Numerical Method for Prediction of Material’s Properties Effect on Temperature Difference in the Gap of Contact Surface During Conduction Heat Transfer

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Abstract

Gaps are always present in the contact surfaces of two solid material. Air trapped in the gaps inhibit heat transfer and causes the contact resistance. Temperature difference between the contact surfaces is very important in many practical applications, however, it is difficult to be measured. In this study, the temperature difference in the gap is estimated by numerical methods of one-dimensional conduction heat transfer without taking into account the surface roughness, elasticity of the material, and the contact pressure. The numerical methods is used to estimate the gap temperature difference in the contact surface of thermoplastic types of polypropylene material, limestone, steel, and copper. The results show that the temperature difference in the gap depends on the properties of the contact material and the air, such as thermal conductivity, density, and specific heat. The larger the thermal conductivity of solid material the lower the temperature difference in the gap and the shorter the transient time for the gap temperature difference to reach its maximum. Once it becomes maximum, the temperature difference in the gap fluctuated periodically with the amplitude becoming increasingly larger. The greater
Introduction

Heat transfer through a contact area between solid materials is important for many practical applications such as cooling of micro-electronic devices, heat transfer in the nut-bolts connection, bearings, thermoelectric systems and so on. Some researchers have reported that the heat transfer at the contact area is influenced by the roughness of the surface [9]. Studies on heat transfer through the contact surface for the purpose of preheating on the object to be merged are still very rare.

The result of numerical simulation on conduction heat transfer at contact area shows that the thermal contact conductance increases linearly with the pressure on the contact area [8]. Work has also been done even with three-dimensional numerical method on the wavy surface of a square or hexagonal pattern [6]. The result shows that the thermal contact conductance increases linearly with the slope of the wavy contact surface [13]. In practical application heat transfer in contact area of elastic solids is estimated by taking surface roughness into account [9]. A mathematical model for the effect of surface roughness, contact pressure, gas density, heat capacity, and mechanical properties of the material on the temperature difference in the gap has been developed [10]. Temperature difference on the contact surface increases with time during transient and reaches a maximum value during steady state. During transients temperature difference cannot be considered to be constant [4]. Transient method has been developed and successfully employed to measure the dependence of the thermal contact conductance on the contact pressure. [5]. Finite element method is employed to determine the effect of surface morphology on the thermal contact resistance, while the contact surface temperature is determined by the method of regression. The results showed that the contact surface temperature decreases linearly with increase in surface roughness and nonlinearly decreases with increase in ratio of non-contact and nominal contact area [1]. There is a strong relationship between the average gap distance and surface roughness for all gases that fills in the gap [11]. For heat transfer at interface, radiation is negligible at temperatures below 600 °C [7].

In practical application such as in melting process the contact pressure, contact area, the elasticity of materials, surface roughness, and flatness of contact surface are impossible to be measured. The most important thing that must be identified prior to melting process is the properties of the material because it determine how the process will be handle. Thus it is necessary to develop a simple formula to predict heat transfer at contact area based on the properties of materials.

This paper provides a new simple formula for solving complex conduction heat
transfer problem in the solid contact. The equation is derived from the combination of governing equation and the thermodynamic equation calculated by numerical methods. Calculation of heat transfer in solids contact area is merely determined by the properties of the material without taking into account the elasticity, roughness, flatness, and contact pressure of the contact area. Contact surfaces are considered to be perfect such that the gap in the contact surfaces is very narrow. In this case, Grashof number is very small, below 2500, that convection heat transfer can be neglected [12]. Thus, heat is transfered in the gas trapped in the gap only by conduction.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>A</td>
<td>area (m²)</td>
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<tr>
<td>Cp</td>
<td>specific heat (J kg⁻¹ K⁻¹)</td>
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<tr>
<td>Gr</td>
<td>Grashof number</td>
</tr>
<tr>
<td>Ko</td>
<td>thermal inertia ratio (ρ ρ Cpg/ρ ρ Cps)</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity (W m⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>L</td>
<td>length (m)</td>
</tr>
<tr>
<td>m</td>
<td>mass (kg)</td>
</tr>
<tr>
<td>q’</td>
<td>heat flux per unit volume (W m⁻³)</td>
</tr>
<tr>
<td>Q</td>
<td>heat flux (W)</td>
</tr>
<tr>
<td>t</td>
<td>time (s)</td>
</tr>
<tr>
<td>S</td>
<td>gap width or gas layer thickness (m)</td>
</tr>
<tr>
<td>T</td>
<td>temperature (K)</td>
</tr>
<tr>
<td>u</td>
<td>heat propagation speed (m s⁻¹)</td>
</tr>
<tr>
<td>V</td>
<td>volume (m³)</td>
</tr>
<tr>
<td>x</td>
<td>horizontal axis</td>
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</table>

Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>β</td>
<td>thermal inertia</td>
</tr>
<tr>
<td>δ</td>
<td>stating ratio S/dx</td>
</tr>
<tr>
<td>ρ</td>
<td>density (kg m⁻³)</td>
</tr>
<tr>
<td>α</td>
<td>thermal diffusivity of material (m² s⁻¹)</td>
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</table>

Subscripts

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<tr>
<td>g</td>
<td>gas</td>
</tr>
<tr>
<td>s</td>
<td>solid</td>
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2. Problem Formulation

The physical model considered in this work is shown schematically in figure 1. Figure 1a shows two homogeneous solid materials in contact taking into account surface roughness. Figure 1b illustrates that the surface roughness is not taken into account but it is represented by a gap which is the sum of all the very small cavities formed by the surface roughness of the two solid materials in contact.

The mathematical model in this paper is built based on figure 1b. The gap filled by gas in the contact surface of the solid objects disrupts the flow of heat from one side to the other. This gap is considered to be a source of negative energy that is when the temperature of the surrounding solid object raises the gas in the gap will absorb the heat. Consequently, the temperature in the gap drops. This is in accordance with the first law of thermodynamics, gas will reduce the flow of heat energy. Heat of the gap will be forwarded to the solid material which temperature is lower than the gas temperature. Once the heat flux quantities is estimated the temperature difference in the gap can be obtained.

Governing equations for one-dimentional conduction heat transfer with heat source is as follows:
\[ \frac{k_s}{\partial^2 T_s}{\partial x^2} - q' = \rho_s C_p_s \left( \frac{\partial T_s}{\partial t_s} \right) \]  

where:  
\[ q' = \frac{m_g C_p_g}{V} \frac{dT}{dt} \]  
\[ m_g = \rho_g V_g \]  

Equation (2) and equation (3) give  
\[ q' = \frac{\rho_s V_g C_p_g}{V} \frac{dT}{dt} \]  

Replacing \( V_g = A \cdot dx \) and \( V = A \cdot dx \) then:  
\[ q' = \frac{\rho_s V_g C_p_g}{V} \frac{dx}{dt} \]  

The gas trapped in the gap (S) is very thin. Heat convection in the gas can be ignored because the Grashof number which is a function of \( S^3 \) becomes very small. Thus the heat is transferred through the gas only by conduction. Substituting equation (5) into equation (1) gives  
\[ k_s \frac{\partial^2 T_s}{\partial x^2} + \frac{\rho_g V_g C_p_g}{V} \frac{dx}{dt} \frac{dT_s}{dt} = \rho_s C_p_s \left( \frac{\partial T_s}{\partial t_s} \right) \]  

In this case, \( \frac{dx}{dt} \) is the conduction heat propagating speed due to the rate of absorption of heat by the gas in the gap. When there is no heat absorption, \( \frac{dx}{dt} \) equals zero. This indicates that the temperature on the left side of the gap equals to the temperature on the right side. At constant pressure, heat absorption speed is directly proportional to the heat energy that flows into the gas and inversely proportional to the gas density, contact area, gas specific heat, and gas temperature change, or it can be written as the following formula [3]:  
\[ Q = \frac{m C_p dT}{dt} \]  
then:  
\[ \frac{dx}{dt} = \frac{Q}{\rho A C_p dT} \]  

Fig. 1. Physical model of heat transfer in contact area of solid materials. (a) Two solid materials with surface roughness in contact; (b) sum of all the very small cavities formed by the surface roughness is considered as one gap.
3. Numerical

The equation (6) is rearranged to the new form as in equation (8):

\[
\frac{\partial T}{\partial t} = \frac{k_s}{\rho_s C_{ps}} \frac{\partial^2 T}{\partial x^2} + \frac{\rho_g C_{pg} dT_g}{\rho_s C_{ps} dx} \frac{dx}{dt}
\]  

(8)

Partial differential equation (8) is converted into ordinary differential equation (9) to become

\[
\frac{dT}{dt} = \frac{k_s}{\rho_s C_{ps}} \frac{d^2T}{dx^2} + \frac{\rho_g C_{pg} dT_g}{\rho_s C_{ps} dx} \frac{dx}{dt}
\]  

(9)

Solution of equation (9) using one dimensional numerical method can be described as in figure 2. Position of the gap is in i to z with thickness of \( \delta \Delta x \). In this case \( \Delta x \) is the distance from i to i+1. Therefore, the rest of \( \Delta x \), from z to i+1 is a solid.

Fig. 2. Discretization scheme

Here \( \delta \) is a dimensionless scale which is the ratio between S and dx or \( \delta = S/dx \).

From figure 2 it could be demonstrated that when:

- \( \delta = 0 \) means no gap, all solid
- \( 0 < \delta < 1 \) means the gap is less than \( \Delta x \)
- \( \delta = 1 \) means the gap equals to \( \Delta x \)
- \( \delta > 1 \) means the gap is greater than \( \Delta x \)

The temperature in the nodal is

\[
T_g = T_i
\]

\[
T_{i+1} = T_{g+1} + (1 - \delta) (T_{i+1} - T_i)
\]

\[
T_{g+1} = \delta T_{i+1} + (1 - \delta) T_i
\]

Equation (9) is converted into a numerical equation that result is equation (10),

\[
\frac{dT}{dt} = \frac{k_s}{\rho_s C_{ps}} \frac{T_{i-1} + T_{i+1} - 2T_i}{\Delta x^2} + \frac{\rho_g C_{pg}}{\rho_s C_{ps} \Delta x} \frac{dx}{dt} (T_{g+1} - T_g)
\]

\[
\frac{dT}{dt} = \frac{k_s}{\rho_s C_{ps}} \frac{T_{i-1} + T_{i+1} - 2T_i}{\Delta x^2} + \frac{\rho_g C_{pg}}{\rho_s C_{ps} \Delta x} \frac{dx}{dt} (\delta T_{i+1} + (1 - \delta) T_i - T_i)
\]

\[
\frac{dT}{dt} = \frac{k_s}{\rho_s C_{ps}} \frac{T_{i-1} + T_{i+1} - 2T_i}{\Delta x^2} + \frac{\rho_g C_{pg}}{\rho_s C_{ps} \Delta x} \frac{dx}{dt} (T_{i+1} - T_i)
\]  

(10)

By notating the value \( (\rho_s C_{ps}) \) with \( \beta_s \) which is a thermal inertia of the solid, the value \( (\rho_g C_{pg}) \) with \( \beta_g \) which is a thermal inertia of the gas in the gap, and \( \frac{dx}{dt} = u \), then the equation (10) becomes equation (11).
\[
\frac{dT}{dt} = \frac{\alpha_s}{\Delta x^2} \left( T_{i+1} + T_{i-1} - 2T_i \right) + \frac{\beta_s}{\beta_i} \frac{\delta}{\Delta x} u \left( T_{i+1} - T_i \right) \tag{11}
\]

if \( \frac{\alpha_s}{\Delta x^2} = \lambda_s \); \( \frac{\beta_s}{\beta_i} = Ko \) and \( A = Ko \frac{\delta}{\Delta x} u \)

then equation (11) turns into equation (12):
\[
\frac{dT}{dt} = (\lambda_s + A) T_{i+1} - (2 \lambda_s + A) T_i + \lambda_s T_{i-1} \tag{12}
\]

Equation (12) is conduction heat transfer through the gap in contact surface. The conduction heat transfer equation for solids without gap can be obtained by making the value of \( \delta = 0 \) so that \( A = 0 \) and then put it into the equation (12). Thus the equation (12) becomes
\[
\frac{dT}{dt} = \lambda_s \left( T_{i+1} + T_{i-1} - 2T_i \right) \tag{13}
\]

4. Results and Discussion

To determine the effect of material properties on the contact surface gap temperature during conduction heat transfer, equation (12) and (13) are applied to various metal and non-metal materials such as steel, copper, polypropylene and limestone. The gas that fills the gap is air. The properties of these materials are taken from the results of a preliminary study as well as from [4] and [2] as shown in Table 1. Other variables are defined as \( \delta = 0.01 \), the heat propagation velocity \( u = 0.1 \text{ m/s} \) and \( \Delta x = 0.001 \text{ m} \).

Table 1. Properties of materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Polypropylene</th>
<th>Limestone</th>
<th>Steel</th>
<th>Copper</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductivity, ( k ) (W/m² K)</td>
<td>0.2</td>
<td>1.3</td>
<td>37.7</td>
<td>379</td>
<td>0.03</td>
</tr>
<tr>
<td>Density, ( \rho ) (kg/m³)</td>
<td>900</td>
<td>2500</td>
<td>7822</td>
<td>8954</td>
<td>1.14</td>
</tr>
<tr>
<td>Heat Capacitance, ( C_p ) (j/kg.K)</td>
<td>1600</td>
<td>900</td>
<td>444</td>
<td>383</td>
<td>1009</td>
</tr>
<tr>
<td>Thermal diffusivity, ( \alpha = k/\rho C_p ) (m²/s)</td>
<td>( 1.4\times10^{-7} )</td>
<td>( 5.8\times10^{-7} )</td>
<td>( 1.1\times10^{-4} )</td>
<td>( 1.1\times10^{-4} )</td>
<td>( 2.6\times10^{-5} )</td>
</tr>
<tr>
<td>Ko = ( \rho g C_p / \rho_s C_p )</td>
<td>( 8\times10^{-4} )</td>
<td>( 5.1\times10^{-4} )</td>
<td>( 3.3\times10^{-4} )</td>
<td>( 3.35\times10^{-4} )</td>
<td>-</td>
</tr>
</tbody>
</table>

The temperature distribution is calculated in the arrangement of contact solid materials as shown in figure 3. The thick of each solid material is 1 mm. Position of gaps are given by X1, X2, X3, X4 and X5. Heat energy enters the system from the left, \( X = 0 \) when \( t \geq 0 \) and the temperature is maintained at 150°C. The heat energy left the system at the right at \( X = L \) when \( t \geq 0 \) and the temperature is maintained at 30°C.

The estimated temperature difference are presented in figure 4 up to 7. The temperature difference (\( \Delta T \)) was obtained from the difference between the
Numerical method for prediction of material’s properties effect

Figure 3. Structure of contact materials

Figures 4 to 7 present the instantaneous change of temperature difference in each gap of the five areas of contact for heating of polypropylene, limestone, steel, and copper. All figures show that there are three different characteristics of temperature difference change in the gap for each material. The first is a transient increase in gap
temperature difference, the second is the magnitude of the gap temperature difference at steady state, and the third is a periodic fluctuation of gap temperature difference during steady state.

Figure 4 shows that transient increase in the gap temperature difference of polypropylene becomes steady in 80 seconds and the temperature difference during steady state ranged from 0.3°C to 0.5°C. Temperature difference in the gap of limestone requires 20 seconds to reach steady state and the gap temperature difference is ranged from 0.04°C to 0.08°C (Figure 5). For steel, as shown in Figure 6, takes a shorter transient that is 2 seconds and the gap temperature difference is also smaller, i.e., around 0.0015°C to 0.0025°C. Copper only takes 0.1 seconds to reach a steady state. Temperature difference in the gap ranged from 0.0002°C to 0.0004°C (Figure 7). From equation (9) and Table 1 it can be seen that the cause of the difference between the transient and the temperature difference in the gap is thermal inertia ratio between air and solid material (K₀) and the thermal diffusivity (α) or thermal conductivity (k) of solid materials. At large K₀ and small k the transient time is long and the gap temperature difference is large. This occurs in polypropylene (Figure 4) and limestone (Figure 5). On the contrary, if K₀ is small and k is large, the transient time is short and the gap temperature difference is small as in steel (Figure 6) and copper (Figure 7).

As shown in Figures 4 to 7 that during steady state, the temperature difference in the gap still fluctuate periodically. This is explained in equation (9) that the fluctuations is due to the equilibrium between the rate of increase of temperature due to thermal diffusivity in solid (αₙ = kₙ/ρₙCₚₙ), the first term of the right hand side of the equation, and the rate of heat absorbed due to thermal inertia of the air in the gap (K₀ = ρₐCₚₐ/ρₙCₚₙ), the second term of the right hand side of the equation. When the rate of temperature increase due to thermal diffusivity in solid is in equilibrium with that due to the heat absorbed by the air in the gap, the gap temperature difference remains steady (dTₛ/dt = 0). Otherwise, the rate of temperature increase in solid is not in the same time with that in air gap. Therefore the gap temperature difference may increases and decreases or fluctuate periodically. When K₀ is large and kₙ is small, the rate of temperature increase in the gap is low and therefore the frequency of fluctuations is low. Because the number of heat absorbed in the gap is large due to large K₀ while the rate of heat transfer in solid is low due to the small kₙ the temperature in the gap tends to increase continously for the heat can not be forwarded soon by a solid wall. As a result, the amplitude of the temperature difference at the gap is high such as occurs in polypropylene (Figure 4) and limestone (Fig. 5). In contrast, at small K₀ and large kₙ the rate of gap temperature increase is high due to low thermal inertia so that the frequency of temperature difference fluctuation is high. Due to rapid heat transfer by the large kₙ the temperature in the gap has not enough time to increase so that the amplitude of temperature difference is small as in steel (Figure 6) and copper (Figure 7). At the condition of very large kₙ as in copper, the frequency of gap temperature difference fluctuation is very high and even the temperature difference in the gap can be negative.
The temperature difference fluctuation in the gap may disturbs heating of solid material especially in melting process. During melting the temperature should be constant even it could have to rise in order to overcome the melting surface tension.

5. Conclusion

The new equation for one-dimentional conduction heat transfer developed in this study which considers gap as a source of negative energy is calculated by using numerical method. The equation can predict the temperature difference across the gap with fairly good results though surface roughness, the contact pressure, and elasticity of the material are not taken into account. The results obtained are as follows:
1. Greater value of the thermal conductivity result in smaller temperature difference in the gap.
2. Temperature difference in the gap tends to fluctuate. The amplitude and the frequency of fluctuation depend on the thermal inertia of the air in the gap and the thermal conductivity of solid wall. The amplitude is high and the frequency is low when the thermal inertia of the air in the gap is large and thermal conductivity of solid material is small, and vice versa. For all kinds of materials the amplitude of temperature difference fluctuation tend to increase with time and for large thermal conductivity the temperature difference in the gap can be negative.

References


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