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Estimating IBNR Claims Reserves for General Insurance Using Archimedean Copulas

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Abstract

Claims reserving for general insurance business has developed significantly over the recent past. There has always been a slight mystery in short-term insurance contracts of how to go about reserving for claims, which have not yet come in, and are still in some sense of figment of the future. Insurance claims variables are non-normally distributed and therefore a measure that will capture the dependence among the variables better than the usual correlation is employed. One such method is the use of copulas. The object of this paper, therefore, is to use the Archimedean copula, Clayton copula and Frank copula to estimate outstanding incurred but not reported (IBNR) claim reserves. A comparison of the estimates of outstanding claim reserves obtained from different Archimedean copulas is also presented.

Keywords: IBNR claim reserves, Archimedean copula, Clayton copula, Frank copula

1. INTRODUCTION

In general insurance, a claim is a demand for payment of damages that may be covered under a policy and a reserve is an estimate of the amount of money set aside for the eventual payment of a claim. Payment of a claim is what consummates the insurance contract. A claim is incurred when it happens, regardless of when in the future it is paid. Reserves are classified as liabilities on the company's balance sheet as they represent future obligations of an insurance company. They are important since they are a measure of a company's financial solvency and improper reserving can therefore present a false picture of a company's financial condition. A 'reported' claim is one that has already been processed to the extent that a central record on it is held. However, claims occur almost every day but are usually not reported the same day. This may be due to the normal time lag in reporting claims, difficulties in determining the size of the claim and so on. The only certainty is that, such claims will come in and that there is a duty to make provision for them. These claims, not yet known to the insurer, but for which a liability is believed to exist at the reserving date are referred to as Incurred but not Reported (IBNR) claims. Past claims data, which should be adequate and accurate, is used to construct estimates for the future payments and it consists of a triangle of incremental claims grouped by time of origin (when the claim or accident was incurred) and development time (time elapsed since the accident). The problem is thus to complete this run-off triangle. The only inherent uncertainty is described by the distribution of possible outcomes, and one needs to arrive at the best estimate of the reserve.

Many classical methods of dealing with the reserve problem have been developed all of which are based on different coefficient calculations and deal with the classical development triangle (for example, Chain ladder method, Bornhuetter-Fergusson and separation technique). In almost any method, analyzing the upper triangle is based on well-known techniques from statistics. However, the essential problem to be solved is the management of the risk associated with the future (the lower triangle). Most methods estimate the lower triangle cell-by-cell, and do not pay enough attention to the structure describing the dependencies between these cells. Each cell can be considered as a univariate random variable being part of the multivariate random variable describing the lower triangle. Hence, the IBNR reserve must be considered as a (univariate) random variable being the sum of the dependent components of the random vector describing the lower triangle. Goovaerts, et. al. (2001) studied various IBNR evaluation techniques and found out that estimating the correlations from the past data, and using them for multivariate simulations of the lower triangle is a dangerous technique because the insurer is especially interested in the tail of the distribution function and that a multivariate simulation technique will only be possible if the whole dependency structure of the lower triangle is known. They observed that in practice, situations where only the distribution functions of each cell can be estimated with enough accuracy, but where only limited information of the dependency structure can be obtained (because of inadequate data) are encountered. Since the ’true’ multivariate distribution function of the lower triangle could not be determined in most cases, because the mutual dependencies are not known, or difficult to cope with they concluded that the only conceivable solution is to find upper and lower bounds for this sum of dependent random variables which use as much as possible of the available information.

Understanding relationships among multivariate outcomes is a basic problem in statistical science. Multivariate relationship is limited by the basic setup that requires the analyst to identify one dimension of the outcome as the primary measure of interest (the dependent variable) and other dimensions as supporting variables (the independent variables). In insurance, this relationship is not of primary interest as we mostly deal with joint distribution functions where we need to understand the distribution of several variables interacting simultaneously and not in isolation of one another. In the IBNR problem it is necessary to consider the claim size and development time as the two variables interacting simultaneously. The normal distribution has long dominated the study of multivariate distributions. More recent texts on multivariate analysis, such as (Krzanowski, 1988) have begun to recognize the need for examining alternatives to the normal distribution setup. This is certainly true for actuarial science applications such as long tailed claims variables (Hogg and Klugman, 1984), where the normal distribution does not provide an adequate approximation to many datasets. There has been a tendency to use correlation as if it was an all-purpose dependence measure but it is often misused and applied to problems for which it is not suitable. However, empirical research in finance and insurance show that the distributions are seldom in the class of spherical and elliptical distributions (Embrechts, McNeil and Straumann, 1999). Therefore, correlation is a rather imperfect measure of dependency in many circumstances and thus the copula comes in handy as an alternative measure of dependency.

A construction of multivariate distribution that does not suffer from these drawbacks is based on the copula function. Copulas thus are extremely helpful because they give a natural way of allowing for dependency that is free from the drawbacks of correlation. They are invariant to reasonable transformations of random variables and/or their distribution functions. The notion of a function characterizing the dependence structure between several random variables comes from the work of Hoeffding in the early forties. Other authors independently introduced related notions afterwards but it was Sklar (1959) that defined copula as a function that links the multivariate distribution to functions of the univariate marginal distributions. Literature on the statistical properties and applications of copulas in finance and insurance has been developing rapidly in the recent past. Wang (1997) proposes copula for modelling aggregate loss distributions of correlated insurance policies. Frees and Valdez (1998) and Klugman and Parsa (1999) use copula to model bivariate insurance claim data. More recently, Pettere and Kollo (2006) used the Archimedean class of copulas to model claim size of a Latvian insurance company and later used the bivariate Clayton copula to estimate IBNR reserves. This paper, therefore, focuses on how different types of copulas can be used to fit IBNR claims data and then compares the estimates of outstanding claim reserves obtained from different Archimedean copulas for any statistical significance.

In section 2 we outline the methodology that will be used. Discussion of results obtained as well as estimation and comparison of IBNR reserves estimates will be in section 3. Conclusions based on the results will be in the final section.

2. METHODOLOGY

Suppose that an $n-$dimensional random vector $X=(X_1,X_2,...,X_n)$ has distribution function

$$F(x_1,x_2,...,x_n)=\Pr(X_1\le x_1,X_2\le x_2,...,X_n\le x_n)$$

one can decompose F into univariate marginals of X_i, $i=1,2,...,n$ and another distribution function called copula.

Definition: Copula
Informally, a copula C is a joint distribution function defined on the unit square with uniform marginals (or margins).
Formally, define I as the unit interval, $I=[0,1]$, F as any one-dimensional distribution function and C a distribution function of the uniform distribution $[0,1]$, then

$$F(x)=C[F(x)],\quad x\in\Re.$$

This can be carried over to two and higher dimensional distribution functions F (with $F_1,F_2,...,F_n$ denoting the one-dimensional marginal distribution functions). That is,

$$F(x_1,x_2,...,x_n)=C[F_1(x_1),F_2(x_2),...,F_n(x_n)] \tag{2.1}$$

where $(x_1,x_2,...,x_n)\in\Re^n$.

Here C denotes the distribution function on $[0,1]^n$ with uniform margins. That is,

$$C(u_1,u_2,...,u_n)=\Pr(U_1\le u_1,U_2\le u_2,...,U_n\le u_n)$$

where U is a (ex-ante) uniform random variable, whereas u is the corresponding (ex-post) realization. Thus equation (2.1) breaks up a multivariate distribution into

i. The one-dimensional marginal distribution functions $F_1,F_2,...,F_n$.

ii. The dependence structure (Copula).

Proposition: Properties of copulas

A copula is any function $C:[0,1]^n \to [0,1]$ satisfying the properties

1. $\forall(u_1,u_2,...,u_n)$ in $[0,1]^n$, if atleast one component u_i is zero, then $C(u_1,u_2,...,u_n)=0$.

2. For $u_i \in [0,1]$, $C(1,...,u_i,...,1)=u_i$, $\forall i \in (1,2,...,n)$

3. $\forall[u_{11},u_{12}]\times[u_{21},u_{22}]\times...\times[u_{n1},u_{n2}]$ $n-$dimensional rectangles in $[0,1]^n$

$$\sum_{i1=1}^{2}...\sum_{in}^{2}(-1)^{i1+i2+...+in}C\left(u_{1_{i1}},u_{2_{i2}},...,u_{n_{in}}\right)\geq 0 .$$

Theorem: Sklar (1959)

Let $F_1(x_1),F_2(x_2),...,F_n(x_n)$ be a given continuous marginal distribution functions. Then, for every $(x_1,x_2,...,x_n)\in \Re^n$,

1. If $F(x_1,x_2,...,x_n)$ is a joint distribution with function with margins $F_1(x_1),F_2(x_2),...,F_n(x_n)$, then there exists a unique copula C such that

$$F(x_1,x_2,...,x_n)=C(F_1(x_1),F_2(x_2),...,F_n(x_n)) \tag{2.2}$$

2. Conversely, if C is any copula, the function F, defined in equation (2.2) is a joint distribution function with margins $F_1(x_1),F_2(x_2),...,F_n(x_n)$.

For non-continuous $F_1(x_1),F_2(x_2),...,F_n(x_n)$, C is uniquely defined on $RangeF_1(x_1)\times RangeF_2(x_2)\times...\times RangeF_n(x_n)$.

If we consider the random variables X and Y such that

$$F(x,y)=\Pr(X\leq x,Y\leq y),\quad F_1(x)=\Pr(X\leq x),\quad F_2(x)=\Pr(Y\leq y)$$

then we call C the copula of X and Y. Sklar's theorem, through the statement

$$F(x,y)=C(F_1(x),F_2(y))$$

splits the joint probability distribution into the marginals and a copula, so that the latter only represents the dependence between X and Y. From this modelling separation it follows that also in the estimation or calibration phase one can identify the marginals and, at a second stage, specify the copula function.

2.1. Bivariate Archimedean copula concepts

If we consider the random variables X and Y such that

$$F(x,y) = \Pr(X \le x, Y \le y), \quad F_1(x) = \Pr(X \le x), \quad F_2(x) = \Pr(Y \le y)$$

then $F(x,y)$ can be written in terms of a copula and it's marginal distributions as

$$F(x,y) = C(F_1(x), F_2(y)) = C(u,v) = \Pr(U \le u, V \le v) \tag{2.3}$$

then we call C the copula of X and Y. If we represent the copula in the following form,

$$C(u,v) = \psi^{-1}(\psi(u), \psi(v)), \tag{2.4}$$

where $\psi : [0,1] \to [0,\infty]$ is a continuous strictly decreasing and convex function such that $\psi(1) = 0$ and $\psi(0) = \infty$ and is such that the function ψ has an inverse $\psi^{-1} : [0,\infty] \to [0,1]$ with the same properties like ψ, except that $\psi^{-1}(0) = 1$ and $\psi^{-1}(\infty) = 0$ then the copula is referred to as Archimedean. A list of Archimedean copulas and their properties can be found in Nelsen (2007).

In this paper we have examined three copulas Clayton, Frank and Gumbel copulas and their properties are shown in table 2.1 below.

Table 2.1: Archimedean copulas and associated properties

	Clayton	Frank	Gumbel
Generator $\psi(t)$	$t^{-\theta} - 1$	$\ln\left(\frac{e^{\theta t} - 1}{e^{\theta} - 1}\right)$	$(-\ln t)^{-\theta}$
Inverse generator $\psi(s)$	$(1+s)^{-1/\theta}$	$\theta^{-1}\ln\left[1 + e^{s}\left(e^{-\theta} - 1\right)\right]$	$\exp\left(-s^{1/\theta}\right)$
Bivariate copula $C_{\psi}(u,v)$	$\left(u^{-\theta} + v^{-\theta} - 1\right)^{1/\theta}$	$\frac{1}{\theta}\left(1 + \frac{(e^{\theta u} - 1)(e^{\theta v} - 1)}{e^{\theta} - 1}\right)$	$\left(u^{-\theta} + v^{-\theta} - 1\right)^{1/\theta}$

Table 2.1: Archimedean copulas and associated properties (continued)

Parameter θ	$[-1,\infty)\setminus\{0\}$	$[-\infty,\infty)\setminus\{0\}$	$[1,\infty)$
Kendall's τ	$\frac{\theta}{\theta+2}$	$1-\frac{4}{\theta}(1-D_1(\theta))$	$1-\theta^{-1}$
Spearman's ρ	Complicated form	$1-\frac{12}{\theta}(D_1(\theta)-D_2(\theta))$	No closed form

where $D_k(x)=\frac{k}{x^k}\int_0^x \frac{t^k}{e^t-1}dt$ is the 'Debye' function.

We will use these Archimedean copulas for the estimation of IBNR reserves in section 3.

3. IMPLEMENTATION AND RESULTS

Appropriate distribution models will first be fitted to the random variables of interest and the best fit determined by the Kolmogorov-Smirnov test. An appropriate Archimedean copula model for the data will then be found using the method due to Genest and Rivest (2001) and goodness of fit of the model determined both graphically and analytically using QQ-plots and Kolmogorov-Smirnov test statistic respectively. Average claim size in each development time unit will then be found by simulation using the obtained copulas. The number of claims occurring in each time unit will also be found and a distribution fitted. This distribution will then be used to estimate the average number of claims in each time unit. Finally, reserves will be obtained by multiplying average claim size in each development time unit to the average number of claims reported in each time unit and to the number of time units. Comparison of the estimated reserves will then be done using the Mann-Whitney U test statistic. In this paper, we have used published data from Taylor and Ashe (1983) which was also used by Verrall (1991, 1998), Mack (1993) and Renshaw (1989, 1994). The data is in incremental form. The development time has length one year and used claims are for a period of ten years. A claim will be characterized by two random variables, development time and claim size. The analysis and fitting of copulas will be done using S-plus Finmetrics module and calculations done using excel.

3.1. Fitting Distributions to Claim Size and Development Time

To have an idea about the desirable shape of the families of distributions that can be used we calculated descriptive statistics for both random variables and are shown as in table 3.1 below.

Table 3.1: Descriptive Statistics for Development Time and Claim Size

Statistic	Development time	Claim size
Mean	4.00	624,693.04
Median	4.00	527,326.00
Mode	1.00	67,948.00
Std. Deviation	2.47	349,555.06
Sample Variance	6.11	122,188,737,802.30
Skewness	0.59	0.58
Kurtosis	-0.58	-0.32
Range	9.00	1494452.00
Minimum	1.00	67,948.00
Maximum	10.00	1,562,400.00
Sample Size	55	55

From the descriptive statistics, the median and the mode values for both variables are not close and the range is also big. As a result the variables will not be exactly normally distributed. They exhibit a slightly longer tail to the right than the normal distribution as can be seen from the skewness values and also slightly more peaked as indicated by their respective kurtosis values. The best fitting distributions for the random variables seen in the graphs below was obtained by comparing the cumulative distribution functions (CDFs) of the random variables with the CDFs of a hypothesized distribution. For development time the best fitting distribution was obtained by a lognormal distribution with $\mu = 1.16$ and $\sigma = 0.71$ with a Kolmogorov test statistic value of 0.13 corresponding to a $p-value$ of 0.3019 which allowed us not to reject the hypothesized distribution. Claim size was also best fit by a lognormal distribution with $\mu = 13.16$ and $\sigma = 0.66$ with a Kolmogorov test statistic value of 0.11 with a corresponding $p-value$ of 0.5033.

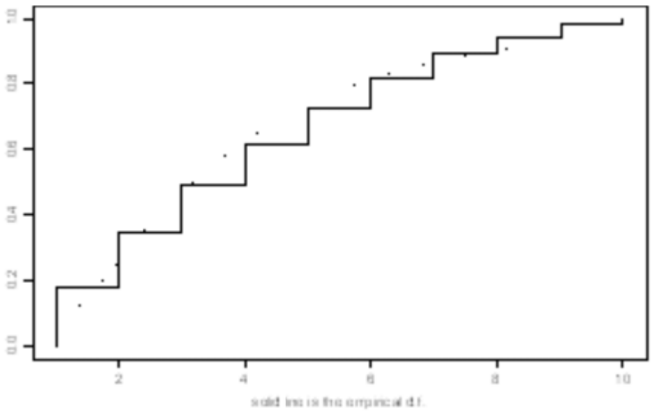

Figure 3.1: Empirical and Hypothesized CDFs for Development Time

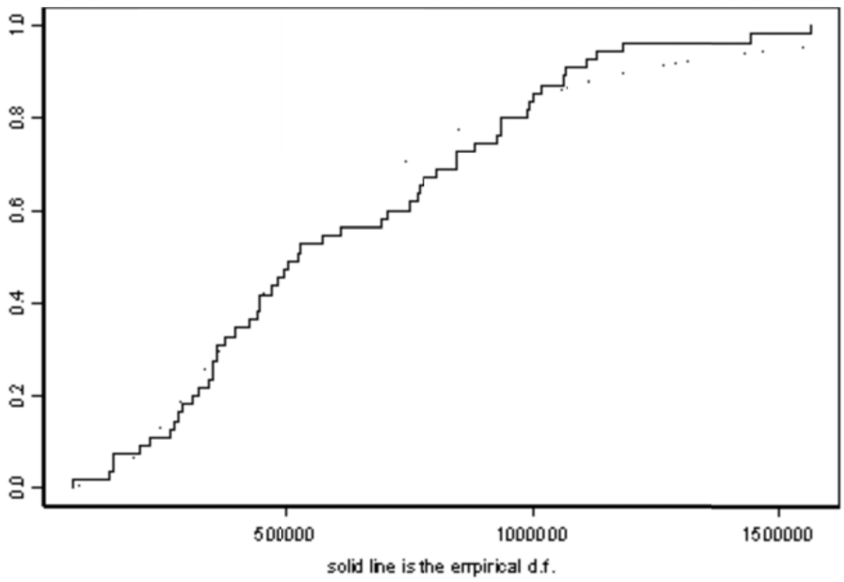

Figure 3.2: Empirical and Hypothesized CDFs for Claim Size

Relationship between the two variables was also determined using Kendall's tau and linear correlation coefficient. Kendall's tau value between the random variables was -0.215 compared to a linear correlation coefficient of -0.381. This implies a slight negative relationship. Figure 3.3 below shows this relationship between the variables.

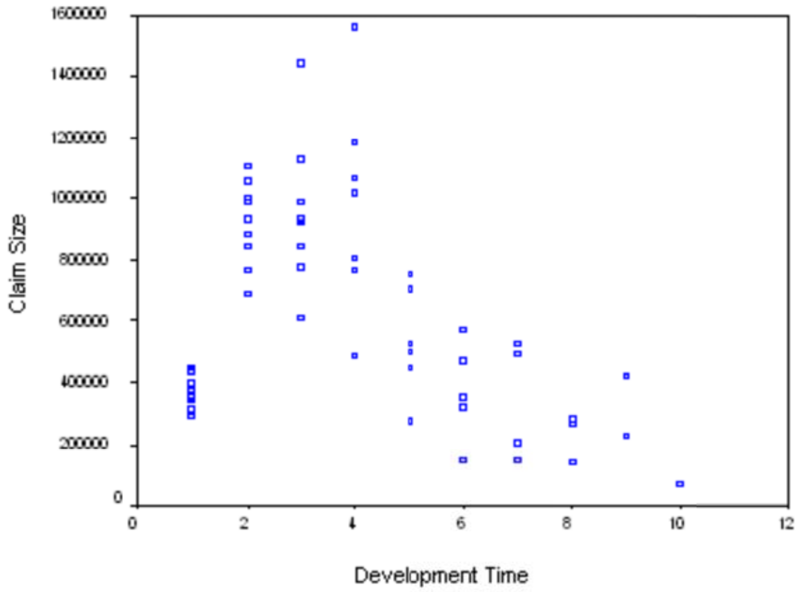

Figure 3.3: Location of the Claim Sizes in Development Time

We fitted the copulas to our data using a non-parametric fitting procedure due to Genest and Rivest (1993) outlined in the thesis by de Matteis (2001). The copula parameter estimates that were obtained using Kendall's tau obtained above were -0.053, 0.823 and -1.870 for the Clayton (Family 1), Gumbel (Family 4) and Frank (Family 5) copulas respectively of which only the Clayton's and Frank's parameter estimates were valid. To see the fit of these copulas graphically the conditional distribution of Y given X against standard uniform quantiles are plotted.

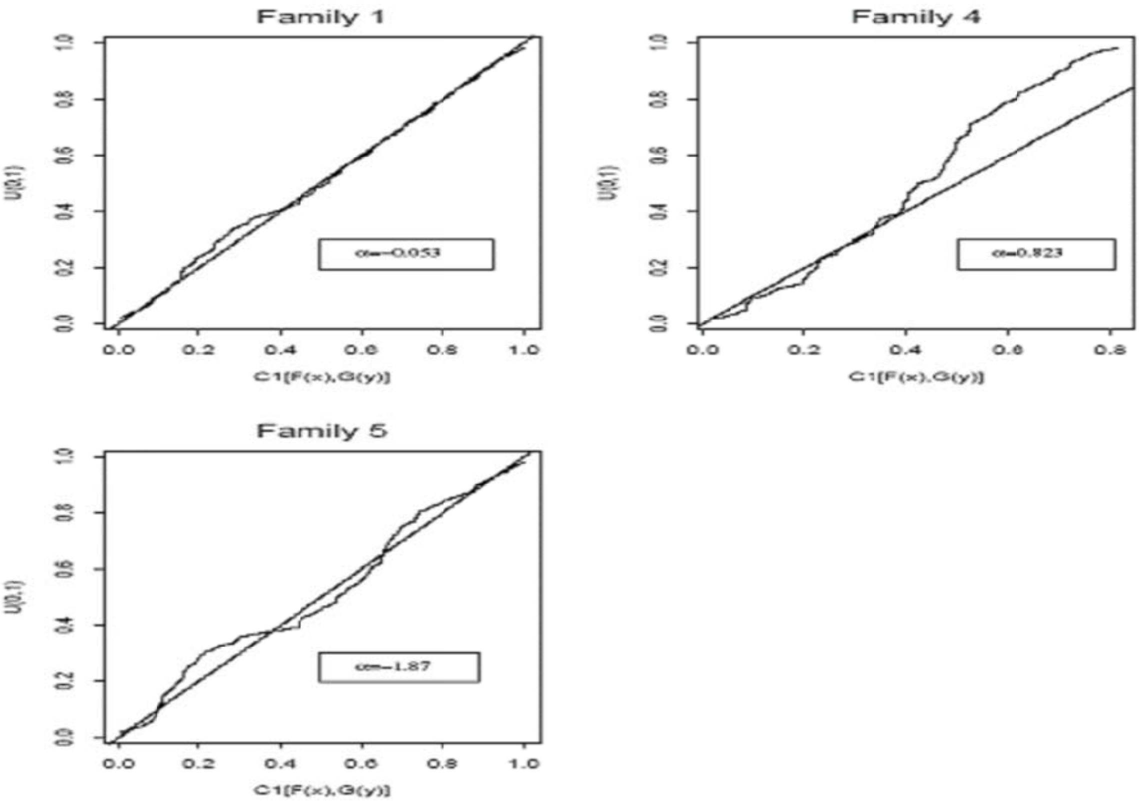

Figure 3.4: Graphs for the Approximation of the Three Copulas

From the QQ-plots above the best fitting copula for the data is the Clayton followed by the Frank copula. The choice of the two is justified by the $p-value$ of the Kolmogorov test statistic which was 0.949 for the Clayton copula and 0.502 for the Frank copula.

3.2. Estimating IBNR

200 points were simulated 10 times from the Clayton and Frank copulas and average claim sizes for each development time obtained. These averages are shown in table 3.2.

Table 3.2: Simulated Average Claim Sizes for each Development time

Development Time	Clayton	Frank
1	426,029.75	407,558.12
2	496,147.31	494,981.86
3	567,657.85	519,469.52
4	608,822.63	610,444.86
5	694,730.07	749,242.52
6	710,670.22	732,481.70
7	879,374.33	821,578.75
8	925,298.47	920,883.87
9	1,033,734.40	1,087,198.60
10	1,186,543.90	1,054,629.50

The numbers of claims happening in each year of origin were also examined and a distribution fitted as shown below.

Table 3.3: Statistics of Number of claims happening in a year

Mean	442.30
Std. Error of Mean	77.06
Median	526.50
Mode	22.00
Std. Deviation	243.67
Sample Variance	59377.12
Skewness	-0.72
Kurtosis	-0.89
Range	692.00
Minimum	22.00
Maximum	714.00
Sample Size	10.00

The best fitting distribution for the number of claims happening in a year was obtained by the lognormal distribution with $\mu = 5.766$ and $\sigma = 1.110$ with a Kolmogorov test statistic value of 0.227 with a corresponding $p-value$ of 0.358. Average number of claims happening in each year of origin is then found by multiplying the value of the lognormal density function for the number of claims happening in each year of origin by the average number of claims happening in a year and also the average plus 1, 2 and 3 standard deviations which will be represented by Dev, Dev 1, Dev 2 and Dev 3 respectively and by the length of the interval (in our case 1 year). Results are shown in table 3.4 below.

Table 3.4: Average number of claims for the Four Variations

Year of origin	Lognormal PDF	PDF*Dev	PDF*Dev 1	PDF*Dev 2	PDF*Dev 3
1	0.00050192	0.22199692	0.34430061	0.46660431	0.58890801
2	0.00038692	0.17113263	0.26541391	0.35969518	0.45397646
3	0.00041703	0.18445271	0.28607236	0.38769200	0.48931164
4	0.00050444	0.22311434	0.34603365	0.46895296	0.59187227
5	0.00054863	0.24266025	0.3763479	0.51003555	0.64372319
6	0.00069588	0.30778815	0.47735640	0.64692465	0.81649289
7	0.00089046	0.39385231	0.61083547	0.82781863	1.04480179
8	0.00146250	0.64686520	1.00323953	1.35961386	1.71598819
9	0.00206313	0.91252108	1.41525193	1.91798277	2.42071362
10	0.00089534	0.39600942	0.61418099	0.83235256	1.05052412

IBNR reserves are then calculated by multiplying the simulated average claim size in each development time by the average number of claims in each development time unit and to the number of time units for all the four variations. The results are shown in table 3.5 below.

Table 3.5: Estimated IBNR Reserves using the Clayton Copula

Year of origin	PDF*Dev	PDF*Dev 1	PDF*Dev 2	PDF*Dev 3
1	0.00	0.00	0.00	0.00
2	203,056.37	314,925.24	426,794.11	538,662.98
3	409,536.34	635,160.22	860,784.10	1,086,407.99
4	701,823.27	1,088,475.38	1,475,127.49	1,861,779.61
5	1,759,584.00	2,159,472.85	2,559,361.70	2,959,250.54
6	1,457,568.12	2,260,579.10	3,063,590.09	3,866,601.08
7	2,138,756.42	3,317,051.20	4,495,345.97	5,673,640.75
8	3,906,531.44	6,058,738.01	8,210,944.57	10,363,151.14
9	6,091,041.94	9,055,224.23	12,019,406.53	14,983,588.82
10	2,812,846.66	4,362,514.73	5,912,182.80	7,461,850.87
Total	**19,480,744.55**	**29,252,140.96**	**39,023,537.37**	**48,794,933.79**

Finally we compare the four total estimated IBNR reserves from both copulas using the Mann-Whitney U statistic. The test statistic value was -0.577 corresponding to a $p-value$ of 0.686 implying that the estimates obtained from the two copulas are not significantly different.

4. CONCLUSION

In this paper we have considered two variables, development time and claim size, which are both lognormally distributed. A critical analysis of their bivariate distribution revealed that only two copula, the Clayton and Frank, out of the three Archimedean copulas examined, could be used to study the bivariate data. It is observed that estimates of outstanding IBNR claim reserves from the Clayton copula were slightly lower than the estimates from the Frank copula and this could be attributed to the fact that the Clayton copula is a better fit compared to the Frank copula. However, the Mann-Whitney U statistic test showed that the estimates obtained from the two copulas are not significantly different. Therefore, from these results we conclude that if only one copula which statistically fit a given set of multivariate data is required, then any of the two copulas can be picked for estimation since each one will statistically give the same results.

REFERENCES

[1] R. L. Bornhuetter and R. E. Ferguson, The Actuary and IBNR, Proceedings of the Casualty Actuarial Society, 1973, 181 – 195.

[2] U. Cherubini, E. Luciano and W. Vecchiato, Copula Methods in Finance, Wiley, Chichester, 2004.

[3] R. De Matteis, Fitting Copulas to Data, Diploma Thesis, Institute of Mathematics, University of Zurich, 2001.

[4] P. Embrechts, A. McNeil, and D. Straumann, Correlation and Dependence in Risk Management, Properties and Pitfalls, Cambridge University Press, 2001.

[5] P. England, and R. Verrall, Standard Errors of Prediction in Claims Reserving: A Comparison of Methods, General Insurance Convention and ASTIN Colloquium, Glasgow, Scotland, 1998.

[6] E. W. Frees and E. A. Valdez, Understanding Relationships Using Copulas, North American Actuarial Journal, 2 (1988), 1 – 25.

[7] C. Genest and L. Rivest (1993). Statistical Inference Procedures for Bivariate Archimedean Copulas. Journal of the American Statistical Association, 88 (1993), 1034 – 1043.

[8] M. J. Goovaerts, J. L. M. Dhaene, V. E. Borre and R. Redant, Some remarks on IBNR evaluation techniques, Belgian Actuarial Bulletin, 2001, 58 – 60.

[9] C. Hipp, Dependence Concepts in Finance and Insurance: Copulas, Karlsruhe University, 2005.

[10] S. A. Klugman, H. Panjer, G. Venter and G. Willmot, Loss Models: From Data to Decisions, Unpublished Monograph, 1984.

[11] S. A. Klugman and R. Parsa, Fitting bivariate loss distributions with Copulas. Insurance, Mathematics and Economics, 24 (1999), 139 – 148.
[12] W. J. Krzanowski, Principles of Multivariate Analysis: A User's Perspective, Oxford University Press, Oxford, 1988.
[13] T. Mack, Distribution free calculation of the standard error of chain ladder reserve estimates, ASTIN Bulletin, 23 (1993), 213 – 225 .
[14] R. B. Nelsen, Dependence Modeling with Archimedean Copulas, Department of Mathematical Sciences, Lewis and Clark College, 2005.
[15] R. B. Nelsen, An Introduction to Copulas. Springer Series in Statistics, 2007.
[16] G. Pettere and T. Kollo, Modelling Claim Size in Time via Copulas. Proceedings of the 28th International Congress of Actuaries, ICA 2006.
[17] G. Pettere, Modelling Incurred But Not Reported Claim Reserve Using Copula, Proceedings of the 28th International Congress of Actuaries, ICA 2006.
[18] A. E. Renshaw, Chain Ladder and Interactive Modelling (Claims Reserving and GLIM), Journal of the Institute of Actuaries, 116 (1989), 559 – 587.
[19] A. E. Renshaw, On the second moment properties and the implementation of certain GLIM based stochastic claims reserving models, Actuarial Research Paper, No. 65, Department of Actuarial Science and Statistics, City University, London, 1994.
[20] G. C. Taylor and F. R. Ashe, Second Moments of Estimates of Outstanding Claims, Journal of Econometrics, 23 (1983), 37 – 61.
[21] R. J. Verrall, On the Estimation of Reserves from Loglinear Models, Insurance, Mathematics and Economics, 10 (1991a), 75 – 80.
[22] R. J. Verrall (1991b). Chain Ladder and Maximum Likelihood, Journal of the Institute of Actuaries, 18 (1991b), 489 – 499.

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