Applied Mathematical Sciences, Vol. 7, 2013, no. 143, 7105 - 7111
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ams.2013.311649

The Technology of Interactive Resource Allocation in Accordance with the Customizable System of Rules

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Abstract

The article deals with the general linear problem of resource allocation. Statement of the problem is designed to complement and combine well-known methods for solving linear problems of resource allocation. The formulation of the problem is represented by the system of mandatory and orienting rules. The technology of search for solution is designed for expert working in the interactive mode of computational experiment.

Keywords: General linear problem of resource allocation, Target displacement of solution, Customizable system of rules

1 Introduction

In practice, an efficient solution of resource allocation problem is usually not the result of solving a linear programming problem. A concretization of the efficiency concept can vary and depends on many factors: resource stocks , the fulfillment of contractual deliveries to customers, corporate performance indicators, etc.

Typically, a concretization is defined by experts on resource planning (experts planners). In each situation the planning result depends on the skills and awareness of the expert planner. When awareness is changed, an expert can change his view on efficiency. Breadth of choice of feasible solutions, which are analyzed by the expert, has a significant impact on his final decision. In this sense the methodical fullness of search for feasible solutions is crucial.

Linear problems of resource allocation are traditionally solved by methods of linear programming (LP) [1, 3]. Applicability of LP-methods is based on assumption that the data values used in the calculation are the same as the actual values at the time of decision application. It is also known that optimal plans for the practical data, as a rule, do not exist due to the incompatibility of constraints. The input data can vary quite considerably between the time of plan calculation and the time of its application. In extreme cases, the set of main variables and the system of resource constraints may also change. This requires an operational data correction.

Solution of the LP problem is typically calculated by means of the simplex method [1] or the interior point method [3]. The solution can be found only if the constraints are compatible. In case of incompatibility, the Chebyshev point is often calculated as the compromise solution. The search for this point is also performed using the simplex method.

It is known that in practice, use of LP methods is quite problematic. One problem is the mathematical incorrectness of the LP problem due to the instability of solution for small changes of the input data [4]. Another problem is the non-applicability of solution found as a Chebyshev point in case of constraints incompatibility. Such solutions can not be applied because they violate constraints that must be met: it is impossible to allocate more resource than available.

Expert planner, using a program that can solve only the LP problem and the problem of search for Chebyshev point, is too limited in the choice of means to obtain the desired results. Traditional LP software does not allow an expert intervention in the search for solution. If given system of constraints is incompatible, programs propose to adjust the input data.

To make up for these shortcomings, the authors have proposed the informal statement and method for solving *the general linear problem of resource allocation*, which has been called *the method of target displacement of solution*.

This method is implemented in the *technology of interactive resource allocation in accordance with the customizable system of rules*. This technology allows an expert to search for plans in accordance with his knowledge of the applicability and efficiency of the plans. Software implementation of the technology has been developed and tested in a number of applications [2].

2 Rules of resource allocation

Expert planner defines rules of resource allocation in the form of requirements on the values of resource functions $F_i(\overline{x})$ – linear forms, whose values depend on vector of allocation $\overline{x}$ and numerical coefficients.

In general case, a simple rule can be written in one of three forms:

$F_i(\overline{x}) = c_i\ [\xleftarrow{priority} p_i]$

$F_i(\overline{x}) \le c_i\ [\xleftarrow{priority} p_i]$

$F_i(\overline{x}) \ge c_i\ [\xleftarrow{priority} p_i]$

where F_i – resource function, c_i – constant, p_i – priority of the rule ($0 < p_i \le \infty$); square brackets denote optionality of priority.

A composite rule is a logical combination of simple rules. In terms of Boolean algebra, a simple rule is an elementary formula, and a composite rule, in general case, is formed from simple rules by means of logical operations [conjunction, disjunction, negation ($\wedge, \vee, \neg$)].

Expert planner performs step-by-step search for solution. At each step he customizes rules that determine the change of solution. (Any rule may remain unchanged during the search).

The rules can be *mandatory* or *orienting*. Mandatory rules have an absolute priority ($p_i = \infty$), that is, they can not be violated. Orienting rules specify the desired values of resource functions, setting the direction for displacement of solution.

Let $\overline{x^0}$ be a given vector of allocation, and

$\{\, F_i(\overline{x}) = F_i(\overline{x^0}) + h_i\ [\xleftarrow{priority} p_i],\ \ h_i \ne 0 \,\}$ – a given composite rule: the simple orienting rules, related by conjunction, are enclosed in curly brackets.

Let say that the vector of allocation $\overline{x}$ satisfies the given orienting rules ($\overline{x}$ is more efficient than $\overline{x^0}$), if

$F_i(\overline{x^0}) < F_i(\overline{x}) \le F_i(\overline{x^0}) + h_i$ is true for $\forall h_i > 0$, and

$F_i(\overline{x^0}) + h_i \le F_i(\overline{x}) < F_i(\overline{x^0})$ is true for $\forall h_i < 0$.

For example, implementation of the composite orienting rule "the supply of fuel should be increased by 100 tons for the consumer K and increased by 500 tons for the consumer N" means that the supply is increased for both consumers, but not necessarily by the specified amounts exactly.

An optimization rule is defined as the special type of a composite rule. It can be written as

$Q_{\min}(\overline{x}) = F_i(\overline{x}) : P_1 \wedge ... \wedge P_k$ or $Q_{\max}(\overline{x}) = F_i(\overline{x}) : P_1 \wedge ... \wedge P_k$,

where $P_1 ... P_k$ are the simple mandatory rules (constraints).

This expresses a standard formulation of the LP problem. Note that it includes the mandatory rules only.

The developed technology allows formulating and trying to solve LP problem at any step during the search for solution: an expert can set an optimization rule to any resource function and select a subsystem of rules to define a system of constraints. For example, (in case of constraints compatibility) an optimization solution can be considered as the starting point for target displacement of the solution.

3 The general linear problem of resource allocation

Let a_{ij} (i=1...m, j=1...n) be a consumption of i^{th} resource for unit of intensity of j^{th} activity;
b_i (i=1...m) – a stock of i^{th} resource;
x_j (j=1...n) – intensity of j^{th} activity to be determined.
Total consumption of i^{th} resource is expressed by a linear form
$a_{i1}x_1+...+a_{in}x_n$.
A composite rule defining the system of resource constraints is
$\{ a_{i1}x_1+...+a_{in}x_n \leq b_i \}$ (i=1...m).
Further, a set of efficiency indicators may be defined:
$\{c_{i1}x_1+...+c_{in}x_n\}$ (i=1...k),
where c_{ij} – i^{th} specific efficiency indicator for unit of intensity of j^{th} activity.
A simple rules may be defined for any efficiency indicator also.
Any rule may have a priority p_i ($0 < p_i \leq \infty,\ 1 \leq i \leq m+k$).
In general case, a two-sided constraint (conjunction of two simple rules) may be defined for each resource function. Therefore, let rewrite the overall system as
$\{\ [b_i \leq]\, a_{i1}x_1 + ... + a_{in}x_n\, [\leq B_i]\quad [\xleftarrow{priority} p_i];\quad x_j \geq 0\ \}$ (i=1…m+k, j=1…n)
(all the coefficients from resource constraints and efficiency indicators are denoted as a_{ij}; square brackets denote optionality of constraints and priorities; the variables x_j are non-negative in accordance with resource allocation problem).

The general problem - search for the vector of allocation $\bar{x} = \langle x_1...x_n \rangle$, *providing the values of resource functions, which are estimated by the expert planner as the most efficient and realizable.*
The informality of the problem statement is stipulated by orientation to the computational experiment mode, which involves the possibilities of changing the input data and system of rules, governing the search for solutions.
In general case, the expert planner solves a set of particular problems, having the formal statements and algorithms, and performs comparative analysis of solutions.

4 The target displacement of solution

The informal method of target displacement of solution is designed for expert planner, who forms the system of rules and analyzes solutions in step-by-step dialogue with specialized software.
On the first step the expert can choose the initial solution (starting point) arbitrarily.

By default, the software proposes a compromise solution – Chebyshev point. If the system of constraints is compatible, such solution ensures equal reserves for resource constraints; otherwise it ensures the minimization of the maximum deficit:

$$\min_{\mathbf{x}} \max_{i}(a_{i1}x_1+\ldots+a_{in}x_n-b_i) \mid x_j \geq 0 \quad (i=1...s,\ j=1...n),$$

where s is a number of constraints in the system reduced to the form $A\overline{x} \leq b, \overline{x} \geq \theta$.

The expert also can specify an optimization rule and try to solve the LP problem.

In case of constraints compatibility, the solution can be considered as the initial one.

Then the expert analyzes the received values of resource functions, and estimates realizability and efficiency of a solution. If the values satisfy, a solution is the final one, and the target displacement is not needed. If not, the expert planner specifies the requirements for changes of some values, that is, modifies the system of rules for resource allocation. The rules define the direction and magnitude for displacement to the next point. [When the next point satisfies the expert, this is the final solution.]

Solutions can be entered into a database of possible plans for future analysis. Thus the trajectory of the solution is stored, enabling rollback.

A step of target displacement of solution is calculated as follows.

Let $\overline{x^0} = \langle x_1^0,...,x_n^0 \rangle$ ($x_j^0 \geq 0$, $j = 1...n$) be a current point (received on the previous step), and an expert planner has defined a composite rule for displacement from $\overline{x^0}$ to a target point $\overline{x} = \langle x_1,...,x_n \rangle$ ($x_j \geq 0$, $j = 1...n$):

$$\{ F_i(\overline{x}) = F_i(\overline{x^0}) + h_i [\xleftarrow{priority} p_i] \},$$

where $F_i(\overline{x}) = a_{i1}x_1 + ... + a_{in}x_n$, $i = 1...l$; $0 < p_i < \infty$ for $h_i \neq 0$, $p_i = \infty$ for $h_i = 0$.

Formally, the system of rules for displacement can be inconsistent. Therefore, the simple rules with $h_i \neq 0$ are treated as orienting rules, and the value h_i is called *the desired step* (which is often different from *the actual step* of the function that can be obtained for the given set of rules). If actual and desired steps have the same sign for all the simple rules, the new point anyway increases the efficiency of the solution. The point $\overline{x}$ is searched as follows.

First, the $\overline{x^0}$ projection to the hyperplane

$a_{i1}x_1+...+a_{in}x_n= F_i(\overline{x^0}) + h_i$ for $\forall h_i \neq 0$ is calculated.

The direction vector of the normal to this hyperplane is $\langle a_{i1},...,a_{in} \rangle$. So we should change variables by ha_{i1} ... ha_{in}, where h is to be calculated, to find the projection. A displacement along the normal gives the function increment $h(a_{i1}^2+...+a_{in}^2)$, which is to be equal h_i, so

$$h = \frac{h_i}{a_{i1}^2 + ... + a_{in}^2} \quad \text{(naturally, } a_{i1}^2+...+a_{in}^2 \neq 0\text{)}.$$

Thus, the projection is

$$\langle x_1^0 + \frac{a_{i1}h_i}{a_{i1}^2 + \ldots + a_{in}^2},\ \ldots\ , x_n^0 + \frac{a_{in}h_i}{a_{i1}^2 + \ldots + a_{in}^2} \rangle .$$

When projections are found for $\forall h_i \neq 0$, we receive the desired increment for each variable:

$dx_{ji} = \dfrac{a_{ij}h_i}{{a_{i1}}^2 + \ldots + {a_{in}}^2}$ (j=1...n; let indices of the functions go from 1 to s, $1 \le s \le l$).

Now we compute the *average head of normals*, using formulas

$$x_j = x_j^0 + \frac{p_1 dx_{j1} + \ldots + p_s dx_{js}}{\sum_{i=1}^{s} p_i} \ (j = 1 \ldots n) .$$

The average head of normals is "closer" to the hyperplanes which define the rules with higher priorities. If the priorities are not set, we consider them equal to 1, and get the formulas

$$x_j = x_j^0 + \frac{dx_{j1} + \ldots + dx_{js}}{s} \ (j = 1 \ldots n) .$$

Then, if $\exists k : h_k = 0 \, (1 \le k \le l)$, the average head of normals is projected to the hyperplane $a_{k1}x_1+\ldots+a_{kn}x_n= F_k(\overline{x^0})$; otherwise, the average head itself pretends to be result of the step.

Next, the condition ($x_j \geq 0$, j = 1...n) is verified, and potential negative variables are changed to zero.

Finally, we compare the actual and desired steps. If they have the same sign for all the functions in the scope, the calculated point is the target $\langle x_1, \ldots, x_n \rangle$. If not, an expert should correct the system of rules.

Conclusion

The most important new feature is the ability to perform step-by-step search for the most efficient and realizable solution of the general linear problem of resource allocation. At any step an expert planner can analyze the values of resource functions and customize the system of orienting and mandatory rules, governing the search. If the value of some "objective" function is estimated as most efficient, an expert can set the mandatory rule of fixing the function value ($h_k = 0$).

The developed technology significantly extends the traditional arsenal of facilities for solving linear problems of resource allocation.

Acknowledgements. The authors are grateful to Boris N. Kurov and other research associates from the Institute of Informatics Problems of the Russian Academy of Sciences for years of collaboration and discussion of the technology.

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Received: November 21, 2013