The Minimum Test Collection Problem

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Abstract

In this paper we consider an approach to solve the minimum test collection problem. This approach is based on an explicit reduction from the problem to the satisfiability problem.

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Investigation of different regularities plays an important role for the detection of various knowledge (see e.g. [1] – [8]). In particular, the minimum test collection problem can be used for rapid identification of unknown pathogens (see e.g. [9]). Let

\[ D = \{D[1], \ldots, D[n]\}, \]
\[ T = \{T[1], \ldots, T[m] \mid T[i] \subseteq D\}. \]

The minimum test collection problem (MTC):

Instance: Given a set \( D \) and a positive integer \( d \).

Question: Is there a set \( S \subseteq T \) such that \( |S| \leq d \) and for any \( 1 \leq i < j \leq n \), there is \( T[k] \in S \) such that

\[ |\{D[i], D[j]\} \cap T[k]| = 1? \]
Note that MTC is \textbf{NP}-complete (see e.g. [10]). Encoding hard problems as instances of SAT and solving them with different efficient satisfiability algorithms has caused considerable interest (see e.g. [11] – [15]). In this paper, we consider an approach to solve the MTC problem. Our approach is based on an explicit reduction from the problem to the satisfiability problem. Let

\[ \varphi[1] = \land_{1 \leq i \leq d} \lor_{1 \leq j \leq m} x[i, j], \]
\[ \varphi[2] = \land_{1 \leq i \leq d} \land_{1 \leq j, j[2] \leq m} (\neg x[i, j[1]] \lor \neg x[i, j[2]]), \]
\[ \varphi[3] = \land_{1 \leq i \leq d} \land_{1 \leq j \leq m} \land_{1 \leq k \leq n, D[k] \in T[j]} (\neg x[i, j] \lor y[i, k]), \]
\[ \varphi[4] = \land_{1 \leq i < j \leq n} \land_{1 \leq k \leq d} z[i, j, k], \]
\[ \varphi[5] = \land_{1 \leq i < j \leq n} ((\neg z[i, j, k] \lor y[k, i] \lor y[k, j]) \land (\neg z[i, j, k] \lor \neg y[k, i] \lor \neg y[k, j])). \]

Let \( \xi = \land_{i=1}^5 \varphi[i] \). It is easy to check that there is a set \( S \subseteq T \) such that \( |S| \leq d \) and for any \( 1 \leq i < j \leq n \), there is \( T[k] \in S \) such that \( |\{D[i], D[j]\} \cap T[k]| = 1 \) if and only if \( \xi \) is satisfiable. It is clear that \( \xi \) is a CNF. So, \( \xi \) gives us an explicit reduction from MTC to SAT. Now, using standard transformations (see e.g. [16]) we can obtain an explicit transformation \( \xi \) into \( \zeta \) such that \( \xi \iff \zeta \) and \( \zeta \) is a 3-CNF. Clearly, \( \zeta \) gives us an explicit reduction from MTC to 3SAT. In papers [17, 18] the authors considered some satisfiability algorithms. Our computational experiments have shown that these algorithms can be used to solve MTC.

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**References**


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