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# The Convolution and Neutrix Convolution Involving the Function $x^s \ln^p(1+x_+)$

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**Abstract**

The neutrix convolutions $\ln^p(1+x_+) \circledast x^r$ and $(1+x)^{-1}\ln^p(1+x_+) \circledast x^r$ are evaluated for $r = 0, 1, 2, \ldots$ and $p = 1, 2, \ldots$ . Further results are also given.

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In the following, $\mathcal{D}$ denotes the space of infinitely differentiable functions with compact support and $\mathcal{D}'$ denotes the space of distributions defined on $\mathcal{D}$.

The convolution of certain pairs of distributions in $\mathcal{D}'$ is usually defined as follows, see for example Gel'fand and Shilov [6].

**Definition 1.** Let $f$ and $g$ be distributions in $\mathcal{D}'$ satisfying either of the following conditions:

(a) either $f$ or $g$ has bounded support,

(b) the supports of $f$ and $g$ are bounded on the same side.

Then the *convolution* $f * g$ is defined by the equation

$$\langle (f*g)(x), \varphi(x) \rangle = \langle g(x), \langle f(t), \varphi(x+t) \rangle \rangle$$

for arbitrary test function $\varphi$ in $\mathcal{D}$.

The classical definition of the convolution is as follows:

**Definition 2.** If $f$ and $g$ are locally summable functions then the *convolution* $f * g$ is defined by

$$(f * g)(x) = \int_{-\infty}^{\infty} f(t)g(x-t)\,dt$$

for all $x$ for which the integrals exist.

It follows easily from this definition that if $f * g$ exists then $g * f$ exists and

$$f * g = g * f$$

and if $(f * g)'$ and $f * g'$ (or $f' * g$) exists, then

$$(f * g)' = f * g'(or f' * g)$$

Note that if $f$ and $g$ are locally summable functions satisfying either of the conditions (a) or (b) in Definition 1, then Definition 1 is in agreement with Definition 2.

Definition 1 is rather restrictive and so a neutrix convolution was introduced in [2]. In order to define the neutrix convolution, we first of all let $\tau$ be the function in $\mathcal{D}$, see Jones [7], satisfying the following conditions:

(i) $\tau(x) = \tau(-x)$,
(ii) $0 \le \tau(x) \le 1$,
(iii) $\tau(x) = 1, |x| \le \frac{1}{2}$,
(iv) $\tau(x) = 0, |x| \ge 1$.

The function $\tau_n$ is now defined by

$$\tau_n(x) = \begin{cases} 1, & |x| \le n, \\ \tau(n^n x - n^{n+1}), & x > n, \\ \tau(n^n x + n^{n+1}), & x < -n, \end{cases}$$

**Definition 3.** Let $f$ and $g$ be distributions in $\mathcal{D}'$ and let $f_n = f\tau_n$ for $n = 1, 2, \dots$ . Then the *neutrix convolution* $f \circledast g$ is defined to be the neutrix limit of the sequence $\{f_n * g\}$, provided the limit $h$ exists in the sense that

$$\underset{n\to\infty}{\text{N}-\lim}\langle f_n * g, \varphi\rangle = \langle h, \varphi\rangle$$

for all $\varphi$ in $\mathcal{D}$, where $N$ is the neutrix, see van der Corput [1], having domain $N' = \{1, 2, \dots, n, \dots\}$ and range the real numbers with negligible functions finite linear sums of the functions

$$n^\lambda \ln^{r-1} n, \quad \ln^r n \quad (\lambda > 0,\ r = 1, 2, \dots)$$

and all functions which converge to zero as $n$ tends to infinity.

Note that the convolution $f_n * g$ in this definition is in the sense of Definition 2, the support of $f_n$ being bounded. Note also that the neutrix convolution in this definition, is in general non-commutative.

It was proved in [2] that if the convolution $f * g$ exists by Definition 1, then the neutrix convolution $f \circledast g$ exists and

$$f * g = f \circledast g,$$

showing that Definition 3 is a generalization of Definition 1.

Further, it was proved in [4] that if the neutrix convolution $f' \circledast g$ exists and $\mathrm{N-}\lim_{n\to\infty}\langle (f\tau_n') * g, \varphi\rangle$ exists and equals $\langle h, \varphi\rangle$ for all $\varphi$ in $\mathcal{D}$. Then the neutrix convolution $f \circledast g'$ exists and

The following results were proved in [3]:

$$\begin{aligned}(1+x)^s \ln(1+x_+) * {x_+}^r &= \sum_{i=0}^{r}\binom{r}{i}(-1)^{r-i}\Big\{\frac{(1+x)^{r+s+1}\ln(1+x_+)}{r+s-i+1} \\ &\quad -\frac{[H(x)+x_+]^{r+s+1}-[H(x)+x_+]^i}{(r+s-i+1)^2}\Big\} \\ x^s \ln(1+x_+) * {x_+}^r &= \sum_{i=0}^{r}\binom{r}{i}\sum_{j=0}^{s}\binom{s}{j}(-1)^{s-j+r-i}\Big[\frac{(1+x)^{r+j+1}\ln(1+x_+)}{r+j-i+1} \\ &\quad -\frac{[H(x)+x_+]^{r+j+1}-[H(x)+x_+]^i}{(r+j-i+1)^2}\Big], \\ (1+x)^s\ln(1+x_+)\circledast x^r &= \sum_{i=0}^{r}\sum_{k=1}^{r+s-i+1}\binom{r}{i}\binom{r+s-i+1}{k}\frac{(1+x)^i(-1)^{r+i+k}}{k(r+s-i+1)}, \\ x^s\ln(1+x_+)\circledast x^r &= \sum_{i=0}^{r}\sum_{j=0}^{s}\sum_{k=1}^{r+j-i+1}\binom{r}{i}\binom{s}{j}\binom{r+j-i+1}{k} \\ &\quad \times\frac{(-1)^{s-j+r-i+k+1}(1+x)^i}{k(r+j-i+1)},\end{aligned}$$

for $r, s = 0, 1, 2, \dots$, where $H(x)$ denotes Heaviside's function.

We need the following lemma, which can be easily proved by induction.

**Lemma.**

$$\int t^r \ln^p t\,dt = \sum_{j=0}^{p}\binom{p}{j}\frac{(-1)^{p-j}(p-j)!}{(r+1)^{p-j+1}}t^{r+1}\ln^j t \tag{1}$$

*for* $r = 0, 1, 2, \ldots$ *and* $p = 1, 2, \ldots$ .

We now prove

**Theorem 1.** *The convolution* $\ln^p(1 + x_+) * x_+{}^r$ *exists and*

$$\begin{aligned} \ln^p(1 + x_+) * x_+{}^r &= \sum_{i=0}^{r}\sum_{j=1}^{p}(-1)^{r+p-i+j}(p-j)!\binom{r}{i}\binom{p}{j}\frac{(1+x)^{r+1}\ln^j(1+x_+)}{(r-i+1)^{p-j+1}} \\ &\quad + \sum_{i=0}^{r}(-1)^{r+p-i}p!\binom{r}{i}\frac{H(x)[(1+x)^{r+1}-(1+x)^i]}{(r-i+1)^{p+1}}, \end{aligned} \tag{2}$$

*for* $r = 0, 1, 2, \ldots$ *and* $p = 1, 2, \ldots$ .

*In particular,*

$$\ln^p(1 + x_+) * H(x) = \sum_{j=1}^{p}(-1)^{p-j}(p-j)!\binom{p}{j}(1+x)\ln^j(1+x_+) + (-1)^p p! x_+, \tag{3}$$

*for* $p = 1, 2, \ldots$ *and*

$$\begin{aligned} \ln(1 + x_+) * x_+{}^r &= \sum_{i=0}^{r}(-1)^{r-i}\binom{r}{i}\frac{H(x)(1+x)^{r+1}}{r-i+1} \\ &\quad + \sum_{i=0}^{r}(-1)^{r-i+1}\binom{r}{i}\frac{H(x)[(1+x)^{r+1}-(1+x)^i]}{(r-i+1)^2}, \end{aligned} \tag{4}$$

*for* $r = 0, 1, 2, \ldots$ .

**Proof.** When $x < 0$, it is clear that

$$\ln^p(1 + x_+) * x_+{}^r = 0. \tag{5}$$

When $x > 0$, we have on putting $u = 1 + t$

$$\begin{aligned} \ln^p(1 + x_+) * x_+{}^r &= \int_0^x \ln^p(1+t)(x-t)^r\,dt \\ &= \int_1^{1+x} \ln^p u(1+x-u)^r\,du \\ &= \sum_{i=0}^{r}\binom{r}{i}(1+x)^i(-1)^{r-i}\int_1^{1+x} u^{r-i}\ln^p u\,du \\ &= \sum_{i=0}^{r}\sum_{j=1}^{p}(-1)^{r+p-i-j}(p-j)!\binom{r}{i}\binom{p}{j}\frac{(1+x)^{r+1}\ln^j(1+x)}{(r-i+1)^{p-j+1}} \\ &\quad + \sum_{i=0}^{r}(-1)^{r-i+p}p!\binom{r}{i}\frac{(1+x)^{r+1}-(1+x)^i}{(r-i+1)^{p+1}} \end{aligned} \tag{6}$$

on using equation (2) and equation (3) follows from equations (6) and (7).

Equation (4) follows on putting $r = 0$ in equation (3) and equation (5) follows on putting $p = 1$ in equation (3).

**Corollary 1.1** *The convolution* $\ln^p(1+x_-) * x_-{}^r$ *exists and*

$$\begin{aligned}\ln^p(1+x_-) * x_-{}^r &= \sum_{i=0}^{r}\sum_{j=1}^{p}(-1)^{r+p-i+j}(p-j)!\binom{r}{i}\binom{p}{j}\frac{(1-x)^{r+1}\ln^j(1+x_-)}{(r-i+1)^{p-j+1}} \\ &\quad + \sum_{i=0}^{r}(-1)^{r-i+p}p!\binom{r}{i}\frac{H(-x)[(1-x)^{r+1}-(1-x)^i]}{(r-i+1)^{p+1}}, \qquad (7)\end{aligned}$$

*for* $r = 0, 1, 2, \ldots$ *and* $p = 1, 2, \ldots$ *.*

*In particular,*

$$\ln^p(1+x_-) * H(-x) = \sum_{j=0}^{p}(-1)^{p-j}(p-j)!\binom{p}{j}(1-x)\ln^j(1+x_-) + (-1)^p p!x_-, \quad (8)$$

*for* $p = 1, 2, \ldots$ *and*

$$\begin{aligned}\ln(1+x_-) * x_-{}^r &= \sum_{i=0}^{r}(-1)^{r-i}\binom{r}{i}\frac{H(-x)(1-x)^{r+1}}{r-i+1} \\ &\quad + \sum_{i=0}^{r-1}(-1)^{r-i+1}\binom{r}{i}\frac{H(-x)[(1-x)^{r+1}-(1-x)^i]}{(r-i+1)^2}, \qquad (9)\end{aligned}$$

*for* $r = 0, 1, 2, \ldots$ *.*

**Proof.** Equations (8) to (10) follow from equations (3) to (5) respectively on replacing $x$ by $-x$.

**Theorem 2.** *The convolution* $(1+x)^{-1}\ln^p(1+x_+) * x_+{}^r$ *exists and*

$$\begin{aligned}&(1+x)^{-1}\ln^p(1+x_+) * x_+{}^r = \\ &= \frac{r}{p+1}\Big[\sum_{i=0}^{r-1}\sum_{j=1}^{p+1}(-1)^{r+p-i+j}(p-j+1)!\binom{r-1}{i}\binom{p+1}{j}\frac{(1+x)^r\ln^j(1+x_+)}{(r-i)^{p-j+2}} \\ &\quad + \sum_{i=0}^{r-1}(-1)^{r-i+p}(p+1)!\binom{r-1}{i}\frac{H(x)[(1+x)^r-(1+x)^i]}{(r-i)^{p+2}}\Big], \qquad (10)\end{aligned}$$

*for* $r, p = 1, 2, \ldots$ *.*

*In particular*

***Proof.*** *Since*

$$[\ln^{p+1}(1+x_+)]' * x_+{}^r = \ln^{p+1}(1+x_+) * (x_+{}^r)',$$

*we have*

$$(p+1)(1+x_+)^{-1}\ln^p(1+x_+) * x_+{}^r = r\ln^{p+1}(1+x_+) * x_+{}^{r-1}$$

*and equation (11) follows on using equation (3).*

*Next, since*

$$[\ln^{p+1}(1+x_+)]' * H(x) = \ln^{p+1}(1+x_+) * H'(x),$$

*we have*

$$\begin{aligned}(p+1)(1+x_+)^{-1}\ln^p(1+x_+) * H(x) &= \ln^{p+1}(1+x_+) * H'(x) \\ &= \ln^{p+1}(1+x_+) * \delta(x) = \ln^{p+1}(1+x_+)\end{aligned}$$

*and equation (12) follows.*

*Putting $p = 0$ in equation (11) gives equation (13).*

**Corollary 2.1** *The convolution $(1-x)^{-1}\ln^p(1+x_-) * x_-{}^r$ exists and*

$$\begin{aligned}(1-x)^{-1}\ln^p(1+x_-) * x_-{}^r = \\ = \frac{r}{p+1}\Big\{\sum_{i=0}^{r-1}\sum_{j=1}^{p+1}(-1)^{r+p-i+j}(p-j+1)!\binom{r-1}{i}\binom{p+1}{j}\frac{(1-x)^r\ln^j(1+x_-)}{(r-i)^{j+1}} \\ -\sum_{i=0}^{r-1}(-1)^{r-i+p}\binom{r-1}{i}(p+1)!\frac{H(-x)[(1-x)^r-(1-x)^i]}{(r-i)^{p+2}}\Big\}, \quad (11)\end{aligned}$$

*for $r, p = 1, 2, \ldots$ .*

*In particular*

***Proof.*** *Equations (14) to (16) follow from equations (11) to (13) respectively on replacing $x$ by $-x$.*

*For our next theorem, we define the coefficients $c_{p,j}$ by*

$$\ln^p(1+t) = \sum_{j=1}^{\infty} c_{p,j}t^j,$$

*where $c_{p,j} = 0$ for $j = 1, 2, \ldots, p-1$.*

***Theorem 3.*** *The neutrix convolution $\ln^p(1+x_+) \circledast x^r$ exists and*

$$\begin{aligned}\ln^p(1+x_+) \circledast x^r &= \\ = \sum_{i=0}^{r}\sum_{j=1}^{p}\sum_{k=1}^{r-i+1}\binom{r}{i}\binom{p}{j}\binom{r-i+1}{k}\frac{(-1)^{r+p-i-j}(p-j)!c_{j,k}(1+x)^i}{(r-i+1)^{p-j+1}}, \quad (12)\end{aligned}$$

*for* $r = 0, 1, 2, \dots$ *and* $p = 1, 2, \dots$ .

*In particular,*

$$\ln(1+x_+) \circledast x^r = \sum_{i=0}^{r} \sum_{k=1}^{r-i+1} \binom{r}{i} \binom{r-i+1}{k} \frac{(-1)^{r-i+1}(1+x)^i}{k(r-i+1)}, \tag{13}$$

*for* $r = 0, 1, 2, \dots$ .

***Proof.*** *Putting* $[(\ln^p(1+x_+)]_n = \ln^p(1+x_+)\tau_n(x)$ *and* $u = 1+t$, *we have*

$$\begin{aligned} [\ln^p(1+x_+)]_n * x^r &= \int_0^n \ln^p(1+t)(x-t)^r\,dt \\ &\quad + \int_n^{n+n^{-n}} \ln^p(1+t)(x-t)^r \tau_n(t)\,dt \\ &= \int_1^{n+1} \ln^p u(1+x-u)^r\,dt \\ &\quad + \int_n^{n+n^{-n}} \ln^p(1+t)(x-t)^r \tau_n(t)\,dt \\ &= I_1 + I_2, \end{aligned} \tag{14}$$

*where*

$$\begin{aligned} I_1 &= \sum_{i=0}^{r} \binom{r}{i} (-1)^{r-i}(1+x)^i \int_1^{n+1} u^{r-i} \ln^p u\,du \\ &= \sum_{i=0}^{r} \sum_{j=1}^{p} \binom{r}{i} \binom{p}{j} (-1)^{r+p-i-j}(p-j)!(1+x)^i \frac{(n+1)^{r-i+1}\ln^j(n+1)}{(r-i+1)^{p-j+1}} \\ &\quad + \sum_{i=0}^{r} \binom{r}{i} (-1)^{r-i+p} p!(1+x)^i \frac{(n+1)^{r-i+1}-1}{(r-i+1)^{p+1}}. \end{aligned} \tag{15}$$

*Now*

$$(n+1)^{r-i+1}\ln^j(n+1) = (n+1)^{r-i+1}[\ln n + \ln(1+1/n)]^j \tag{16}$$

*and*

$$(n+1)^{r-i+1}\ln^j(1+1/n) = \sum_{k=0}^{r-i+1} \sum_{m=1}^{\infty} \binom{r-i+1}{k} c_{j,m} n^{k-m}. \tag{17}$$

*It follows from equations (21) and (22) that*

$$\begin{aligned}\underset{n\to\infty}{\mathrm{N-lim}}(n+1)^{r-i+1}\ln^j(n+1) &= \underset{n\to\infty}{\mathrm{N-lim}}(n+1)^{r-i+1}[\ln n+\ln(1+1/n)]^j \\ &= \underset{n\to\infty}{\mathrm{N-lim}}(n+1)^{r-i+1}\ln^j(1+1/n) \\ &= \underset{n\to\infty}{\mathrm{N-lim}}\sum_{k=0}^{r-i+1}\sum_{m=1}^{\infty}\binom{r-i+1}{k}c_{j,m}n^{k-m} \\ &= \sum_{k=0}^{r-i+1}\binom{r-i+1}{k}c_{j,k}. \qquad (18)\end{aligned}$$

*Then from equations (20) and (23), we have*

$$\underset{n\to\infty}{\mathrm{N-lim}}\, I_1 = \\ = \sum_{i=0}^{r}\sum_{j=1}^{p}\sum_{k=1}^{r-i+1}\binom{r}{i}\binom{p}{j}\binom{r-i+1}{k}\frac{(-1)^{r+p-i-j}(p-j)!c_{j,k}(1+x)^i}{(r-i+1)^{p-j+1}}. \qquad (19)$$

*Next, since $I_2 = O(n^{-n})$, it follows that*

*Equation (18) now follows from equation (17) on putting p=1.*

***Corollary 3.1*** *The neutrix convolution $\ln^p(1+x_-) \circledast x^r$ exists and*

$$\ln^p(1+x_-) \circledast x^r = \\ = \sum_{i=0}^{r}\sum_{j=1}^{p}\sum_{k=1}^{r-i+1}\binom{r}{i}\binom{p}{j}\binom{r-i+1}{k}\frac{(-1)^{p-i-j}(p-j)!c_{j,k}(1-x)^i}{(r-i+1)^{p-j+1}}, \qquad (20)$$

*for $r = 0, 1, 2, \ldots$ and $p = 1, 2, \ldots$ .*
*In particular,*

$$\ln(1+x_-) \circledast x^r = \sum_{i=0}^{r}\sum_{k=1}^{r-i+1}\binom{r}{i}\binom{r-i+1}{k}\frac{(-1)^{i+1}(1-x)^i}{k(r-i+1)}, \qquad (21)$$

*for $r = 0, 1, 2, \ldots$ .*

***Proof.*** *Equations (26) and (27) follow from equations (17) and (18) on replacing $x$ by $-x$.*

***Theorem 4.*** *The neutrix convolution $(1+x)^{-1}\ln^p(1+x_+) \circledast x^r$ exists and*

$$(1+x)^{-1}\ln^p(1+x_+) \circledast x^r = (p+1)^{-1}\Big\{\sum_{i=1}^{r}\binom{r}{i}(-1)^i c_{p+1,i}x^{r-i} \\ + \sum_{i=0}^{r-1}\sum_{j=1}^{p+1}\sum_{k=1}^{r-i}\binom{r-1}{i}\binom{p+1}{j}\binom{r-i}{k}\frac{(-1)^{r+p-i-j}r(p-j+1)!c_{j,k}(1+x)^i}{(r-i)^{p-j+2}}\Big\}, \qquad (22)$$

*for* $r, p = 0, 1, 2, \dots$ .

*In particular,*

$$\begin{aligned} H(x)(1+x)^{-1} \circledast x^r &= \sum_{i=1}^{r} \binom{r}{i} \frac{(-1)^{i+1} x^{r-i}}{i} \\ &\quad + \sum_{i=0}^{r-1} \sum_{k=1}^{r-i} \binom{r-1}{i} \binom{r-i}{k} \frac{(-1)^{r-i+k} r(1+x)^i}{k(r-i)}, \end{aligned} \tag{23}$$

*for* $r = 0, 1, 2, \dots$ .

***Proof.*** *We have*

$$\begin{aligned} \ln^{p+1}(1+x_+)\tau_n'(x) * x^r &= \int_n^{n+n^{-n}} \ln^{p+1}(1+t)\tau_n'(t)(x-t)^r \, dt \\ &= -\ln^{p+1}(1+n)(x-n)^r \\ &\quad - \int_n^{n+n^{-n}} [(p+1)(1+t)^{-1} \ln^p(1+t)(x-t)^r \\ &\qquad -r \ln^{p+1}(1+t)(x-t)^{r-1}]\tau_n(t) \, dt \\ &= J_1 + J_2, \end{aligned} \tag{24}$$

*where*

$$J_1 = \sum_{i=0}^{r} \sum_{j=0}^{p+1} \binom{r}{i} \binom{p+1}{j} (-1)^{i-1} x^{r-i} n^i \ln^{p-j+1} n \ln^j(1+1/n)$$

*and so*

$$\begin{aligned} \operatorname*{N-lim}_{n\to\infty} J_1 &= \operatorname*{N-lim}_{n\to\infty} \sum_{i=0}^{r} \binom{r}{i} (-1)^{i-1} x^{r-i} n^i \ln^{p+1}(1+1/n) \\ &= \operatorname*{N-lim}_{n\to\infty} \sum_{i=0}^{r} \sum_{m=1}^{\infty} \binom{r}{i} (-1)^{i-1} c_{p+1,m} x^{r-i} n^{i-m} \\ &= \sum_{i=0}^{r} \binom{r}{i} (-1)^{i-1} c_{p+1,i} x^{r-i}. \end{aligned} \tag{25}$$

*Next, since* $J_2 = O(n^{-n})$, *it follows that*

*It now follows from equations (30) to (32) that*

*Using equation (1), we now have*

$$(p+1)(1+x)^{-1}\ln^p(1+x_+) \circledast x^r + \sum_{i=0}^{r}\binom{r}{i}(-1)^{i-1}c_{p+1,i}x^{r-i} = \\ = r\ln^{p+1}(1+x_+)\circledast x^{r-1}. \quad (26)$$

*Equation (28) now follows from equations (17) and (34).*

*Equation (29) follows on putting $p = 0$ in equation (28).*

***Corollary 4.1*** *The neutrix convolution $(1-x)^{-1}\ln^p(1+x_-)\circledast x^r$ exists and*

$$(1-x)^{-1}\ln^p(1+x_-)\circledast x^r = (p+1)^{-1}\Big\{\sum_{i=1}^{r+1}\binom{r}{i}(-1)^{r+i}c_{p+1,i}x^{r-i} \\ + \sum_{i=0}^{r-1}\sum_{j=1}^{p+1}\sum_{k=1}^{r-i}\binom{r-1}{i}\binom{p+1}{j}\binom{r-i}{k}\frac{(-1)^{p-i-j}r(p-j+1)!c_{j,k}(1-x)^i}{(r-i)^{p-j+2}}\Big\}, \quad (27)$$

*for $r, p = 0, 1, 2, \ldots$ .*

*In particular,*

$$H(-x)(1-x)^{-1}\circledast x^r = -\sum_{i=1}^{r}\binom{r}{i}\frac{x^{r-i}}{i} \\ -\sum_{i=0}^{r-1}\sum_{k=1}^{r-i}\binom{r-1}{i}\binom{r-i}{k}\frac{(-1)^{i+k}r(1-x)^i}{k(r-i)}, \quad (28)$$

*for $r = 0, 1, 2, \ldots$ .*

*Equations (35) and (36) follow from equations (28) and (29) on replacing $x$ by $-x$.*

# References

[1] J.G. van der Corput, *Introduction to the neutrix calculus,* J. Analyse Math., 7(1959-60), 291-398.

[2] B. Fisher, *Neutrices and the convolution of distributions,* Zb. Rad. Prirod.-Mat. Fak., Ser. Mat., Novi Sad, 17(1987), 119-135.

[3] B. Fisher and F. Al-Sirehy, *The convolution and neutrix convolution involving the function $x^s\ln(1+x_+)$*, to appear.

[4] B. Fisher and K. Taş, *The convolution of functions and distributions*, J. Math. Anal. Appl., **306**(2005), 364-374.

[5] B. Fisher and M. Telci, *Some neutrix convolutions of functions and distributions*, J. Analysis, **11**(2003), 23-32.

[6] I.M. Gel'fand and G.E. Shilov, *Generalized functions,* Vol. I, Academic Press 1964.

[7] D.S. Jones, *The convolution of generalized functions,* Quart. J. Math. Oxford Ser., **24**(1973), 145-163.

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