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New Proposed Uniform-Exponential Distribution

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Abstract

In this paper we propose new formula of Uniform-Exponential Distribution (U_ED) with discussion some of its properties, like the moment generated function, mean, mode, median , variance , the r-th moment about the mean, the r-th moment about the origin, reliability, hazard functions, coefficients of variation, of sekeness and of kurtosis. Finally, we estimate the parameters .

Keyword: Exponential distribution, Uniform distribution, Maximum Likelihood estimation

1-Introduction

There are many researches based on the Beta distribution and described as a new distribution as the distributions: Beta-Pareto which is presented by Akinsete, Famoye and Lee (2008), Beta generalized exponential constructed by Barreto-Souza, , Santos, and Cordeiro (2009), Beta-half-Cauchy is presented by Cordeiro, and Lemonte (2011), while Beta Generalized Logistic derived by Morais, Cordeiro ,and Audrey (2011) , Beta –hyperbolic Secant(BHS) by Mattheas, David(2007), Beta Fre´chet by Nadarajah, and Gupta(2004) , Beta normal distribution and its application Eugene, N., Lee, C. and Famoye, F. (2002) and Beta exponential by Nadarajah, S. and Kotz, S., 2004**.** Uniform Exponential Distribution (UED) and Exponential Pareto distribution(EPD) proposed by Abed Al-Kadim and Abdalhussain Boshi (2013) of the form

$F(x) = \int_a^{bb-1+F^{\#}(x;\lambda)} f(x)dx$

Where $F^{\#}(x;\lambda)$is the c.d.f. of the exponential distribution and $f^*(a,b)$ is the pdf of the continuous uniform distribution and the form

$F_{e.p}(x) = \int_0^{\frac{1}{1-F^{\#}(x)}} f^{\Im}(x)\, dx,$

where $F^{\#}$is the pareto distribution, $(X) = 1-\left(\frac{p}{x}\right)^{\theta}$, and $f^{\Im}$ is the exponential distribution, that is we use another distribution instead Beta distribution.

While in this paper we introduce the Uniform-Exponential Distribution(U_ED) of the form

$G(x) = \int_a^{bF(x)} f(x)dx$

Where $F(x)$is the c.d.f. of the exponential distribution and $f(a,b)$ is the pdf of the continuous uniform distribution, then the c.d.f of U_ED becomes as

$F(x;a,b,\lambda) = 1 - \frac{be^{-\lambda x}}{b-a}$ (1)

So the p.d.f. is given by :

$g(x) = (\frac{b}{b-a})\lambda e^{-\lambda x}$ (2)

for $\ln(z(a,b))^{1/\lambda} \leq x < \infty , \lambda > 0 , 0 \leq a < b$

which is similar to Exponential distribution , it is like a weighted distribution and we can rewrite it as

$g(x) = z(a,b)\lambda e^{-\lambda x}$ (3)

where $z(a,b) = \frac{b}{b-a}$ as the weighted function. Now we can prove that $g(x)$ is pdf as follows:

$$\int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} g(x)dx = \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} \frac{\lambda b e^{-\lambda x}}{b-a} dx$$

$\int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} g(x)dx = \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} \frac{\lambda b e^{-\lambda x}}{b-a} dx$

$$= \frac{b}{b-a}\left[e^{\ln\left(\frac{b-a}{b}\right)}\right] = \frac{b}{b-a}\left[\left(\frac{b-a}{b}\right)\right] = 1 \quad \blacksquare$$

That is

$$\lim_{x\to\infty} F(x) = \lim_{x\to\infty}\left(1-\frac{be^{-\lambda x}}{b-a}\right) = 1, \text{ and } \lim_{x\to w} F(x) = 0$$

Figure1and **Figure2** show us the shape of the df of U_ED which behaves as an Exponential distribution.
We take the sympoles a,b,a1,b1,$a2$,b2 as parameters for uniform distribution.

And **Figure3** shows us the graph of this pdf for different parameters, a=0, b= 1,λ= 0.5,a1=0 ,b1= 1, $\lambda 1$=0.6 ', $a2$=0 ,b2= 1 , $c2$= 1.Which means that pdf behaves as the pdf of Exponential distribution and takes the value =1 at $a2$=0 ,b2= 1 , $\lambda 2$= 1. **Figure4** shows us the pdf takes many shapes of Exponential distribution but for different values of λ. While **Figure5** shows us the pdf for different values of a, b, a1,b1, $a2$,b2.

2- Properties of U_ED

Proposition1

The moment generating function of U_ED is of the form

$$M(t) = \left(1-\frac{t}{\lambda}\right)^{-1} e^{\frac{t}{\lambda}ln(\frac{b}{b-a})} \quad \textbf{(4)}$$

Proof

$$M(t) = E(e^{tx}) = \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} \frac{\lambda b\, e^{tx} e^{-\lambda x}}{b-a} dx = \frac{b\, e^{\ln(\frac{b}{b-a})^{-(\lambda-t)/\lambda}}}{(b-a)\left(1-\frac{t}{\lambda}\right)}$$

$$M(t) = \left(1-\frac{t}{\lambda}\right)^{-1} \left(\frac{b}{b-a}\right)^{t/\lambda} \quad \blacksquare$$

Proposition2

The rth central moment about the origin , and the rth central moment about the mean of U_ED are as follows:

$$E(X^r) = w^r + \frac{rw^{r-1}}{\lambda} + \frac{r(r-1)w^{r-2}}{\lambda^2} + \cdots . + \frac{r!}{\lambda^r} \quad \textbf{(5)}$$

$$E(X-\mu)^r = \sum_{i=0}^{r}\binom{r}{i}(-\mu)^i\left[w^s + \frac{sw^{s-1}}{\lambda} + \frac{s(s-1)w^{s-2}}{\lambda^2} + \cdots . + \frac{s!}{\lambda^s}\right] \quad \textbf{(6)}$$

For $r = 1, 2, 3, \ldots$,where $w = \frac{1}{\lambda}\ln(\frac{b}{b-a})$ and $s = r - i$

Proof

$$\boldsymbol{E}(\boldsymbol{X}^{\boldsymbol{r}}) = \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} \frac{\lambda b x^r e^{-\lambda x}}{b-a} dx$$

$$= \frac{\lambda b}{b-a} \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} x^r e^{-\lambda x} dx$$

Let $u = x^r$, $du = r\, x^{r-1}\, dx$,

$$dv = \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} e^{-\lambda x} dx$$

Then

$$\boldsymbol{E}(\boldsymbol{X}^{\boldsymbol{r}}) = -\frac{x^r}{\lambda} e^{-\lambda x}\Big|_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} + \frac{r}{\lambda} \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} x^{r-1} e^{-\lambda x} dx$$

Now let

$$u_1 = x^{r-1}, \ du_1 = (r-1)\, x^{r-2}\, dx,$$

$$dv_1 = \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} e^{-\lambda x} dx \ , v_1 = -\frac{1}{\lambda} e^{-\lambda x}$$

Then

$$\boldsymbol{E}(\boldsymbol{X}^{\boldsymbol{r}}) = -\frac{x^r e^{-\lambda x}}{\lambda}\Big|_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} - \frac{r\, x^{r-1} e^{-\lambda x}}{\lambda}\Big|_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} + \frac{r(r-1)}{\lambda} \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} x^{r-2} e^{-\lambda x} dx$$

So let

$$u_2 = x^{r-2}, \ du_2 = (r-2)\, x^{r-3}\, dx,$$

and let

$$dv_2 = \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} e^{-\lambda x} dx \ , v_2 = -\frac{1}{\lambda} e^{-\lambda x}$$

Then $\boldsymbol{E}(\boldsymbol{X}^{\boldsymbol{r}}) = -\frac{x^r e^{-\lambda x}}{\lambda}\Big|_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty}$

$$-\frac{r\, x^{r-1} e^{-\lambda x}}{\lambda}\Big|_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} - \frac{r(r-1) x^{r-2} e^{-\lambda x}}{\lambda}\Big|_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} + \frac{r(r-1)(r-2)}{\lambda} \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} x^{r-3} e^{-\lambda x} dx$$

We continue using the integration by parts to get the following formula

$$\boldsymbol{E(X^r)} = \left[-\frac{x^r e^{-\lambda x}}{\lambda} - \frac{x^{r-1} e^{-\lambda x}}{\lambda} - \frac{x^{r-2} e^{-\lambda x}}{\lambda} - \cdots + \frac{r! e^{-\lambda x}}{\boldsymbol{\lambda^{r+1}}}\right]_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty}$$

$$\boldsymbol{E(X^r)} = \frac{\boldsymbol{w^r}}{\boldsymbol{\lambda}} e^{-\lambda w} + \frac{\boldsymbol{r}}{\boldsymbol{\lambda^2}} \boldsymbol{w^{r-1}} e^{-\lambda w} + \frac{\boldsymbol{r(r-1)}}{\boldsymbol{\lambda^3}} \boldsymbol{w^{r-2}} e^{-\lambda w} + \cdots . + \frac{\boldsymbol{r!}}{\boldsymbol{\lambda^{r+1}}} e^{-\lambda w}$$

$$\boldsymbol{E(X^r)} = \boldsymbol{w^r} + \frac{\boldsymbol{r w^{r-1}}}{\boldsymbol{\lambda}} + \frac{\boldsymbol{r(r-1) w^{r-2}}}{\boldsymbol{\lambda^2}} + \cdots . + \frac{\boldsymbol{r!}}{\boldsymbol{\lambda^r}}$$

For $\boldsymbol{r = 1, 2, 3}, \ldots$,where $\boldsymbol{w = \frac{1}{\lambda} \ln(\frac{b}{b-a})}$.

Also we can get this result by using this formula

$$\boldsymbol{M^{(r)}(t)}\Big|_{\boldsymbol{t=0}} = \boldsymbol{E(X^r)} \quad \boldsymbol{r = 1, 2, 3}, \ldots.$$

Since

$$(\boldsymbol{X - \mu})^r = \sum_{\boldsymbol{i=0}}^{r} \binom{r}{i} (-\boldsymbol{\mu})^i \mathrm{X}^{r-i}$$

$$\boldsymbol{E\,(X - \mu)^r} = \frac{\lambda b \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} (x - \mu)^r \, e^{-\lambda x} dx}{b - a} =$$

$$\sum_{\boldsymbol{i=0}}^{r} \binom{r}{i} (-\boldsymbol{\mu})^i \frac{\lambda b}{b-a} \int_{\ln(\frac{b}{b-a})^{1/\lambda}}^{\infty} \mathrm{X}^{r-i} \quad e^{-\lambda x} dx$$

Let $s = r - i$,then using the same method followed above, we get

$$\boldsymbol{E\,(X - \mu)^r} = \sum_{\boldsymbol{i=0}}^{r} \binom{r}{i} (-\boldsymbol{\mu})^i \left[\boldsymbol{w^s} + \frac{s}{\lambda} \boldsymbol{w^{s-1}} + \frac{s(s-1)}{\lambda^2} \boldsymbol{w^{s-2}} + \cdots . + \frac{s!}{\lambda^s}\right] \qquad \blacksquare$$

Result1

The mean, variance, coefficients of variation, skewness, and kurtosis of U_ED as follows:

$\boldsymbol{\mu = \frac{1}{\lambda} + \frac{1}{\lambda} \ln(\frac{b}{b-a})}$ **(7),**

$\boldsymbol{\sigma^2 = \frac{1}{\lambda^2}}$ **(8) ,**

$\boldsymbol{CV = 1}$ **(9) ,**

$\boldsymbol{CS = 2(10}$ **(10),**

$\boldsymbol{CK = 9}$ **(11)**

Proof

Using (5), we get the mean respectively as follows

$$E(X) = w + \frac{1}{\lambda} = \ln(\frac{b}{b-a})^{1/\lambda} + \frac{1}{\lambda} = \frac{1}{\lambda}\left[1 + \ln(\frac{b}{b-a})\right]$$

Using (6), we get the variance

$$\sigma^2 = E(X-\mu)^2 \text{e}$$

$$\sigma^2 == \sum_{i=0}^{2}\binom{2}{i}(-\mu)^i\left[w^{2-i} + \frac{2-i}{\lambda}w^{2-i-1} + \cdots + \frac{(2-i)!}{\lambda^{2-i}}\right]$$

$$= \left[w^2 + \frac{2}{\lambda}w + \frac{2}{\lambda^2}\right] - 2\left[w + \frac{1}{\lambda}\right]\mu + \mu^2$$

$$= \left[\frac{1}{\lambda^2}\ln^2(\frac{b}{b-a}) + \frac{2}{\lambda^2}\ln(\frac{b}{b-a}) + \frac{2}{\lambda^2}\right] - \left[\frac{2}{\lambda^2}\ln\left(\frac{b}{b-a}\right) + \frac{2}{\lambda^2}\ln^2\left(\frac{b}{b-a}\right) + \frac{2}{\lambda^2} + \frac{2}{\lambda^2}\ln\left(\frac{b}{b-a}\right)\right] + \left[\frac{1}{\lambda^2} + \frac{2}{\lambda^2}\ln\left(\frac{b}{b-a}\right) + \frac{1}{\lambda^2}\ln^2\left(\frac{b}{b-a}\right)\right] = \frac{1}{\lambda^2}$$

Now $E(X-\mu)^3 = \sum_{i=0}^{3}\binom{3}{i}(-\mu)^i\left[w^{3-i} + \frac{3-i}{\lambda}w^{3-i-1} + \cdots + \frac{(3-i)!}{\lambda^{3-i}}\right] = \frac{2}{\lambda^3}$

$$E(X-\mu)^4 = \sum_{i=0}^{4}\binom{4}{i}(-\mu)^i\left[w^{4-i} + \frac{4-i}{\lambda}w^{4-i-1} + \cdots + \frac{(4-i)!}{\lambda^{4-i}}\right] = \frac{9}{\lambda^4}$$

Then coefficients of variation, skewness, and kurtosis of U_ED as follows:

$$CV = \frac{\sigma}{\mu} = 1,\ CS = \frac{E(X-\mu)^3}{\sigma^3} = \frac{2/\lambda^3}{1/\lambda^3} = 2$$

$$, CK = \frac{E(X-\mu)^4}{\sigma^4} = \frac{4/\lambda^4}{1/\lambda^4} = 9 \quad \blacksquare$$

From this result we can conclude that distribution, U_ED, is another formula for Exponential distribution. Each of these coefficients is constant.

Proposition3

The mode and the median of U_ED are defined as:

$$x_{mode} = 0 \quad (12)$$

$$x_{median} = ln(\frac{2b}{b-a})^{\lambda} \quad (13)$$

Proof

When we take limit of the pdf $g(x)$ to check its shape, for example, if it has one or more beaks.

$$\lim_{x\to\infty} g(x) = \frac{\lambda b \lim_{x\to\infty} e^{-\lambda x}}{b-a} = 0$$

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{\lambda b}{b-a} \lim_{x\to 0} e^{-\lambda x} = \frac{\lambda b}{b-a}$$

We conclude that $g(x)$ decreases to zero as $x \to \infty$, it has the constant value $\frac{\lambda b}{b-a}$ as $x \to 0$. That it has the maximum value at $x = 0$, and then it decreases to zero as $x \to \infty$. And

$\frac{d[g(x)]}{dx} = 0 = \frac{\lambda^2 b e^{-\lambda x}}{b-a}$, $\therefore x = \infty$. That is $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \acute{g}(x) = 0$

Then $g(x)$ has the maximum value at $x = 0$,that $x_{mode} = 0$.

$$F(x_{median}) = \frac{1}{2}$$

$$1 - \frac{b e^{-\lambda x_{median}}}{b-a} = \frac{1}{2}$$

$x_{median} = ln(\frac{2b}{b-a})^{\lambda}$ ■

Proposition 4

The reliability function and hazard function are given as:

$R(x) = \frac{b}{b-a} e^{-\lambda x}$ **(14)**

$h(x) = \lambda$ **(15)**

Proof

Since $R(x) = 1 - F(x)$,and $h(x) = \frac{g(x)}{R(x)}$, then $R(x) = \frac{b}{b-a} e^{-\lambda x}$. And then

$h(x) = \frac{\frac{\lambda b}{b-a} e^{-\lambda x}}{\frac{b}{b-a} e^{-\lambda x}} = \lambda$ ■

From this proposition we note that hazard function is a function to the scale parameter , it is as constant function.

Figure6 shows us the hazard function as an identity function of $\boldsymbol{\lambda}$, while **Figure7** shows us the hazard function as a constant function.

3-Estimation

We know that there are three parameters, $\boldsymbol{\lambda, a, b}$, so the question is can we estimate each of them using1) the maximum likelihood method? , 2) the moment method

To answer that question ,we try to do that as follows:

3-1 The maximum likelihood method

$$\boldsymbol{l(\lambda, a, b; x_1, x_2, \dots, x_n) = l}$$

$\boldsymbol{l = (\frac{\lambda b}{b-a})^n e^{-\lambda \sum_{i=1}^{n} x_i}}$ **(16)**

$\boldsymbol{lnl == nln\lambda + nlnb - nl\,n(b-a) - \lambda \sum_{i=1}^{n} x_i}$ **(17)**

$$\boldsymbol{\frac{\partial lnl}{\partial a} = 0, \frac{\partial lnl}{\partial b} = 0, \frac{\partial lnl}{\partial \lambda} = 0}$$

$$\boldsymbol{\frac{\partial lnl}{\partial a} = \frac{n}{\hat{b} - \hat{a}} = 0} \qquad (18)$$

$$\boldsymbol{\frac{\partial lnl}{\partial b} = \frac{n}{\hat{b}} - \frac{n}{\hat{b} - \hat{a}} = 0} \qquad (19)$$

$$\boldsymbol{\frac{\partial lnl}{\partial \lambda} = \frac{n}{\hat{\lambda}} - \hat{\lambda} \sum_{i=1}^{n} x_i = 0} \qquad (20)$$

There is no estimator for $\boldsymbol{a}$, or $\boldsymbol{b}$ from **(18), (19)**, but from **(20)** we get

$\boldsymbol{\hat{\lambda}_{ml} = \frac{n}{\sum_{i=1}^{n} x_i} = \frac{1}{\bar{X}}}$ **(21)**

Now when we take $\boldsymbol{w = \frac{1}{\lambda} \ln(\frac{b}{b-a}) = \frac{1}{\lambda} \ln[z(a, b)]}$, where $\boldsymbol{z(a, b) = \frac{b}{b-a}}$, and we know that $\boldsymbol{\ln(z(a, b))^{1/\lambda} \leq x < \infty}$ which means that

$\boldsymbol{\ln(z(a, b))^{1/\lambda} \leq y_1 \leq \cdots \leq y_n < \infty}$ **,** where $\boldsymbol{y_1, \dots, y_n}$ are the order statistics of the random sample of thr random variable $\boldsymbol{X}$.

$$\boldsymbol{\frac{\partial lnl}{\partial z} = \frac{n}{\hat{z}} = 0} \qquad (22)$$

Then $\ln(\hat{z}(a,b))^{1/\hat{\lambda}} \le y_1, \quad \ln(\hat{z}(a,b))^{\bar{\bar{X}}} \le y_1$,

$$\hat{z}_{ml} = e^{y_1/\bar{X}} = e^{min(X_1,\dots,X_n)/\bar{X}}v \qquad (22)$$

3-2 The moment method

$$\frac{1}{n}\sum_{i=1}^{n} X_i = \frac{1}{\hat{\lambda}}[1 + \ln(\hat{z})] \qquad (23)$$

$$\frac{1}{n}\sum_{i=1}^{n} X_i^2 = \frac{2}{\hat{\lambda}^2}[1 + \ln(\hat{z})] + \frac{1}{\hat{\lambda}^2}\ln^2(\hat{z}) \qquad (24)$$

From (23) we get

$$\hat{\lambda}_{mo} = \frac{1}{\bar{X}}[1 + \ln(\hat{z})] \qquad \textbf{(25)}$$

Substitute **(25)** in **(24)** to get

$$\hat{\lambda}_{mo} = \left(\frac{1}{n}\sum_{i=1}^{n} X_i^2 - \bar{X}^2\right)^{-1/2} \quad (26)$$

Then $\ln(\hat{z}) = \frac{\bar{X}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n} X_i^2 - \bar{X}^2}} - 1 \ (27)$

And $\hat{z} == e^{\frac{\bar{X}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n} X_i^2 - \bar{X}^2}} - 1}$ **.** That is $\hat{\lambda}_{mo} > \hat{\lambda}_{ml}$.

4- Conclusions

We can derive new formula of Uniform-Exponential Distribution (U_ED) based on another distribution instead Beta distribution, wit discussion some of its properties

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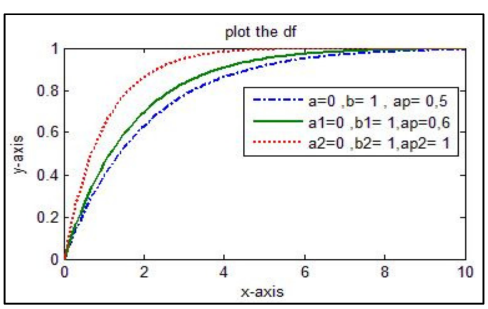

Figure1 The df of U_ED for different pa rameters, a=0,b=1 ,λ= 0.5,a1=0 ,b1= 1, , $\lambda 1$=0.6, $a2$=0,b2= 1 , $\lambda 2$= 1.

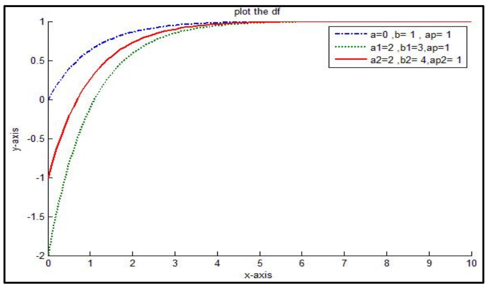

Figure2 The df of U_ED for different parameters, a=0,b=1,λ= 1,a1=2 , b1=3, λ1=1, α2=2,b2=4 , λ2= 1.

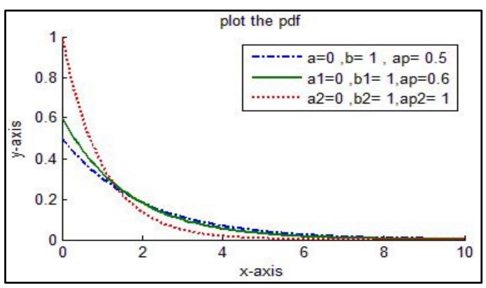

Figure3 The pdf of U_ED for different parameters a=0,b=1,λ= 0.5 ,a1=,0 b1=3, λ1=0.6, α2=0,b2=1 , λ2= 1.

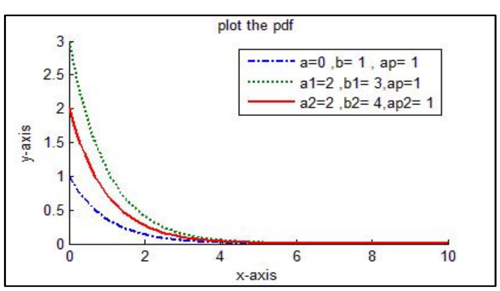

Figure4 The pdf of U_ED for different parameters, a=0,b=1,λ=1,a1=2, b1= 3 , λ1=1 , α2=2 ,b2=4 , λ2=1.

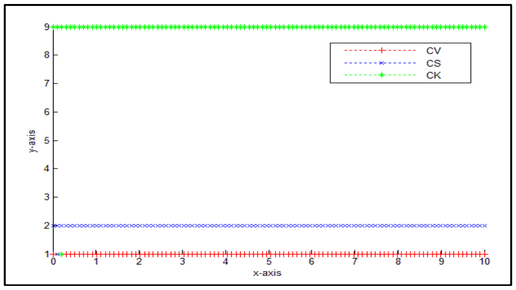

Figure5 The coefficients of variation, skewness, and kurtosis of U_ED

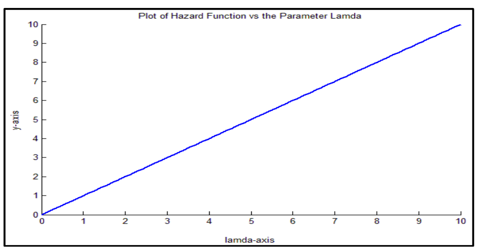

Figure6 The plot of hazard function as function of $\boldsymbol{\lambda}$

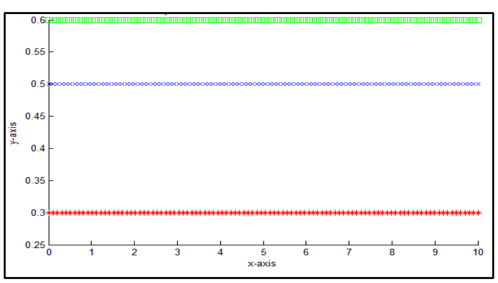

Figure7 The plot of hazard function as a constant function.

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