Applied Mathematical Sciences, Vol. 7, 2013, no. 144, 7187 - 7192
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ams.2013.311670

Why 20? Why 40? A Possible Explanation of a Special Role of Numbers 20 and 40 in Traditional Number Systems

Olga Kosheleva[1] and Vladik Kreinovich[2]

[1]Department of Teacher Education
[2]Department of Computer Science
University of Texas at El Paso
500 W. University, El Paso, TX 79968, USA

Abstract

Both historical and linguistic evidence shows that numbers 20 and 40 played a special role in many traditional numerical systems. The fact that, e.g., the same number 20 appears in unrelated cultures such as Romans and Mayans is an indication that this number must have a general explanation related to human experience. In this paper, we provide a possible explanation of 20 and 40 along these lines: namely, we show that these numbers can be identified as the smallest sample sizes for which we can extract statistically significant information.

Mathematics Subject Classification: 01A12, 01A15

Keywords: traditional numbers systems, Mayan arithmetic, traditional Russian arithmetic

1 Formulation of the Problem

Numbers like 20 and 40 have a special role in several traditional numerical systems. Sometimes, we know it from the old records – e.g., we know that

Mayans used a 20-based system; see, e.g., [1, 2, 5, 6, 7, 8, 10, 11]. Sometimes, we know it from the unusual names of the corresponding numbers in the language:

- in Latin-originated languages such as French and Spanish, most multiples of ten have names like "three tens" (*trenta* in Spanish), "five tens" (*cincuenta* in Spanish), etc., except for the number 20, which has a special name (*veinte* in Spanish);

- in Russian, similarly, most names literally mean "three tens" (*tridcat'*) or "five tens" (*piat'desiat*), with the exception of the number 40, which has a special name *sorok*.

A natural question is: why? The fact that, e.g., the number 20 appears in such completely unrelated cultures as the Romans and the Mayans indicates that the special role of this number is probably not accidental: it probably reflects some important feature of general human experience.

2 Towards a Possible Explanation

Our main idea. Numbers describe groups (groups of people, groups of dogs, etc.). In data processing terms, numbers describe *sample sizes.* Samples are important: we learn knowledge from our experiences, from observing samples of the corresponding data.

When the sample is too small, we cannot extract any reliable information from this sample. For example, after observing only one dog (or even two dogs), we cannot make reliable conclusions about dogs in general:

- we may encounter a friendly dog, but this does not mean that all dogs are friendly;

- we may encounter a vicious dog, but this does not mean that all dogs are vicious.

Since based on samples of small size, we cannot make reliable conclusions, there should be the smallest sample size $n_{\min}$ based on which we can make meaningful conclusions. Because of the special important of this smallest sample size, it is reasonable to expect that the corresponding number is specially marked in a traditional number system.

In this paper, we will show that this idea can provide a possible explanation of the special role of numbers 20 and 40.

Analysis of the problem. Based on the sample of values $x_1, \ldots, x_n$, the first thing we usually do is estimate the mean μ and standard deviation σ; see, e.g., [9]. In many cases, the distribution is close to normal, so, in data processing, we can use formulas corresponding to normal distributions. In particular, we can usually estimate the mean as the arithmetic mean of the sample values $\mu \approx \hat{\mu} \stackrel{\text{def}}{=} \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$, and the standard deviation as $\sigma \approx \hat{\sigma}$, where $(\hat{\sigma})^2 \stackrel{\text{def}}{=} \frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \hat{\mu})^2$.

These estimates are based on a finite sample and are, therefore, approximate. In particular, the mean square deviation σ_e of the difference $\sigma - \hat{\sigma}$ between the actual (unknown) value σ and its estimate $\hat{\sigma}$ is known, for reasonably large n, to be approximately equal to $\sqrt{2} \cdot \frac{\sigma}{\sqrt{n}}$.

Based on this standard deviation, we can conclude that with high confidence, the actual value σ deviates from the estimate $\hat{\sigma}$, by no more than $k_0 \cdot \sigma_e$, where usually, we take $k_0 = 2$ (corresponding to confidence 95%) or $k_0 = 3$ (corresponding to confidence 99.9%). So, based on processing a sample of size n, we conclude that the actual value σ is within the interval $[\sigma - k_0 \cdot \sigma_e, \sigma + k_0 \cdot \sigma_e]$.

We are looking for the smallest sample size n which can enable us to get reliable estimates – at least the crude ones. It is known (see, e.g., [3]) that we base our crude estimates on half-order of magnitude (a possible explanation for this is given in [4]), i.e., by the factor of three. It is therefore reasonable to select a sample size for which the resulting estimates are at most half-order of magnitude different from the actual value σ. In other words, we need to select n for which $\sigma - k_0 \cdot \sigma_e \geq \frac{\sigma}{3}$ and $\sigma + k_0 \cdot \sigma_e \leq 3\sigma$.

Substituting the above expression for σ_e into the first inequality, we get

$$\sigma - k_0 \cdot \sqrt{2} \cdot \frac{\sigma}{\sqrt{n}} \geq \frac{\sigma}{3}.$$

Dividing both sides by $\sigma > 0$, we conclude that

$$1 - k_0 \cdot \frac{\sqrt{2}}{\sqrt{n}} \geq \frac{1}{3}.$$

Moving the term containing n to the right-hand side and all other terms to the left-hand side, we get

$$k_0 \cdot \frac{\sqrt{2}}{\sqrt{n}} \leq \frac{2}{3}.$$

Squaring both sides, we get $k_0^2 \cdot \frac{2}{n} \leq \frac{4}{9}$. Multiplying both sides by n and by $\frac{9}{4}$, we get $n \geq 4.5 \cdot k_0^2$.

For the second inequality, we similarly get $\sigma + k_0 \cdot \sqrt{2} \cdot \dfrac{\sigma}{\sqrt{n}} \le 3\sigma$, hence $1 + k_0 \cdot \dfrac{\sqrt{2}}{\sqrt{n}} \le 3$; so, $k_0 \cdot \dfrac{\sqrt{2}}{\sqrt{n}} \le 2$ therefore $k_0^2 \cdot \dfrac{2}{n} \le 4$, and $n \ge 0.5 \cdot k_0^2$.

For both inequalities to be satisfied, we must satisfy both inequalities $n \ge 4.5 \cdot k_0^2$ and $n \ge 0.5 \cdot k_0^2$. The right-hand side of the first inequality is always greater than the right-hand side of the second one. So, for both inequalities to be satisfied, it is sufficient to have $n \ge 4.5 \cdot k_0^2$. Now:

- For $k_0 = 2$, this inequality takes the form $n \ge 18$. Thus, the smallest sample size that satisfies this inequality is $n_{\min} = 18$. If we take into account that our computations were approximate, this is a good approximation to the number 20.

- For $k_0 = 3$, the above inequality takes the form $n \ge 40.5$. Thus, the smallest sample size that satisfies this inequality is $n_{\min} \ge 41$. This is an even better approximation to the number 40.

Conclusion. Numbers ≈ 20 and ≈ 40 indeed naturally appear as sample sizes for which we can extract reliable conclusions from observations. This may be an explanation of why there numbers played a special role in the traditional number systems.

Discussion.

- Our derivation of the numbers 20 and 40 uses basic facts from statistics. Of course, we do not claim that our ancestors consciously used formulas of modern statistics – but they did not need to know statistics to come up with these sample sizes. The formulas leading to 20 and 40 simply describe which samples are large enough so that we can extract meaningful knowledge from them. Our ancestors could have come up with these numbers from experience – generalizations based on too small samples were useless, and these were the smallest sizes for generalization worked.

- We can also speculate why in the South (Romans, Mayans), people used the number 20 corresponding to the lower confidence of 95%, while in the North (Russia), they used the number 40 corresponding to the higher confidence level of 99.9%. A possible explanation may be that in the South, the environment is more friendly, so possible mistakes are less critical, while in the harsher Northern climates, a mistake can be fatal – so it is desirable to have more reliable conclusions.

- Instead of looking for estimates which are one half-order of magnitude close to σ, we could alternatively look for estimates which are two, three, etc., half-orders of magnitude close, i.e., for which, for some integer $d = 2, 3, \ldots$, we get $\sigma - k_0 \cdot 3^{-d} \cdot \sigma$ and $\sigma + k_0 \cdot \sigma_e \leq 3^d \cdot \sigma$. By applying an analysis similar to the one we did for $d = 1$, we can conclude that both inequalities are satisfied if and only if $n \geq \dfrac{2k_0^2}{(1 - 3^{-d})^2}$. We can thus compute the smallest value $n_{\min}$ for which both inequalities are satisfied (i.e., for which we can reconstruct σ modulo d half-orders of magnitude):

- For $d = 2$ and $k_0 = 2$, we get $n_{\min} = 10$ – which may explain the special role of the number 10 in the traditional number systems.
- For $d = 2$ and $k_0 = 3$, we get $n_{\min} = 23$ – which is another good approximation to 20.
- For $d \geq 3$ and $k_0 = 2$, we get $n_{\min} = 9$ (which is close to 10).
- For $d \geq 3$ and $k_0 = 3$, we get $n_{\min} = 19$ (which is also close to 20).

One can see that 20 appears in many more cases than 40; this explains why more traditional number system assign special role to number 20 than to number 40.

Acknowledgments. This work was supported in part by the National Science Foundation grants HRD-0734825 and HRD-1242122 (Cyber-ShARE Center of Excellence) and DUE-0926721, by Grants 1 T36 GM078000-01 and 1R43TR000173-01 from the National Institutes of Health, and by a grant N62909-12-1-7039 from the Office of Naval Research.

The authors are thankful to Gennady N. Solopchecko for valuable discussions.

References

[1] C. B. Boyer and U. C. Merzbach, *A History of Mathematics*, Wiley, New York, 1991.

[2] M. D. Coe, *The Maya*, Thames & Hudson, London, New York, 2011.

[3] J. R. Hobbs, "Half orders of magnitude", In: L. Obrst and I. Mani (eds.), *Proceeding of the Workshop on Semantic Approximation, Granularity, and Vagueness, A Workshop of the Seventh International Conference on Prin- ciples of Knowledge Representation and Reasoning KR'2000*, Breckenridge, Colorado, April 11, 2000, pp. 28–38.

[4] J. Hobbs and V. Kreinovich, "Optimal choice of granularity in commonsense estimation: why half-orders of magnitude", *International Journal of Intelligent Systems*, 2006, Vol. 21, No. 8, pp. 843–855.

[5] G. Ifrah, *A universal history of numbers: From prehistory to the invention of the computer*, London, 1998.

[6] O. Kosheleva, "Mayan and Babylonian arithmetic can be explained by the need to minimize computations", *Applied Mathematical Sciences*, 2012, Vol. 6, No. 15, pp. 697–705.

[7] G. Morley, *Ancient Maya*, Stanford University Press, Stanford, 1946.

[8] O. Neugebauer, *The Exact Sciences in Antiquity*, Dover Publ., New York, 1969.

[9] D. J. Sheskin, *Handbook of Parametric and Nonparametric Statistical Procedures*, Chapman and Hall/CRC Press, Boca Raton, Florida, 2011.

[10] B. L. van der Waerden, *Science Awakening*, P. Noorhoff, Groningen, 1954.

[11] B. L. van der Waerden, *Geometry and Algebra in Ancient Civilizations*, New York, 1983.

Received: November 29, 2013