beginning of each time period before the realization of BFR and the terms charges. Since we assumed lessee is indifferent to the profit and for simplicity of the modeling, we only consider lessor side in this mathematical modeling. The modeling is in lessee’s perspective taking consideration the fact that the maintenance of leased asset is borne by lessee. Therefore the optimization problem is to minimize the lessor’s profit sharing ratio with certain constraints given as in equation (3).

$$
B^p = \sum_{t=0}^{T-1} P u_{t+1}^{}(\omega) \theta_t^p
$$

(2)

$$
\text{min}\{\vartheta_{t}^{pp}\}
$$

(3)
\[ \vartheta_t^{pp}, u_{t+1}(\omega), r_f \geq 0; \forall t \]

given that for all \( t \in [0, T] \) (in month);
\[ X_0 \leq 0.9 \times P \]
\[ a = \frac{0.9P}{T} \]
and
\[ \left[ \frac{(X_0)(\frac{a}{T})}{100(1-(1+|\omega|/100)^-T)} \times T \right] - (X_0) \leq u_{t+1}(\omega) \leq \left[ \frac{(X_0)(\frac{a}{T})}{100(1-(1+|\omega|/100)^-T)} \times T \right] - (X_0) \]

With \( \vartheta_t^{pp} \) represent the lessor’s profit sharing ratio. The terms charges, \( u_{t+1}(\omega) \) is a random variable changing monthly depending on BFR, \( \omega \). \( \omega \) is given by \( \omega^- \leq \omega \leq \omega^+ \), with \( \omega^+ = 10.75\% \), this value is stated by the Central Bank of Malaysia (BNM) (Bank Negara Malaysia 1994) and \( \omega^- \) is assumed to be the lowest BFR in the historical data. \( r_f, P \) and \( a \) are constants. Please refer to Nurfadhлина et. al (2012(a)) for details.

We consider minimization formulation for lessor PSR because considering lessee’s effort in maintaining the leased asset and the tax exemption given to lessor that originally must reduced lessee periodic payments. Beside based on Surat Al Nahl; verse 90 and Hadith to forbid making large profits on the original price (Association of Islamic Banking Institutions Malaysia 2010).

From equation (3), the optimal minimum value for lessor’s PSR, \( \vartheta_t^{pp} \) is obtained. Knowing \( \vartheta_t^{pp} \), lessee’s PSR can be calculated. The constraint in equation (3) is construct based on the assumption that the lessor’s profit must be at least equal to the risk-free return. This is given by equation (4) below.

\[ P\vartheta_t^{pp}u_{t+1}(\omega) \geq X_tr_f \tag{4} \]

with \( X_t = X_0 - ta \).

The optimization problem design here is an uncertain linear problem due to the randomness in terms charges. Thus stochastic programming is use to find the optimal solution for the problem by finding the highest probability of an event occur given it must fulfill all constraints (refer to Nurfadhлина et. al (2012(b)) for details). Therefore, the equation is given by (5).

\[ \{\min \{\vartheta_t^{pp} : P\{u_{t+1}(\omega) : \vartheta_t^{pp} \geq \frac{(X_0 - ta)r_f}{u_{t+1}(\omega)P} \geq \alpha \} : u_{t+1}(\omega) \in U\} \tag{5} \]

with \( U \) is the uncertainty set and the optimal solution for equation (5) is the optimum lessor’s PSR.

Thus, as presented in equation (5), the terms charges \( u_{t+1}(\omega) \) are random variables in uncertain set \( U \). Let \( \omega \) be continuous with the probability distribution function (pdf) of \( \omega \) given by \( F(t) = P(\omega) \leq t \). Consider the
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following.

\[
\min \{ \vartheta_{\text{pp}}^{t} : P(\omega : \vartheta_{\text{pp}}^{t} \geq \frac{c}{u_{t+1}(\omega)}) \geq \alpha \} \\
= \min \{ \vartheta_{\text{pp}}^{t} : P(\omega : (u_{t+1}(\omega)) \geq \frac{c}{\vartheta_{\text{pp}}^{t}}) \geq \alpha \} \\
= \min \{ \vartheta_{\text{pp}}^{t} : 1 - F(\frac{c}{\vartheta_{\text{pp}}^{t}}) \geq \alpha \} \\
= \min \{ \vartheta_{\text{pp}}^{t} : F(\frac{c}{\vartheta_{\text{pp}}^{t}}) \geq 1 - \alpha \} \iff \frac{c}{\vartheta_{\text{pp}}^{t}} \leq F^{-1}(1 - \alpha) \quad (6)
\]

From (6), \( \min \{ \vartheta_{\text{pp}}^{t} : \vartheta_{\text{pp}}^{t} \geq \frac{c}{F^{-1}(1 - \alpha)} \} \). Thus the minimum \( \vartheta_{\text{pp}}^{t} \) for the problem is \( \vartheta_{\text{pp}}^{t} = \frac{c}{F^{-1}(1 - \alpha)} \) and \( \vartheta_{\text{pp}}^{t} \in [0, 1] \). To make sure \( \vartheta_{\text{pp}}^{t} \in [0, 1] \), \( c \leq F^{-1}(1 - \alpha) \) must be true at all \( t \). It is clear the \( c \leq F^{-1}(1 - \alpha) \) because \( r_f \) is in percentage and \( F^{-1}(1 - \alpha) \) is a number of \( u_{t+1}(\omega) \) realization. Thus \( c \leq F^{-1}(1 - \alpha); \forall t \) and \( \vartheta_{\text{pp}}^{t} \in [0, 1] \) hold.

3 CONCLUSIONS AND DISCUSSION

In this paper we introduce an alternative mathematical formulation in AITAB contract determination using stochastic programming method. The construction of profit sharing ratio (PSR) model of AITAB contract is also discussed. The PSR determination is crucial in calculating the AITAB contract value regarding to the facts that of the randomness within the term charges.

The methods used presently (known as Rule 78 and Constant Rate of Return (or CRR)) are not significantly different from the conventional hire-purchase known as financial lease. The problem exists due to lack of Shari’ah framework with more application of conventional finance methods (details refer to Nurfadhlina et al. (2009)). Having understood the problem existed, we suggest a modified mathematical models using Islamic finance methods such as Mudharabah and Musyarakah Mutanaqisah that originated from the Shari’ah.

Using Musyarakah Mutanaqisah concept into AITAB contract resulted in diminishing ownership for the lessor over time \( t \) and at the same time increase lessee’s ownership. This makes sense since monthly rent payments consist of periodic payment for the principal investment made by the lessor at \( t = 0 \). Thereby the lessee is also the possessor of leased asset and has a right on it. So in the case where the lessee defaults on the rent payment and asset is repossessed by the lessor, lessee’s investment and effort will not be totally lost as in practice now. It give more justice to lessee and at the same time protect lessor’s interest.

Widening the lessee role being a partner to the a lessor, the profit generated from the leased asset need to be distributed fairly between them. Here we apply Mudharabah concept to determined fairly profit sharing ratio. Usmani
(2006) suggested to calculate the profit sharing ratio on ownership, but as shown it resulted in less benefit gain by the lessor which is not in line with the condition agreed. Fluctuating the profit rate complicates the process of determining the profit sharing ratio since it is not deterministic which leads to indeterministic gain that may not comply to the condition of AITAB contract. Thus we construct a stochastic programs model to determine the optimal PSR.

The proposed PSR model captures the randomness of the benefits gained as a result of the fluctuate profit rate. PSR must be decided at the beginning of each time interval before realization of profit rate. The constraint involves is that benefit of lessor must at least be equal to risk-free investment return. The reason being lessor may choose to place his investment in a risk-free instrument and gains risk-free return, thus the benefit gains by the lessor must at the very least equal to the return of risk-free investment having the same duration and principal. Though the benefits are directly related to profit rate and PSR, the only variable that can be controlled is the PSR. Thus it is crucial to obtain a sufficient PSR for the lessor at the beginning of each time interval in order to meet the constraint. Hence this will result in a more efficient process.

ACKNOWLEDGEMENT

We would like to acknowledge Professor Dr. Rüdiger Schultz from Fakultät für Mathematik, Universität Duisburg-Essen, Germany for his valuable advices, comments and suggestions, regarding the mathematical aspects of this paper. We would also like to thanks the helpful comments from the editor and reviewers.

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Received: January 15, 2013