

# Modeling and Performance of Movement-Based Registration in Wireless Network

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## Abstract

Various location registration schemes have been proposed to minimize the signaling cost on radio channels. In this study, we consider movement-based registration (MBR). MBR is simple and its implementation is quite straightforward. However, it may exhibit relatively frequent location registrations compared to other methods. To solve this problem, a new MBR scheme combined with implicit registration (MBIR) was proposed and analyzed by using the continuous-time Markov chain (CTMC) model. However, previously developed CTMC models have some inherent flaws. Therefore, we propose a new model using an imbedded Markov chain (IMMC) in order to analyze the exact performance of movement-based registration schemes. The numerical results obtained with the proposed IMMC model are compared with those from previous CTMC models.

**Keywords:** location registration, movement-based registration, Markov chain

## 1 Introduction

Mobile subscribers have been increasing and more accelerated growth of smart phone subscribers is expected with the 4G networks. In mobile communication networks, continuous management of the mobile station (MS) location is required to connect an incoming call to the MS, since the MS is continually moving. Location registration (LR) is a series of process that an MS updates its location record in the mobile network database when its location changes. Effective LR schemes are essential to minimize the signaling cost on radio channels due to limited capacity of radio resources in mobile communication networks.

Various registration methods to minimize the LR cost have been proposed. They include movement-based registration, distance-based registration, zone-based registration, time-based registration and so on [1]-[4].

Among these methods, the movement-based registration (MBR) scheme is known to be practical because it is effective and easy to implement under the framework of current mobile networks [4]-[5]. In the MBR scheme, location registration of an MS is performed whenever the MS moves to a specified number of cells. MBR is simple and its implementation is quite straightforward. However, it may exhibit relatively frequent location registrations compared to other methods.

To address this issue, a new MBR scheme combined with implicit registration (MBIR) was proposed and analyzed by using the continuous-time Markov chain (CTMC) model [6]. However, such analytical model for calculating the LR cost is inherently flawed.

In this study, we first introduce the CTMC model used in previous study and investigate their inherent problems. We then propose a new analytical model using an imbedded Markov chain (IMMC) [7] so as to analyze the exact performance of MBR and MBIR. Numerical results show that the proper IMMC model produces higher improvement ratio of the MBIR for the MBR than the approximated CTMC model.

## 2 MBR and CTMC Model

### 2.1 Movement-Based Registration (MBR)

In MBR, a mobile station (MS) registers its location whenever the number of entering cells reaches the specified movement threshold number  $M$  [5]. An MS has a movement counter to count the number of entered cells. The MS requests LR whenever the counter reaches  $M$ . If LR is carried out, the counter of the corresponding MS is reset to zero.

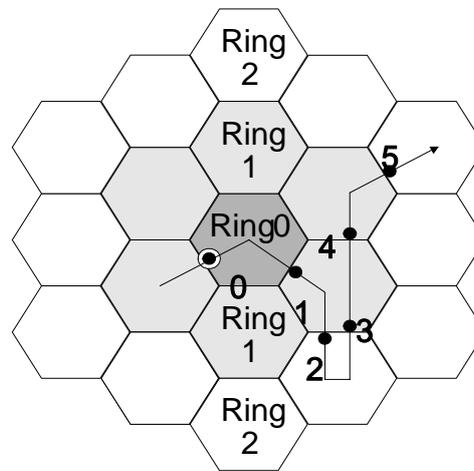


Fig. 1. Location area and rings in the hexagonal configuration ( $M=3$ )

A location area with a specified movement threshold of  $M=3$  under a hexagonal cell configuration when an MS ultimately registers in the ring-0 cell is shown in Fig. 1. In Fig. 1, the MS registers its location at 3, the 3<sup>rd</sup> cell from the ring-0 cell.

MBR can be easily implemented, since the MS suffices to maintain only a counter. However, in general, an MS registers its location frequently in MBR rather than in other registration schemes. Therefore, an improved version of MBR, called MBIR (MBR combined with implicit registration) was proposed [6].

## 2.2 Movement-Based Registration with Implicit Registration (MBIR)

According to CDMA recommendations [1], a base station (BS) can trace the location of the MS when the MS successfully sends an *Origination Message* or a *Page Response Message* to the BS; this is called implicit registration (IR). In other words, if a call originating from an MS or an incoming call to an MS is successfully completed, the BS can detect the cell of the MS from the *Origination Message* or the *Page Response Message* without additional messages.

Therefore, if a mobile cellular network adopting an MBR also adopts an implicit registration, then the network can determine the MS's cell whenever an outgoing call from or incoming call to the MS occurs without an additional registration process. The network can therefore set up a new location area in which the MS's cell is the center cell (ring-0 cell) in order to reduce the number of registrations. Assume that MBIR is adopted in Fig. 1 and the MS originates a call in the 2<sup>nd</sup> cell. Then it performs next location registration at 5, the 5<sup>th</sup> cell from ring-0 cell.

In this way, the MBR performance can be improved using an implicit registration, and the performance of this combined scheme is enhanced as the call generation of the MS increases. This enhanced scheme that combines an MBR

with the implicit registration of outgoing calls was proposed in [6], and we call it MBIR (MBR with IR).

### 3 LR Cost of the MBIR

It is assumed that the mobile communication network is composed of hexagonal cells of the same size, as shown in Fig. 1. In order to analyze the performance, the following are also assumed:

- 1) When the MS leaves a cell, there is an equal probability that any one of six neighboring cells will be selected as the destination.
- 2) The cell residence time  $T_m$  follows a general distribution with mean  $1/\lambda_m$ , and the incoming and outgoing call generations to/from each MS follow Poisson processes with rates of  $\lambda_{ic}$  and  $\lambda_{oc}$ , respectively.

From assumption 2), letting  $T_{ic}$  be the interval between incoming calls to each MS and  $T_{oc}$  be the interval between outgoing calls from each MS,  $T_{ic}$  and  $T_{oc}$  follows exponential distributions with rates of  $\lambda_{ic}$  and  $\lambda_{oc}$ , respectively.

The location area of the movement threshold,  $M=3$ , can be seen in Fig. 1. A location area is composed of  $M$  rings (ring 0, 1, ...,  $M-1$ ). An analytical model was proposed to analyze the registration cost using CTMC model [6]. Let us briefly present the previous analytical model to determine its problems in order to develop a new analytical model.

#### 3.1 Approximate LR Cost of the MBIR

To obtain the LR cost of the MBIR scheme, the MS should be observed at each cell crossing. A two-dimensional random walk mobility model was considered to estimate the LR cost on the radio channel for the location area in Fig. 1. Each MS remains in a cell for a period of time and then randomly moves to one of its neighboring cells. That is, the probability of selecting any cell is equal to  $1/6$ . This kind of MS mobility is described as a random walk in a two-dimensional hexagonal space with six moving directions; it is modeled by a CTMC with a transition rate diagram, as shown in Fig. 2.

In this Markov chain, an MS is in state  $(i, j)$  if its counter value is  $i$  and it is currently residing in a ring- $j$  cell. In Fig. 2, for example, an MS in state  $(1, 1)$  can transit to the one of state  $(2, 0)$ , state  $(2, 1)$ , state  $(2, 2)$  and state  $(0, 0)$ . Especially, an MS in state  $(M-1, j)$  can transit to the state  $(0, 0)$  with transition rate  $\lambda_m + \lambda_{ic} + \lambda_{oc}$ , which is composed of  $\lambda_m$  and  $\lambda_{ic} + \lambda_{oc}$ . Note that the former produces real registration, but the latter produces only an implicit registration.

Let  $Q$  be the transition rate matrix for the transition rate diagram for MBIR in Fig. 2. Let  $\pi_{i,j}$  denote the steady-state probability of state  $(i, j)$ ,  $\pi_{i,j}$  can be obtained using the following balanced equations:

$$\Pi \cdot Q = 0, \quad \sum_i \sum_j \pi_{i,j} = 1$$

Let  $C_{U,MBIR}$  denote the registration cost of MBIR per unit time. The following equation can be derived:

$$C_{U,MBIR} = U \sum_j \pi_{M-1,j} \lambda_m$$

where  $U$  is the unit registration cost required for one registration.

Note that the registration cost for the MBR is a little different from that of the MBIR. The main difference is that an outgoing call in the MBIR causes the MS to be in the ring-0 cell as shown in Fig. 2, but an outgoing call in the MBR does not. Deletion of every  $\lambda_{oc}$  in Fig. 2 removes the effect of outgoing calls and results in the transition rate diagram of the MBR.

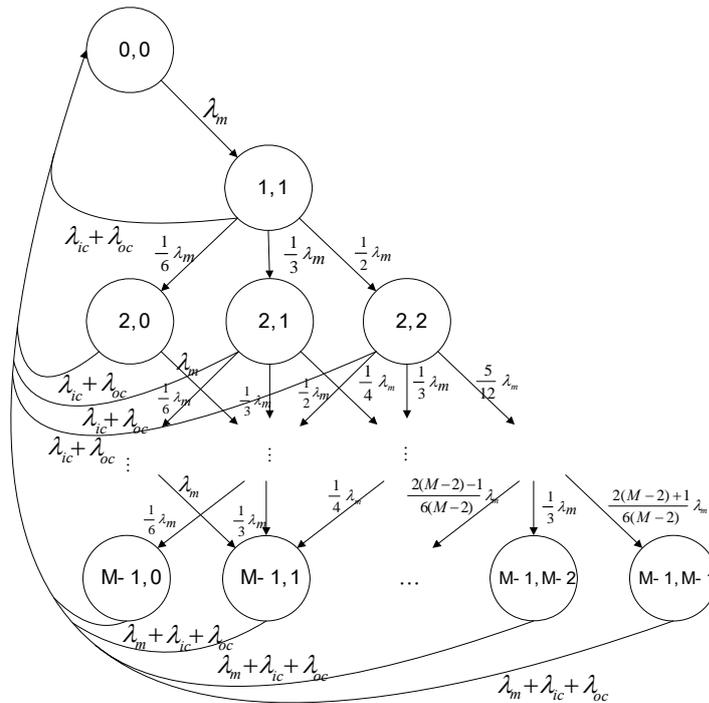


Fig. 2. Transition rate diagram for MBIR using a CTMC

### 3.2 Accurate LR Cost of the MBIR

The method of evaluating the LR cost using the above CTMC model in Section 3.1 has two types of inherent problems:

First, modeling the states of an MS using a CTMC means that the cell residence time of an MS is exponentially distributed. However, in general, it cannot be assumed that the cell residence time of an MS follows an exponential distribution.

Therefore, for other distributions of the cell residence time, the model yields approximate values rather than exact values.

Second, even in the case of an exponential distribution for the cell residence time, the CTMC model described in Section 2.2 makes the mistake of allowing a self-loop. For example, in Fig. 2, when an incoming call is generated in state (0, 0), the MS stays in state (0, 0). In other words, the model cannot consider incoming or outgoing calls in state (0, 0). Nevertheless, if the CTMC model is still used, then an approximated result is obtained that does not reflect the self-loop in which an MS of state (0, 0) transitions back to state (0, 0).

In this section, we provide new mathematical model that uses an imbedded Markov chain (IMMC) in order to analyze the exact performances of the MBR and the MBIR.

The proposed IMMC model can reflect both the general cell residence time and a self-loop. As such, it is expected that the proposed model will yield the exact LR cost of a movement-based LR scheme.

The MBIR scheme can be modeled with an IMMC, as shown in Fig. 3. In the IMMC, an MS is in state (i, j) if its counter value is i and it is currently residing in a ring-j cell.

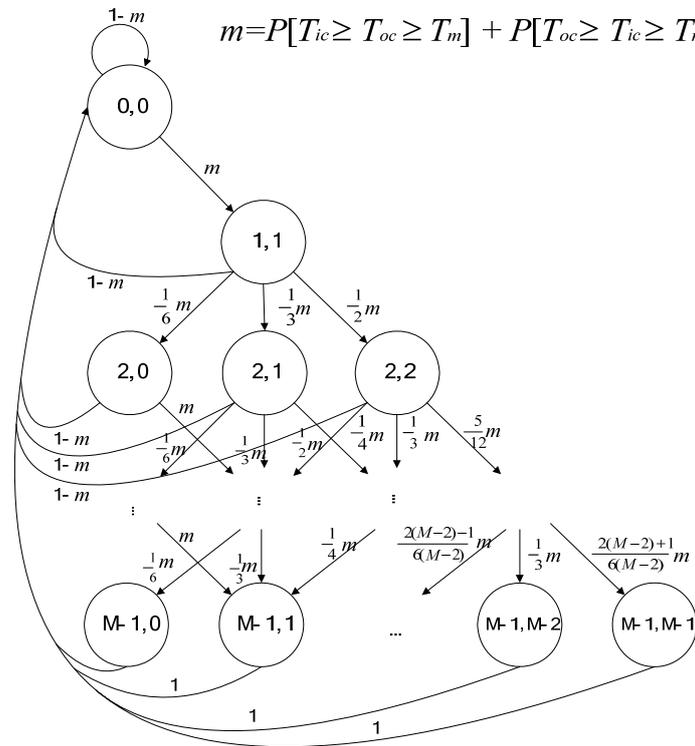


Fig. 3. Transition probability diagram for MBIR using an IMMC

We first examine the transition probability in the IMMC of Fig. 3. Letting m be the probability that an MS in one cell moves to another cell before an

incoming call to the MS arrives or an outgoing call of the MS generates,  $m$  can be written as  $P[T_{ic} \geq T_{oc} \geq T_m] + P[T_{oc} \geq T_{ic} \geq T_m]$ . Assuming that  $T_{ic}$  and  $T_{oc}$  follow exponential distributions with rates of  $\lambda_{ic}$  and  $\lambda_{oc}$ , respectively, we can get the following:

$$\begin{aligned} m &= P[T_{ic} \geq T_{oc} \geq T_m] + P[T_{oc} \geq T_{ic} \geq T_m] \\ &= \int_0^\infty \int_t^\infty \int_y^\infty \lambda_{ic} e^{-\lambda_{ic}x} \lambda_{oc} e^{-\lambda_{oc}y} f_m(t) dx dy dt + \int_0^\infty \int_t^\infty \int_y^\infty \lambda_{oc} e^{-\lambda_{oc}x} \lambda_{ic} e^{-\lambda_{ic}y} f_m(t) dx dy dt \\ &= \int_0^\infty \frac{\lambda_{oc}}{\lambda_{ic} + \lambda_{oc}} e^{-(\lambda_{ic} + \lambda_{oc})t} f_m(t) dt + \int_0^\infty \frac{\lambda_{ic}}{\lambda_{oc} + \lambda_{ic}} e^{-(\lambda_{oc} + \lambda_{ic})t} f_m(t) dt \\ &= \int_0^\infty e^{-(\lambda_{ic} + \lambda_{oc})t} f_m(t) dt = P[T_c \geq T_m] \end{aligned}$$

where  $T_c$  is the interval between any two consecutive calls regardless of incoming call and outgoing calls.

In Fig. 3, for example, an MS in state (1, 1) transits to the one of the following:

- i) state (2, 0) with a transition probability of  $P[T_c \geq T_m] / 6$
- ii) state (2, 1) with a transition probability of  $P[T_c \geq T_m] / 3$
- iii) state (2, 2) with a transition probability of  $P[T_c \geq T_m] / 2$
- iv) state (0, 0) with a transition probability of  $1 - P[T_c \geq T_m]$

Especially, an MS in state  $(M-1, j)$  can transit to the state (0, 0) with transition probability 1, which is composed of  $1 - P[T_c \geq T_m]$  and  $P[T_c \geq T_m]$ . Note that the former produces real registration, but the latter produces only an implicit registration.

Each transition probability is affected by the distribution of  $T_m$ , residence time of an MS in a cell. For example, if it is assumed that  $T_m$  follows an exponential distributions with rate  $\lambda_m$ , then  $P[T_c \geq T_m]$  is given as

$$P[T_c \geq T_m] = \frac{\lambda_m}{\lambda_m + \lambda_c} \quad \text{where} \quad \lambda_c = \lambda_{ic} + \lambda_{oc},$$

since

$$P[T_c \geq T_m] = \int_0^\infty \lambda_m e^{-\lambda_m t_m} \int_{t_m}^\infty \lambda_c e^{-\lambda_c t_c} dt_c dt_m = \int_0^\infty \lambda_m e^{-\lambda_m t_m} [e^{-\lambda_c t_m}] dt_m = \frac{\lambda_m}{\lambda_m + \lambda_c}.$$

If it is assumed that  $T_m$  follows a gamma distribution with mean  $1/\lambda_m$  and variance  $V$ , then  $P[T_c \geq T_m]$  is given as

$$P[T_c \geq T_m] = \left( \frac{\lambda_m \gamma}{\lambda_c + \lambda_m \gamma} \right)^\gamma \quad \text{where} \quad \gamma = \frac{1}{V \lambda_m^2}.$$

The transition probability matrix is  $P_{MBIR}$ , where element  $p_{(i,j),(k,l)}$  describes the probability that an MS moves from state  $(i, j)$  to state  $(k, l)$  in one step. From Fig. 3, the  $\frac{M^2 + M - 2}{2} \times \frac{M^2 + M - 2}{2}$  matrix  $P_{MBIR}$  can be derived. For example, the transition probability matrix  $P_{MBIR}$  for  $M=3$  is as shown in next page.

The steady-state probability for state  $(i, j)$ ,  $\pi_{i,j}$ , can be derived from the

following balance equations:

$$\Pi \cdot P_{MBIR} = \Pi, \quad \sum_i \sum_j \pi_{i,j} = 1$$

$$P_{MBIR} = \begin{bmatrix} 1-m & m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1-m & 0 & m/6 & m/3 & m/2 & 0 & 0 & 0 & 0 \\ 1-m & 0 & 0 & 0 & 0 & 0 & m & 0 & 0 \\ 1-m & 0 & 0 & 0 & 0 & m/6 & m/3 & m/2 & 0 \\ 1-m & 0 & 0 & 0 & 0 & 0 & m/4 & m/3 & 5m/12 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The LR cost of MBIR,  $C_{U,MBIR}$ , can be expressed as

$$C_{U,MBIR} = U \sum_j \pi_{M-1,j} \lambda_m$$

In MBR, the transition probability diagram can be obtained easily since the outgoing call is not considered. The transition probability diagram of the imbedded MC for the MBR is the same as Fig. 3 but  $m$  must be redefined as  $P[T_{ic} \geq T_m]$  instead of  $P[T_{ic} \geq T_{oc} \geq T_m] + P[T_{oc} \geq T_{ic} \geq T_m]$ .

The steady-state probability for state  $(i, j)$ ,  $\psi_{i,j}$ , can be derived from the following balance equations:

$$\Psi \cdot P_{MBR} = \Psi, \quad \sum_i \sum_j \psi_{i,j} = 1$$

The LR cost of MBR,  $C_{U,MBR}$ , can be expressed as

$$C_{U,MBR} = U \sum_j \psi_{M-1,j} \lambda_m$$

where  $U$  is the LR cost for location registration.

### 3.3 Paging Cost and Total Cost

Assuming all cells in a location area are paged simultaneously, whenever an incoming call arrives, the paging cost is the same when either of the MBR and the MBIR schemes are applied. In this case, the paging cost per unit time,  $C_V$ , is as follows for both MBR and MBIR:

$$C_V = V[1 + \sum_{i=1}^{M-1} 6i] \lambda_{ic} = V[1 + 3M(M-1)] \lambda_{ic}$$

where  $V$  is the paging cost for a cell.

The total signaling cost, when considering the LR cost and the paging cost, is as follows:

$$C_T = C_U + C_V$$

## 4 Performance Evaluation

**[Proposition]** For the given threshold  $M$ , the registration cost in the MBIR scheme,  $C_{U,MBIR}$ , equals to or is smaller than that of the MBR scheme,  $C_{U,MBR}$ , i.e.,

$$C_{U,MBIR} = U \sum_j \pi_{M-1,j} \lambda_m \leq U \sum_j \psi_{M-1,j} \lambda_m = C_{U,MBR}$$

[Proof] It is sufficient to show that  $\sum_j \pi_{M-1,j} \leq \sum_j \psi_{M-1,j}$  for every  $M$ . The right side is the steady-state probability that the counter value of an MS is  $M-1$  in the MBR scheme and the left side is the steady-state probability that the counter value of an MS is  $M-1$  in the MBIR scheme.

Comparing the transition probability diagram of the MBR scheme with that of the MBIR scheme, it is apparent that the transition probability diagram is obtained by substituting  $P[T_{ic} \geq T_m]$  with  $P[T_c \geq T_m]$  from each state  $(i, j)$  to each state  $(k, l)$  ( $i, j, k, l=0, 1, 2, \dots, M-1$ ) of the transition probability diagram of Fig. 3, and by substituting  $1 - P[T_{ic} \geq T_m]$  with  $1 - P[T_c \geq T_m]$  from each state  $(i, j)$  ( $i, j=0, 1, 2, \dots, M-1$ ) to state  $(0, 0)$  of the transition probability diagram of Fig. 3.

Noting that  $P[T_{ic} \geq T_m]$  is greater than or equal to  $P[T_c \geq T_m]$ , this indicates that, compared with MBR, the probability that an MS is in state  $(0, 0)$  in the MBIR increases, and the probability that an MS is in state  $(M-1, j)$  ( $i, j=0, 1, 2, \dots, M-1$ ) (the farthest states from state  $(0, 0)$ ) in the MBIR decreases. Therefore, the probability that an MS is in ring  $M-1$  in the MBIR,  $\sum \pi_{M-1,j}$ , is less than or equal to the probability that an MS is in ring  $M-1$  in the MBR,  $\sum \psi_{M-1,j}$ .  $\square$

#### 4.1 Comparison of CTMC and IMMC

To obtain numerical results, we assume that  $U=1.0$ ,  $V=0.1$ ,  $\lambda_m=1$ , and  $\lambda_{ic}=\lambda_{oc}$ . Shown in Fig. 4 is the registration cost per unit time according to the movement threshold  $M$  under a Gamma distribution of the cell residence time when  $\lambda_{ic}=\lambda_{oc}=0.5$ .

In a movement-based LR scheme, the registration cost reduces naturally as the movement threshold increases; this is shown in Fig. 4. However, in this case, the

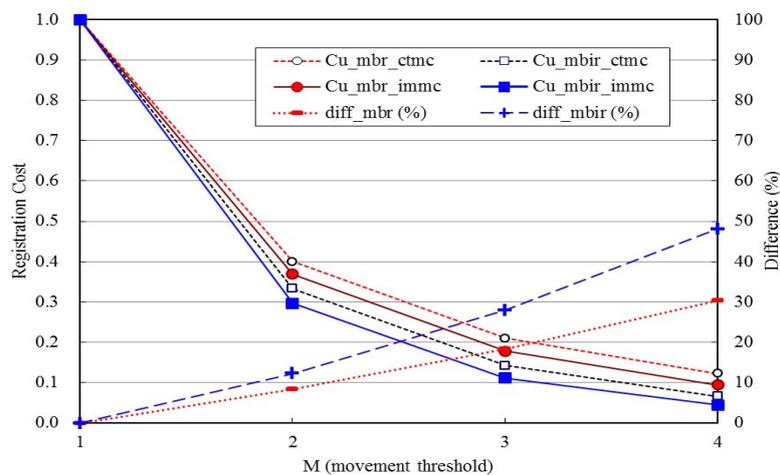


Fig. 4. Registration cost according to the movement threshold  $M$  ( $T_m \sim \text{Gamma}(0.5, 2)$ ,  $\lambda_{ic}=\lambda_{oc}=0.5$ )

registration cost of MBR by CTMC model is greater than that of IMMC model and such a tendency also holds for MBIR. Note that the difference (%) in MBIR is greater than that of MBR. Consequently, in this case, the previous CTMC model underestimates the performance improvement obtained with MBIR when compared to MBR. Therefore, the proposed IMMC model should be used for an exact evaluation of the LR performance of a movement-based LR scheme.

Shown in Table 1 are the LR costs for various cell residence time distributions, as well as the improvements in the CTMC and IMMC models from the MBIR scheme when compared to the MBR method.

Table 1. LR costs for various cell residence time distributions

Modeling & Distribution		M=2	M=3	
CTMC	Cu_mbr	0.400	0.211	
	Cu_mbir	0.333	0.143	
	Effect (%)	-16.7	-32.1	
IMMC	Exp(1)	Same as CTMC		
	Gamma (1/2,2)	Cu_mbr	0.369	0.178
		Cu_mbir	0.297	0.112
		Effect (%)	-19.6	-37.3
	Gamma (2,1/2)	Cu_mbr	0.419	0.232
		Cu_mbir	0.357	0.166
		Effect (%)	-14.7	-28.5
	U(0,2)	Cu_mbr	0.387	0.197
		Cu_mbir	0.302	0.115
		Effect (%)	-22.1	-41.3

The improvement in performance attained from the MBIR scheme by IMMC model is different from that of CTMC model under various cell residence time distributions except for an exponential distribution. For example, in the case of  $M=3$  for Gamma(1/2, 2), the cost is reduced by 32.1% in the CTMC model, whereas it is reduced by 37.3% in the IMMC model. The difference in cost reduction is more evident for a Uniform distribution. From the numerical analyses, it was found that the CTMC model does not suitably reflect the characteristic of cell residence time distribution and the effect of IR. Therefore, the proposed IMMC model should be used for an exact estimation of the LR performance of a movement-based LR scheme.

#### 4.2 Total Signaling Cost by the IMMC Model

Shown in Fig. 5 is the total signaling cost per unit time for various movement thresholds, assuming Gamma(1/2,2) and  $\lambda_{ic}=\lambda_{oc}=0.5$  in the IMMC model. In the environment employed in this numerical example, the total signaling cost reaches a minimum at a movement threshold of  $M=2$  in both the MBR and MBIR schemes. Thus, it would be optimal to implement a movement-based LR scheme with a movement threshold of  $M=2$ . The movement threshold  $M$  necessary to minimize the total signaling cost in other environments could be found.

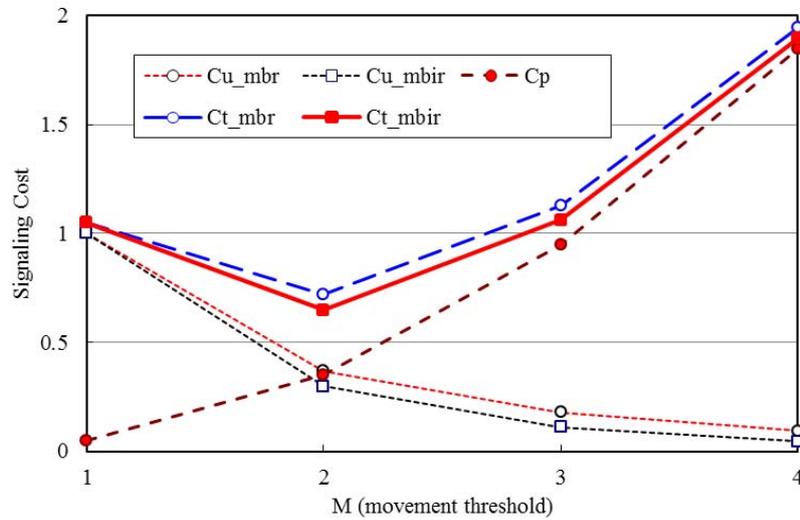


Fig. 5. Total signaling cost for various movement thresholds ( $T_m \sim \text{Gamma}(0.5, 2)$ ,  $\lambda_{ic} = \lambda_{oc} = 0.5$ )

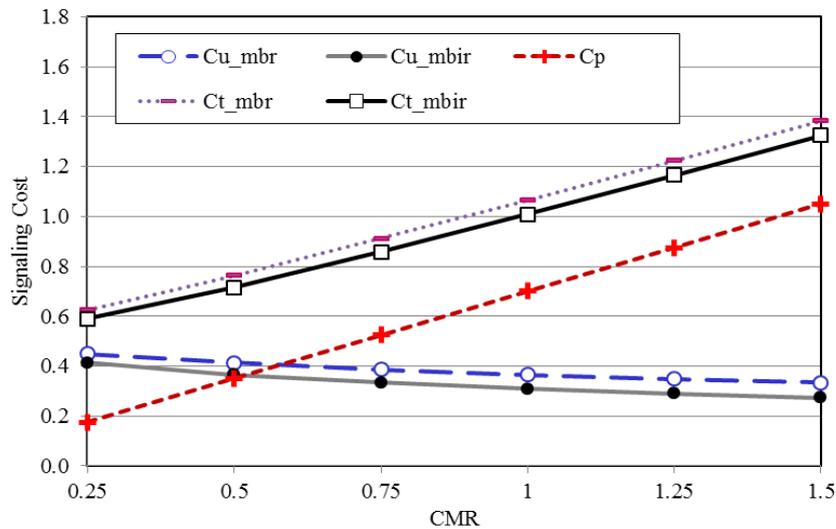


Fig. 6. Total signaling cost according to the CMR ( $T_m \sim \text{Gamma}(0.5, 2)$ ,  $M=2$ )

The total signaling cost per unit time according to the Call-to-Mobility Ratio (CMR) when  $M=2$  is shown in Fig. 6. The CMR is defined as  $\lambda_{ic}/\lambda_m$ . An MS generates calls more frequently as the CMR value increases. For example, an MS with a CMR of 1 generates twice as many incoming calls than an MS with a CMR of 0.5. If  $\lambda_{ic} = \lambda_{oc}$ , an MS with a CMR of 1 generates 0.5 outgoing calls and 0.5 incoming calls per unit time on average when the MS resides in a cell.

As shown in Figs. 5 and 6, the total signaling cost of the MBIR scheme is less than that of the MBR scheme for any movement threshold and CMR value.

## 5 Conclusion

A new model using an IMMC was proposed to analyze the exact performance of MBR and MBIR when the cell residence time is generally distributed and a self-loop is considered. From the obtained numerical results of the performance of movement-based registration, it was shown that the MBIR is always superior to the MBR. In addition, when compared to the IMMC model, the CTMC model underestimates the improvement obtained with the MBIR over the MBR. Such an underestimation is caused by the inherent problems of the CTMC model. Therefore, the IMMC model should be considered in order to exactly analyze the performance of a movement-based registration.

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