Applied Mathematical Sciences, Vol. 7, 2013, no. 141, 7027 - 7036
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ams.2013.37418

Applications of Sumudu Transform to MHD Flows of an Oldroyd-B Fluid

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Abstract

In this paper, Sumudu transform is employed to solve the unsteady MHD flow problems namely, Couette flow, Poiseuille flow and generalized Couette flow of an Oldroyd-B fluid between two parallel plates. The fluid is electrically conducting and magnetic field is applied perpendicular to the plates. Since the Sumudu transform has units preserving properties, therefore aforementioned problems are solved without restoring the frequency domain. But these properties not exist in the Laplace transform, which is the major benefit of Sumudu transform over the Laplace transform. Further, the solutions that have been obtained, have complete agreement with those established by using the Laplace transform. The obtain solutions reveal that the method is very effective and simple.

Mathematics Subject Classification: 76A05

Keywords: Sumudu transform, Oldroyd-B fluid, Velocity profile

1 Introduction

Sumudu transform is an integral transform which was first time introduced by Watagula [1]. Its simple formulation and direct applications to partial differential equations immediately sparked interest in this new tool. This new transform was further developed and applied to many problems by various researchers [1–3] and references there in. The Sumudu transform has very special and useful properties and can help to solve complex applications in science and engineering. Having units preserving properties, it may be used to solve problems without resorting to the frequency domain. This is one of many strength points for this new transform, especially with respect to applications in problems with physical dimensions. In fact, the Sumudu transform which is itself linear, preserves linear functions, and hence in particular does not change units [1,4]. Belgacem et al. [4] have shown it to be the theoretical dual to the Laplace transform, and hence ought to rival it in problem solving.

In this work, we employ the Sumudu transform to the flow of non-Newtonian fluids. Non-Newtonian fluids such as paints, grease, oils, blood, liquid polymers, glycerin etc., are frequently encountered in many disciplinary fields including chemical engineering, foodstuff, biomedicine etc., and also are closely related to many industrial processes. The study of non-Newtonian fluid flow is also of significant interest in oil reservoir engineering. For a variety of reasons, non-Newtonian fluids are classified on the basis of their behavior in shear. A fluid with a linear relationship between the shear stress and the shear rate, giving rise to a constant viscosity, is always characterized to be a Newtonian fluid. As a constant viscosity relation is not always a Newtonian fluid relation because there are fluids like a second-order fluid, a convected Maxwell fluid and an Oldroyd-B fluid that are certainly non-Newtonian with constant viscosity. In recent years, the Oldroyd-B fluid has acquired a special status amongst the many fluids of rate type, as it includes as special cases the classical Newtonian fluid and the Maxwell fluid. As a result of their wide implications, a lot of papers regarding these fluids have been published in the last time [5-11].

Further, the study of electrically conducting of non-Newtonian fluids has applications in many areas. A few examples are the flow of plasma, flow of mercury amalgams and handling of biological fluids. For example, blood (due to the presence of its red cells) is an electrically conducting fluid. Although, its plasma component is Newtonian, it often exhibits non-Newtonian behavior, especially at low shear rates. The presence of an external magnetic field that effects the blood flow is very important, for example in cardiology [12].

The arrangement of the paper is as follows. In Section 2, we document the governing equations. This is followed by the study of the three transient flow problems namely, Couette flow, Poiseuille flow and generalized Couette flow between two parallel plates. Rana *et al.,* [13] initially applied the Sumudu transform technique to the unsteady one-dimensional Newtonian fluid problems. In this work, we employ this technique to the unsteady unidirectional non-Newtonian fluid problems and all the expressions for velocity profiles are constructed for small times.

2 Governing equations

The basic equations which govern the incompressible unidirectional magnetohydrodynamic flow are:

$$\nabla \cdot \mathbf{V} = \mathbf{0}, \tag{1}$$

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + \nabla \cdot \mathbf{S} + \rho \mathbf{f} + \mathbf{J} \times \mathbf{B}, \tag{2}$$

where p is the pressure, ρ is the density of the fluid, d/dt is the material time derivative, $\mathbf{V}$ is the velocity field, $\mathbf{f}$ is the body force, ∇ is the divergence operator, $\mathbf{S}$ is the extra stress tensor, $\mathbf{J}$ is the electric current density and $\mathbf{B}$ is the total magnetic field, $\mathbf{B}=\mathbf{B}_\circ+\mathbf{b}$, $\mathbf{B}_\circ$ represents the imposed magnetic field and $\mathbf{b}$ denotes the induced magnetic field. In the absence of displacement currents, the modified Ohm's law and Maxwell's equations [14] are

$$\mathbf{J} = \sigma[\mathbf{E}+\mathbf{V} \times \mathbf{B}], \tag{3}$$

$$div\mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \mu_m \mathbf{J}, \quad curl\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{4}$$

in which σ is the electrical conductivity, $\mathbf{E}$ is the electric field and μ_m is the magnetic permeability.

From Ohm's law and Maxwell's equations an evolution for the magnetic flux $\mathbf{B}$ can be obtained easily. This is known as the magnetic induction equation and it suggests that the motion of an electrically conducting fluid in an applied magnetic field induces a magnetic field in the medium. We assume that the total magnetic field $\mathbf{B}$ is perpendicular to the velocity field $\mathbf{V}$ and the induced magnetic field $\mathbf{b}$ is negligible compared with the applied magnetic field $\mathbf{B}_\circ$, so that magnetic Reynolds number is small. Since no external electric field is applied and the effect of polarization of the ionized fluid is negligible, the fluid flow region is assumed as no electric field. Under these assumptions, the magnetohydrodynamic force involved in Eq. (1) can be put into the form [14]

$$\mathbf{J} \times \mathbf{B} = -\sigma B_0^2 \mathbf{V}, \tag{5}$$

For incompressible Oldroyd-B fluid, the constitutive equation for extra stress tensor $\mathbf{S}$ is given by [5]

$$\mathbf{S} + \lambda \frac{D\mathbf{S}}{Dt} = \mu \left(1 + \theta \frac{D}{Dt}\right) \mathbf{A}_1, \tag{6}$$

Where μ, λ, θ are material constants respectively known as the viscosity coefficient, the relaxation and retardation times. It is assumed that $\lambda \geq \theta \geq 0$. $\mathbf{A}_1$ is the first Rivlin-Ericksen tensor and D/Dt the upper convected derivative defined as follows:

$$\mathbf{A}_1 = \nabla\mathbf{V} + (\nabla\mathbf{V})^T, \qquad \frac{D\mathbf{S}}{Dt} = \frac{\partial\mathbf{S}}{\partial t} + (\mathbf{V}.\nabla)\mathbf{S} - (\nabla\mathbf{V})\mathbf{S} - \mathbf{S}(\nabla\mathbf{V})^T, \tag{7}$$

where the superscript T denotes a transpose operation.
For the problem under consideration we assume a velocity field and stress tensor of the form

$$\mathbf{V} = (u(y,t), 0, 0), \quad \mathbf{S} = \mathbf{S}(y,t). \tag{8}$$

By using Eq. (8), the continuity equation is satisfied automatically and the equation of motion (2), in the absence of gravitational effect becomes in component form:

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}s_{xy} - \sigma B_0^2 u, \tag{9}$$

$$0 = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial y}s_{yy}, \tag{10}$$

$$0 = -\frac{\partial p}{\partial z}. \tag{11}$$

Substituting Eq. (8) in (6), we find the following system of equations

$$s_{xx} + \lambda\left[\frac{\partial}{\partial t}s_{xx} - 2s_{xy}\frac{\partial u}{\partial y}\right] = -2\mu\theta\left(\frac{\partial u}{\partial y}\right)^2, \tag{12}$$

$$s_{xy} + \lambda\left[\frac{\partial}{\partial t}s_{xy} - s_{yy}\frac{\partial u}{\partial y}\right] = \mu\left(\frac{\partial u}{\partial y}\right) + \mu\theta\left(\frac{\partial^2 u}{\partial y\partial t}\right), \tag{13}$$

$$s_{yy} + \lambda\frac{\partial}{\partial t}s_{yy} = 0. \tag{14}$$

We notice that Eq. (12) can be integrated to yield

$$s_{yy} = f(y)e^{-\frac{t}{\lambda}}, \tag{15}$$

where $f(y)$ is an arbitrary function of y. we shall investigate the possibility of a solution to the problem in which $f(y) = 0$ [6]. In this case Eqs. (9)–(14) give

$$\left(1 + \lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -\frac{1}{\rho}\left(1 + \lambda\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial x} + \nu\left(1 + \theta\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - N\left(1 + \lambda\frac{\partial}{\partial t}\right)u, \tag{16}$$

where $\nu = \mu/\rho$ is the kinematic viscosity and

$$N = \frac{\sigma B_0^2}{\rho}, \tag{17}$$

is the magnetic parameter.

3 Unsteady Couette Flow

Consider the flow between the two rigid plates distance h apart. Initially, both the plates are at rest and the plate at $y = 0$ is fixed for all the time. The fluid motion starts suddenly due to constant velocity of the plate at $y = h$ in its own plane. The governing initial and boundary value problem is

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \nu\left(1+\theta\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - N\left(1+\lambda\frac{\partial}{\partial t}\right)u, \tag{18}$$

$$\begin{aligned} u(0,t) &= 0 \quad \text{for all} \quad t, \\ u(h,t) &= U \quad \text{for} \qquad t>0, \\ \frac{\partial u(y,0)}{\partial t} &= u(y,0)=0 \quad \text{for} \quad 0 \le y < h. \end{aligned} \tag{19}$$

We find the solution of (18) using (19) by sumudu transform technique. The Sumudu transform of $u(t)$ is defined by [1]

$$S\left[u(t)\right] = \int_0^\infty e^{-t}u(wt)dt = G(w), \tag{20}$$

where $G(w)$ is referred to the sumudu of $u(t)$. Then Eq. (18) and the boundary conditions (19) take the following forms

$$\frac{d^2G(y,w)}{dy^2} - p^2G(y,w) = 0, \tag{21}$$

$$G(0,w) = 0, \qquad G(h,w) = U, \tag{22}$$

where

$$p^2 = \left(\frac{Nw^2 + (1+\lambda N)w + \lambda}{w\nu(w+\theta)}\right) \tag{23}$$

The solution of Eq. (21) by using (22) can be written as

$$\frac{G(y,w)}{U} = \frac{\sinh py}{\sinh ph}. \tag{24}$$

Now inverse sumudu transform is defined by [18, 21] is of the form

$$S^{-1}\left[G(w)\right] = \frac{1}{2\pi\iota}\int_{r-\iota\infty}^{r+\iota\infty} e^{wt}G\left(\frac{1}{w}\right)\frac{dw}{w} = \sum residues\left[e^{wt}\frac{G(\frac{1}{w})}{w}\right]. \tag{25}$$

Sumudu inversion of Eq. (24) yields

$$\frac{u}{U} = \frac{1}{2\pi\iota}\int_{r-\iota\infty}^{r+\iota\infty} \frac{\sinh py}{w \sinh ph} e^{wt} dw. \tag{26}$$

Replacing w by $\frac{1}{w}$ in Eq. (23), we will get the following Eq.

$$p^2 = \frac{N + \lambda w^2 + (1 + N\lambda)w}{\nu(1 + w\theta)} \tag{27}$$

In Eq. (26), $w = 0$ is a simple pole. Therefore residue at $w = 0$ is

$$Res(0) = \frac{\sinh my}{\sinh mh} \qquad \text{where} \qquad m = \sqrt{\frac{N}{\nu}}. \tag{28}$$

The other singular points are the zeros of $\sinh ph = 0$. Setting $p = \iota\alpha$ we find that

$$\sin \alpha h = 0, \tag{29}$$

and $\alpha_n = n\pi/h,\ n = 1, 2, 3, ..., \infty$ are the zeros of Eq. (29), then Eq. (27) can be written as

$$\lambda w^2 + (1 + N\lambda + \alpha^2\nu\theta)w + (N + \alpha^2\nu) = 0. \tag{30}$$

Eq. (30) is quadratic in w, therefore its solution can be written as

$$w_{1n}, w_{2n} = \frac{-(1 + \lambda N + \nu\alpha_n^2\theta) \pm \sqrt{(1 + \lambda N + \nu\alpha_n^2\theta)^2 - 4\lambda(N + \nu\alpha_n^2)}}{2\lambda} \tag{31}$$

$n = 1, 2, 3..., \infty$ are the poles. Since all α_ns are symmetrically placed about origin on the real axis, all poles $(w_{1n}.w_{2n})$ lies on the negative real axis. These are the simple poles and the residue at all these poles can be obtained as

$$Res(w_{1n}) = \left(\frac{-2\pi\nu}{h^2}\right) \frac{(-1)^n n(1 + \theta w_{1n})^2 e^{w_{1n}t}}{w_{1n}(1 + N(\lambda - \theta) + 2\lambda w_{1n} + \lambda\theta w_{1n}^2)} \sin \frac{n\pi}{h} y$$

$$Res(w_{2n}) = \left(\frac{-2\pi\nu}{h^2}\right) \frac{(-1)^n n(1 + \theta w_{2n})^2 e^{w_{2n}t}}{w_{2n}(1 + N(\lambda - \theta) + 2\lambda w_{2n} + \lambda\theta w_{2n}^2)} \sin \frac{n\pi}{h} y$$

Adding $Res(0)$, $Res(w_{1n})$ and $Res(w_{2n})$, a complete solution is obtained as

$$\frac{u}{U} = \frac{\sinh py}{\sinh ph} - \left(\frac{2\pi\nu}{h^2}\right)(-1)^n n \sum_{n=0}^{\infty} \left[\frac{(1 + \theta w_{1n})^2 e^{w_{1n}t}}{w_{1n}(1 + N(\lambda - \theta) + 2\lambda w_{1n} + \lambda\theta w_{1n}^2)} + \frac{(1 + \theta w_{2n})^2 e^{w_{2n}t}}{w_{2n}(1 + N(\lambda - \theta) + 2\lambda w_{2n} + \lambda\theta w_{2n}^2)}\right] \sin \frac{n\pi}{h} y \tag{32}$$

The solution obtained in Eq. (32) is identical to those given by Laplace trnasform method [Eq. 25, 9].

4 Unsteady Poiseuille flow

In this section, we discuss another type of unsteady flow situation, that the fluid between two parallel plates which are stationary is set in motion due to sudden application of a constant pressure gradient is termed as the poiseuille flow. Suppose that the fluid is bounded by two parallel plates at $y = \pm h$, and it is initially at rest and fluid starts suddenly due to constant pressure gradient. The governing equations is (16), and the initial and boundary conditions are:

$$u(\pm h, t) = 0 \quad \text{for all} \quad t \tag{33}$$

$$\frac{\partial u(y,0)}{\partial t} = u(y,0) = 0 \quad \text{for} \quad -h \leq y < h, \tag{34}$$

where $2h$ is the distance between two parallel plates. After taking Sumudu transform, Eqs. (16), (33) and (34) gives

$$\frac{d^2G(y,w)}{dy} - p^2 = \frac{w}{\rho\nu(w+\theta)}\frac{dp}{dx}, \tag{35}$$

$$G(+h,w) = 0, \qquad G(-h,w) = 0, \tag{36}$$

where p^2 is given by Eq. (23).
For the solution of Eqs. (35) and (36), we employ the similar procedure of section 3. In order to avoid the detail, the solution is given by

$$\begin{aligned} \frac{u}{-\left(\frac{1}{\rho N}\left(\frac{dp}{dx}\right)\right)} &= 1 - \frac{\cosh my}{\cosh mh} + \frac{Ne^{w_1 t}}{w_1(w_1-w_2)}\left[1 - \frac{\cosh p_1 y}{\cosh p_1 h}\right] + \frac{Ne^{w_2 t}}{s_2(w_2-w_1)}\left[1 - \frac{\cosh p_2 y}{\cosh p_2 h}\right] \\ &- \frac{\pi\nu N}{h^2}\sum_{n=0}^{\infty}(-1)^k(2k+1)\left[\frac{(1+\theta w_{1k})^2 e^{w_{1k}t}}{w_{1k}(d_0+d_1w_{1k}+d_2w_{1k}^2+d_3w_{1k}^3+d_4w_{1k}^4)}\right. \\ &+ \left.\frac{(1+\theta w_{2k})^2 e^{w_{2k}t}}{w_{2k}(d_0+d_1w_{1k}+d_2w_{2k}^2+d_3w_{2k}^3+d_4w_{2k}^4)}\right]\cos\left(\frac{(2k+1)\pi}{2h}y\right) \end{aligned} \tag{37}$$

where

$$w_1 = \frac{-(1+\lambda N)+\sqrt{(1+\lambda N)^2-4\lambda N}}{2\lambda}, \qquad w_2 = \frac{-(1+\lambda N)-\sqrt{(1+\lambda N)^2-4\lambda N}}{2\lambda}$$

$$p_1 = \sqrt{\frac{N+\lambda w_1^2+(1+N\lambda)w_1}{\nu(1+w_1\theta)}}, \qquad p_2 = \sqrt{\frac{N+\lambda w_2^2+(1+N\lambda)w_2}{\nu(1+w_2\theta)}}$$

$$d_0 = N\left[1+N(\lambda-\theta)\right], \qquad d_1[1+3\lambda N+N(\lambda-\theta)(1+\lambda N)]$$

$$d_2 = 3\lambda\left[1+\lambda N\right], \qquad d_3\lambda\left[2\lambda+\theta(1+\lambda N)\right], \quad d_4 = \lambda^2\theta$$

5 Unsteady Generalized Couette Flow

Suppose the fluid is bounded by two parallel plates at $y = 0$ and $y = h$, and it is initially at rest. The fluid starts suddenly due to a pressure gradient and by the motion of the upper plate. The governing equations is (16), and the initial and boundary conditions are:

$$\begin{aligned} u(0,t) &= 0 \quad \text{for} \quad t > 0, \\ u(h,t) &= U \quad \text{for} \quad t > 0, \\ \frac{\partial u(y,0)}{\partial t} &= u(y,0) = 0 \quad \text{for} \quad 0 \leq y < h. \end{aligned} \tag{38}$$

After taking Sumudu transform, Eqs. (16) and (38) gives

$$\frac{d^2G(y,w)}{dy^2} - \left(\frac{Nw^2 + (1 + N\lambda)w + \lambda}{\nu(w + \theta)}\right) G(y,w) = \frac{w^2}{\mu(w + \theta)} \frac{dp}{dx}, \tag{39}$$

$$G(h,w) = U, \qquad G(0,w) = 0. \tag{40}$$

Following the same procedure as in section 3, we find

$$\begin{aligned} \frac{u}{U} &= \frac{\sinh my}{\sinh mh} + \Gamma\left(1 - \frac{\sinh my}{\sinh mh} - \frac{\sinh m(h-y)}{\sinh mh}\right) + N\Gamma\left[\frac{e^{w_1 t}}{w_1(w_1 - w_2)}\right. \\ &\quad \left.\left(1 - \frac{\sinh q_1 y}{\sinh q_1 h} - \frac{\sinh q_1(h-y)}{\sinh q_1 h}\right) + \frac{e^{w_2 t}}{w_2(w_2 - w_1)}\left(1 - \frac{\sinh q_2 y}{\sinh q_2 h} - \frac{\sinh q_2(h-y)}{\sinh q_2 h}\right)\right] \\ &- \left(\frac{2\pi\nu}{h^2}\right)\sum_{n=1}^{\infty}(-1)^n n\left[\frac{(1+\theta w_{1n})^2 e^{w_{1n}t}}{w_{1n}(1 + N(\lambda - \theta) + 2\lambda w_{1n} + \lambda\theta w_{1n}^2)}\right. \\ &+ \left.\frac{(1+\theta w_{2n})^2 e^{w_{2n}t}}{w_{2n}(1 + N(\lambda - \theta) + 2\lambda w_{2n} + \lambda\theta s_{2n}^2)}\right]\sin\left(\frac{n\pi}{h}y\right) \\ &+ \frac{\pi\nu N\Gamma}{h^2}\sum_{n=1}^{\infty}(-1)^n n\left[\frac{(1+\theta w_{1n})^2 e^{w_{1n}t}}{w_{1n}(d_0 + d_1 w_{1n} + d_2 w_{1n}^2 + d_3 w_{1n}^3 + d_4 w_{1n}^4)}\right. \\ &+ \left.\frac{(1+\theta w_{2n})^2 e^{w_{2n}t}}{w_{2n}(d_0 + d_1 w_{1n} + d_2 w_{2n}^2 + d_3 w_{2n}^3 + d_4 w_{2n}^4)}\right]\left[\sin\left(\frac{n\pi}{h}y\right) + \sin\left(\frac{n\pi}{h}(h-y)\right)\right], \end{aligned} \tag{41}$$

where

$$\Gamma = \frac{-\frac{dp}{dx}}{\rho N U}.$$

The obtained result (41) is identical to those obtained by Laplace transform [Eq. 48, 9].

6 Conclusions

In this work, we have successfully applied the Sumudu Transform to solve the unsteady unidirectional MHD flow problems, namely, Couette flow, Poiseuille flow and generalized Couette flow of an Oldroyd-B fluid between two plates. The obtained results are valid for small times and match with those obtained by laplace transform in Literature [9]. But, It shows that the sumudu transform is much simpler and powerful tool than Laplace transform to solve the engineering and physical problems due to having units preserving properties and without resorting the frequency domain. The obtained results reveals that the Sumudu transform method is very effective and simple.

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Received: July 1, 2013