Fuzzy Risk Analysis Based on Ochiai Ranking

Index with Hurwicz Criterion for Generalized

Trapezoidal Fuzzy Numbers

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Abstract

In this paper, we present a new method for ranking generalized trapezoidal fuzzy numbers (GTrFNs) based on Ochiai index and Hurwicz criterion. The proposed ranking method considers all types of decision makers’ perspective such as optimistic, neutral and pessimistic which is crucial in solving decision-making problems. The proposed method can discriminate the ranking result of GTrFNs having the same mode and symmetric spread. Two observations obtained from the proposed method are presented. Some numerical examples are also presented to illustrate the advantageous of the proposed method. Based on the proposed fuzzy ranking method, a fuzzy risk analysis (FRA) algorithm is presented to deal with FRA problems. The proposed method provides a useful way with strong discrimination ability in handling FRA problems.

Mathematics Subject Classification: 90C70

Keywords: Fuzzy risk analysis, Hurwicz criterion, Ochiai index, ranking fuzzy numbers

1 Introduction

Human often face the difficulty of lacking precise information in assessing the risk of a component made by a manufacturer in fuzzy environment. In order to deal with fuzzy risk analysis (FRA) problems, experts generally used fuzzy numbers to represent the evaluating values of the risk of each sub-component [6]. One of the main tasks in dealing with FRA is on the ranking of generalized trapezoidal fuzzy numbers (GTrFNs). [4] presented a method for FRA based on ranking of GTrFNs using the spreads and the defuzzified values of the GTrFNs. [14] proposed a ranking method based on different shapes and different deviation of GTrFNs in dealing with FRA problems. [5] proposed a method for FRA based on ranking of GTrFNs using different spreads, the defuzzified values and different height of the GTrFNs. [7] used the \( \alpha \)-cuts, the belief features and the signal/noise ratios for ranking the GTrFNs in handling the FRA. [6] then used the concepts of positive and negatives side of area and height for ranking GTrFNs in dealing with FRA. [8] further presented a ranking method based on the left and right height. Recently, [1] presented a geometric ranking method for handling with FRA problems.

The concept of ranking fuzzy numbers (RFNs) has been introduced by [13]. Since then, an enormous number of research papers reporting works on RFNs have appeared. [9] proposed a distance index based on the centroid concept and CV index for RFNs. However, in some situations, the ranking result by the distance index contradicts with the result by the CV index. Thus, to overcome the problems, [10] proposed an area between the centroid point and original point as
the ranking index. [4] then, found that Cheng’s [9] distance index and Chu and Tsao’s [10] method cannot rank correctly two fuzzy numbers having the same mode and symmetric spread. Furthermore, [3] introduced distance minimization concept for RFNs but their method cannot discriminate the ranking of embedded fuzzy numbers [12]. In other studies by [17], they proposed a ranking method based on deviation degree of the fuzzy numbers. However, the method cannot rank fuzzy numbers and images consistently and thus, [2] suggests a correction on the left and right deviation degree used in [17]. Furthermore, [11] has pointed out that [2] also has shortcoming in which the method does not able to rank fuzzy numbers in all situations correctly. [18] also found out that Asady’s [2] method neglected decision makers’ perspective and thus they proposed a ranking method based on epsilon-deviation degree and considers decision makers’ perspective.

In this paper, we present a new method for FRA based on the proposed new ranking method using the Ochiai similarity index and Hurwicz criterion. First, we present a new method for ranking GTrFNs. The proposed ranking method considers all types of decision makers’ perspective such as optimistic, neutral and pessimistic which is crucial in solving decision-making problems. The proposed fuzzy ranking method then is applied in handling with FRA problems.

The paper is organized as follows. Section 2 briefly reviews the preliminary concepts of GTrFNs. In Section 3, the new ranking method based on Ochiai index with Hurwicz criterion is presented. In Section 4, the ranking result of the proposed method is compared with the existing methods. In Section 5, we present a new FRA method based on the proposed ranking method. Lastly, the paper is concluded in Section 6.

2 Preliminaries

This section reviews some basic concepts of fuzzy numbers as follows.

The membership function of a fuzzy number $A$ is defined as,

$$
\mu_A(x) = \begin{cases}
\mu_L^L(x) & a \leq x \leq b \\
dc & b \leq x \leq c \\
\mu_R(x) & c \leq x \leq d \\
0 & \text{otherwise}
\end{cases}
$$

where $\mu_L^L : [a, b] \rightarrow [0, w], \ \mu_R^R : [c, d] \rightarrow [0, w], \ \mu_L^R \in (0, 1], \ \mu_R^L$ and $\mu_R^R$ denote the left and the right membership functions of the fuzzy number $A$.

The membership function $\mu_A$ of a fuzzy number $A$ has the following properties:
1. $\mu_A$ is a continuous mapping from the universe of discourse $X$ to $[0,w]$.

2. $\mu_A(x)=0$ for $x<a$ and $x>d$.

3. $\mu_A(x)$ is monotonic increasing in $[a,b]$.

4. $\mu_A(x)=w$ for $[b,c]$.

5. $\mu_A(x)$ is monotonic decreasing in $[c,d]$.

If the membership function $\mu_A(x)$ is a piecewise linear, then $A$ is called as a generalized trapezoidal fuzzy number with membership function defined as

$$
\mu_A(x) = \begin{cases} 
  w\left(\frac{x-a}{b-a}\right), & a \leq x \leq b \\
  w, & b \leq x \leq c \\
  w\left(\frac{d-x}{d-c}\right), & c \leq x \leq d \\
  0, & \text{otherwise}
\end{cases}
$$

and denoted as $A=(a,b,c,d;w)$. If $b=c$, then the trapezoidal becomes a triangular fuzzy number denoted as $A=(a,b,d;w)$.

### 3 A New Ranking Method based on Ochiai Index with Hurwicz Criterion

The new ranking method is developed based on Ochiai similarity measure index from [15] which is defined as,

$$
S_o(X,Y) = \frac{f(X \cap Y)}{\sqrt{f(X \cap Y) + f(X \cap \neg Y) + f(\neg X \cap Y) + f(\neg X \cap \neg Y)}}
$$

which is reduced to

$$
S_o(X,Y) = \frac{f(X \cap Y)}{\sqrt{f(X)f(Y)}},
$$

and known as Ochiai index [16].

Typically, the function $f$ is taken to be the cardinality function. The objects $X$ and $Y$ described by the features are replaced with fuzzy numbers $A$ and $B$ which are described by the membership functions. The fuzzy Ochiai is defined as,

$$
S_o(A,B) = \frac{|A \cap B|}{\sqrt{|A||B|}}.
$$
Fuzzy risk analysis based on Ochiai ranking index

where \( |A| \) denotes the cardinality of \( A \) and \( \cap \) is the t-norm. The procedure for fuzzy Ochiai ranking index with Hurwicz criterion is as follows:

**Step 1:** For each pair of the fuzzy numbers \( A_i \) and \( A_j \), find the fuzzy maximum and fuzzy minimum of \( A_i \) and \( A_j \).

**Step 2:** Calculate the evidences of \( E(A_i > A_j) \), \( E(A_j < A_i) \), \( E(A_i \sim A_j) \) and \( E(A_j > A_i) \) which are defined based on fuzzy Ochiai index as,

\[
E(A_i > A_j) = S_o \left( \text{MAX} \left( A_i, A_j \right), A_i \right), \quad E(A_j < A_i) = S_o \left( \text{MIN} \left( A_i, A_j \right), A_j \right),
\]

where \( S_o(A_i, A_j) = \frac{|A_i \cap A_j|}{\sqrt{|A_i| \sqrt{|A_j|}}} \) is the fuzzy Ochiai index and \( |A| \) denotes the scalar cardinality of fuzzy number \( A \). To simplify, \( C_{ij} \) and \( c_{\mu} \) are used to represent \( E(A_i > A_j) \) and \( E(A_j < A_i) \), respectively. Likewise, \( C_{ji} \) and \( c_{\mu} \) are used to denote \( E(A_j > A_i) \) and \( E(A_i < A_j) \) respectively.

**Step 3:** Calculate the total evidences \( E_{total}(A_i > A_j) \) and \( E_{total}(A_j > A_i) \) which are defined based on the Hurwicz criterion concept as

\[
E_{total}(A_i > A_j) = \beta C_{ij} + (1 - \beta) c_{\mu} \quad \text{and} \quad E_{total}(A_j > A_i) = \beta C_{ji} + (1 - \beta) c_{\mu}. \]

\( \beta \in [0, 0.5) \), \( \beta = 0.5 \) and \( \beta \in (0.5, 1] \) represent pessimistic, neutral and optimistic criteria respectively. To simplify, \( E_o(A_i, A_j) \) and \( E_o(A_j, A_i) \) are used to represent \( E_{total}(A_i > A_j) \) and \( E_{total}(A_j > A_i) \), respectively.

**Step 4:** For each pair of the fuzzy numbers, compare the total evidences in Step 3 which will result the ranking of the two fuzzy numbers \( A_i \) and \( A_j \) as follows:

i. \( A_i > A_j \) if and only if \( E_o(A_i, A_j) > E_o(A_j, A_i) \).

ii. \( A_i < A_j \) if and only if \( E_o(A_i, A_j) < E_o(A_j, A_i) \).

iii. \( A_i \approx A_j \) if and only if \( E_o(A_i, A_j) = E_o(A_j, A_i) \).

By simplification of the proposed ranking method with \( d_{ij} = C_{ij} - c_{\mu} + C_{ji} + c_{ij} \), \( n_{ij} = c_{ij} - c_{ji} \) and \( \beta_{ij} = \frac{c_{ij} - c_{ji}}{d_{ij}} \), this yields the following observations.
Observation 1

For two GTrFNs $A_i$ and $A_j$ with $d_{ij} \neq 0$, the ranking results for Ochiai index are as follows:

1. If $d_{ij} \neq 0$ and $\beta = \beta_{ij}$, then $A_i \approx A_j$.

2. If $d_{ij} > 0$ and
   a. $\beta > \beta_{ij}$, then $A_i > A_j$.
   b. $\beta < \beta_{ij}$, then $A_i < A_j$.

3. If $d_{ij} < 0$ and
   a. $\beta > \beta_{ij}$, then $A_i < A_j$.
   b. $\beta < \beta_{ij}$, then $A_i > A_j$.

Observation 2

For two GTrFNs $A_i$ and $A_j$ with $d_{ij} = 0$, the ranking results for Ochiai index are as follows:

1. If $n_{ij} > 0$, then, for all $\beta \in [0,1]$, $A_i < A_j$.

2. If $n_{ij} < 0$, then, for all $\beta \in [0,1]$, $A_i > A_j$.

3. If $n_{ij} = 0$, then, for all $\beta \in [0,1]$, $A_i = A_j$.

4 A Comparison of the Proposed Ranking Method with the Existing Methods

In this section, six sets of GTrFNs are adopted from [4] for comparison purposes with the existing ranking methods. The ranking results for different methods are shown in Tables 1 and 2.

Set 1: $A_1 = (0.1, 0.3, 0.5; 1)$, $A_2 = (0.3, 0.5, 0.7; 1)$,
Set 2: $A_1 = (0.1, 0.2, 0.4, 0.5; 1)$, $A_2 = (0.1, 0.3, 0.5; 1)$,
Set 3: $A_1 = (0.1, 0.3, 0.5; 1)$, $A_2 = (0.2, 0.3, 0.4; 1)$,
Set 4: $A_1 = (0.1, 0.3, 0.5; 0.8)$, $A_2 = (0.1, 0.3, 0.5; 1)$,
Set 5: $A_1 = (0.3, 0.5, 1; 1)$, $A_2 = (0.1, 0.6, 0.8; 1)$,
Set 6: $A_1 = (0, 0.4, 0.6, 0.8; 1)$, $A_2 = (0.2, 0.5, 0.9; 1)$, $A_3 = (0.1, 0.6, 0.7, 0.8; 1)$
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For Sets 1 and 5 (from Tables 1 and 2), the ranking results of the proposed method are consistent with all the existing methods. For Sets 2 and 3 with fuzzy numbers having the same mode and symmetric spread, [9], [10], [6] and [8] cannot discriminate them. [4], [5] and [1] produce $A_1 \prec A_2$. The ranking result of the proposed method is affected by decision makers’ perspective where pessimistic decision makers give $A_1 \prec A_2$, neutral decision makers $A_1 \approx A_2$ and optimistic $A_1 \succ A_2$. This shows that the proposed ranking method has strong discrimination ability. For Set 4, all the previous methods produce $A_1 \prec A_2$. The ranking result of the proposed method is similar with Sets 2 and 3 which is affected by decision makers’ perspective. For Set 6, the ranking result of the proposed method is consistent with all the existing methods except [4] and [1].

<table>
<thead>
<tr>
<th>Methods</th>
<th>Fuzzy Number</th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>[9]</td>
<td>$A_1, A_2$</td>
<td>0.5831, 0.7071</td>
<td>0.5831, 0.5831</td>
<td>0.5831, 0.5831</td>
</tr>
<tr>
<td>[10]</td>
<td>$A_1, A_2$</td>
<td>0.15, 0.25</td>
<td>0.15, 0.15</td>
<td>0.15, 0.15</td>
</tr>
<tr>
<td>[4]</td>
<td>$A_1, A_2$</td>
<td>0.4456, 0.4884</td>
<td>0.4239, 0.4456</td>
<td>0.4456, 0.4728</td>
</tr>
<tr>
<td>[5]</td>
<td>$A_1, A_2$</td>
<td>0.2579, 0.4298</td>
<td>0.2537, 0.2579</td>
<td>0.2579, 0.2774</td>
</tr>
<tr>
<td>[6]</td>
<td>$A_1, A_2$</td>
<td>0.3, 0.5</td>
<td>0.3, 0.3</td>
<td>0.3, 0.3</td>
</tr>
<tr>
<td>[8]</td>
<td>$A_1, A_2$</td>
<td>0.2553, 0.4444</td>
<td>0.2553, 0.2553</td>
<td>0.2553, 0.2553</td>
</tr>
<tr>
<td>[1]</td>
<td>$A_1, A_2$</td>
<td>0.2787, 0.4788</td>
<td>0.2622, 0.2787</td>
<td>0.2787, 0.2866</td>
</tr>
</tbody>
</table>

Proposed method $d_{ij}$, $n_{ij}$ or $\beta_{ij}$

<table>
<thead>
<tr>
<th></th>
<th>$d_{ij}$, $n_{ij}$ or $\beta_{ij}$</th>
<th>$A_1 \prec A_2$, $\beta \in [0, 0.5]$</th>
<th>$A_1 \prec A_2$, $\beta \in [0.5, 1]$</th>
<th>$A_1 \approx A_2$, $\beta \in [0, 0.5]$</th>
<th>$A_1 \approx A_2$, $\beta \in [0.5, 1]$</th>
<th>$A_1 \succ A_2$, $\beta \in [0, 0.5]$</th>
<th>$A_1 \succ A_2$, $\beta \in [0.5, 1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.75</td>
<td>0.038, 0.5</td>
<td>0.098, 0.5</td>
<td>0.038, 0.5</td>
<td>0.098, 0.5</td>
<td>0.038, 0.5</td>
<td>0.098, 0.5</td>
</tr>
</tbody>
</table>
Table 2: Comparison of Ranking Results for Sets 4, 5 and 6

<table>
<thead>
<tr>
<th>Methods</th>
<th>Set 4</th>
<th>Set 5</th>
<th>Set 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>[9]</td>
<td>0.461, 0.5831, A₁ ≤ A₂</td>
<td>0.7673, 0.7241, A₁ &gt; A₂</td>
<td>0.68, 0.7257, 0.7462, A₁ &gt; A₂ ≤ A₃</td>
</tr>
<tr>
<td>[10]</td>
<td>0.12, 0.15, A₁ ≤ A₂</td>
<td>0.2870, 0.2619, A₁ &gt; A₂</td>
<td>0.2281, 0.2624, 0.2784, A₁ &gt; A₂ ≤ A₃</td>
</tr>
<tr>
<td>[4]</td>
<td>0.3565, 0.4456, A₁ ≤ A₂</td>
<td>0.4128, 0.4005, A₁ &gt; A₂</td>
<td>0.3719, 0.4155, 0.3979, A₁ &gt; A₂ ≤ A₃</td>
</tr>
<tr>
<td>[5]</td>
<td>0.2063, 0.2579, A₁ ≤ A₂</td>
<td>0.4428, 0.4043, A₁ &gt; A₂</td>
<td>0.3354, 0.4079, 0.4196, A₁ &gt; A₂ ≤ A₃</td>
</tr>
<tr>
<td>[6]</td>
<td>0.2824, 0.3, A₁ ≤ A₂</td>
<td>0.5750, 0.5250, A₁ &gt; A₂</td>
<td>0.45, 0.525, 0.55, A₁ &gt; A₂ ≤ A₃</td>
</tr>
<tr>
<td>[8]</td>
<td>0.2462, 0.2553, A₁ ≤ A₂</td>
<td>0.5111, 0.4773, A₁ &gt; A₂</td>
<td>0.4, 0.4667, 0.5057, A₁ &gt; A₂ ≤ A₃</td>
</tr>
<tr>
<td>[1]</td>
<td>0.2250, 0.2787, A₁ ≤ A₂</td>
<td>0.5684, 0.4837, A₁ &gt; A₂</td>
<td>0.4013, 0.5063, 0.4947, A₁ &gt; A₂ ≤ A₃</td>
</tr>
<tr>
<td>Proposed method</td>
<td>(d_{i}=0.208), (\beta_{i}=0.5)</td>
<td>(d_{i}=0), (n_{i}=0.157)</td>
<td>(d_{i}=0.198), (\beta_{i}=8.97)</td>
</tr>
<tr>
<td></td>
<td>(A₁ ≤ A₂), (\beta \in [0.5, 1])</td>
<td>(A₁ &gt; A₂), (\beta \in [0.1])</td>
<td>(d_{i}=0), (n_{i}=0.28)</td>
</tr>
</tbody>
</table>

5 Fuzzy Risk Analysis based on the Proposed Ranking Method

In this section, the proposed ranking method is applied in dealing with FRA problem. Assume \(A₁, A₂, \ldots, A_n\) are \(n\) manufacturers with each manufacturer \(A_i\) produces component \(S_i\). The component \(S_i\) consists of \(p\) sub-components \(S_{i1}, S_{i2}, \ldots, S_{ip}\), where \(1 \leq i \leq n\). Two evaluating items \(\tilde{R}_{ik}\) and \(\tilde{W}_{ik}\) are used to evaluate each sub-component \(S_{ik}\), where \(\tilde{R}_{ik}\) denotes the probability of failure of the sub-component \(S_{ik}\) and \(\tilde{W}_{ik}\) denotes the severity of loss of the sub-component \(S_{ik}\) with \(1 \leq k \leq p\) and \(1 \leq i \leq n\).

The proposed FRA algorithm is presented as follows:

**Step 1:** Calculate the probability of failure \(\tilde{R}_{i}\) of each component \(S_i\) produced by manufacturer \(A_i\) by aggregating the evaluating values \(\tilde{R}_{ik}\) and \(\tilde{W}_{ik}\) as follows:
Fuzzy risk analysis based on Ochiai ranking index

\[
\tilde{R}_i = \frac{\sum_{k=1}^{p} \tilde{R}_{ik} \otimes \tilde{W}_{ik}}{\sum_{k=1}^{p} \tilde{W}_{ik}}
\]

where \( \tilde{R}_i = (r_{i1}, r_{i2}, r_{i3}, r_{i4}, \tilde{R}_i) \) and \( 1 \leq i \leq n \).

**Step 2:** For each pair of the fuzzy numbers \( \tilde{R}_i \) and \( \tilde{R}_j \), determine the evidences \( C_{ij} \), \( c_{ij} \), \( C_{ji} \) and \( c_{ji} \), where

\[
C_{ij} = S_O(MAX(\tilde{R}_i, \tilde{R}_j), \tilde{R}_i), \quad c_{ij} = S_O(MIN(\tilde{R}_i, \tilde{R}_j), \tilde{R}_i), \quad C_{ji} = S_O(MAX(\tilde{R}_i, \tilde{R}_j), \tilde{R}_j), \quad c_{ji} = S_O(MIN(\tilde{R}_i, \tilde{R}_j), \tilde{R}_j).
\]

Then, calculate the values of \( d_{ij} \), \( \beta_{ij} \) and \( n_{ij} \) where \( \beta_{ij} = \frac{c_{ij} - c_{ji}}{d_{ij}} \), \( d_{ij} = C_{ij} - C_{ji} + c_{ij} \) and \( n_{ij} = c_{ij} - c_{ji} \).

**Step 3:** Based on Observations 1 and 2, determine the ranking of \( \tilde{R}_i \) and \( \tilde{R}_j \).

To illustrate the FRA process of the proposed method, we use data from [5]. Assume \( A_1 \), \( A_2 \) and \( A_3 \) are three manufacturers which produce components \( S_1 \), \( S_2 \) and \( S_3 \), respectively. Component \( S_i \), produced by manufacturer \( A_i \), consists of three sub-components \( S_{i1} \), \( S_{i2} \) and \( S_{i3} \), where \( 1 \leq i \leq 3 \). Two evaluating items \( \tilde{R}_{ik} \) and \( \tilde{W}_{ik} \) are used to evaluate each sub-component \( S_{ik} \),

**Figure 1:** Structure of FRA [5]
where $\tilde{R}_{ik}$ denotes the probability of failure of the sub-component $S_{ik}$ and $\tilde{W}_{ik}$ denotes the severity of loss of the sub-component $S_{ik}$ with $1 \leq k \leq 3$. The structure of FRA is shown in Figure 1.

Table 3 shows a nine-member linguistic term which is used for representing the linguistic terms and their corresponding fuzzy numbers. Meanwhile Table 4 shows the linguistic values of evaluating items $\tilde{R}_{ik}$ and $\tilde{W}_{ik}$ of the sub-component $S_{ik}$ produced by manufacturer $A_i$. $\tilde{W}_{ik}$ denotes the degree of confidence of the decision makers’ perspective with respect to sub-components $S_{ik}$ produced by manufacturer $A_i$ for $1 \leq i \leq 3$ and $1 \leq k \leq 3$.

**Table 3: Linguistic Terms and Their Corresponding Fuzzy Numbers [5]**

<table>
<thead>
<tr>
<th>Linguistic Terms</th>
<th>FNs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolutely Low</td>
<td>(0.0, 0.0, 0.0, 0.0; 1.0)</td>
</tr>
<tr>
<td>Very Low</td>
<td>(0.0, 0.0, 0.02, 0.07; 1.0)</td>
</tr>
<tr>
<td>Low</td>
<td>(0.04, 0.1, 0.18, 0.23; 1.0)</td>
</tr>
<tr>
<td>Fairly Low</td>
<td>(0.17, 0.22, 0.36, 0.42; 1.0)</td>
</tr>
<tr>
<td>Medium</td>
<td>(0.32, 0.41, 0.58, 0.65; 1.0)</td>
</tr>
<tr>
<td>Fairly High</td>
<td>(0.58, 0.63, 0.80, 0.86; 1.0)</td>
</tr>
<tr>
<td>High</td>
<td>(0.72, 0.78, 0.92, 0.97; 1.0)</td>
</tr>
<tr>
<td>Very High</td>
<td>(0.93, 0.98, 1.0, 1.0; 1.0)</td>
</tr>
<tr>
<td>Absolutely High</td>
<td>(1.0, 1.0, 1.0, 1.0; 1.0)</td>
</tr>
</tbody>
</table>

**Table 4: Linguistic Evaluating Values of the Sub-components Produced by Manufacturers $A_1$, $A_2$ and $A_3$ [5]**

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Sub-components</th>
<th>Linguistic values of the severity of loss $\tilde{W}_{ik}$</th>
<th>Linguistic values of the probability of failure $\tilde{R}_{ik}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$S_{11}$</td>
<td>$\tilde{W}_{11} = $Low</td>
<td>$\tilde{R}<em>{11} = $Fairly Low $w</em>{R_{11}} = 0.9$</td>
</tr>
<tr>
<td></td>
<td>$S_{12}$</td>
<td>$\tilde{W}_{12} = $Fairly High</td>
<td>$\tilde{R}<em>{12} = $Medium $w</em>{R_{12}} = 0.7$</td>
</tr>
<tr>
<td></td>
<td>$S_{13}$</td>
<td>$\tilde{W}_{13} = $Very Low</td>
<td>$\tilde{R}<em>{13} = $Fairly High $w</em>{R_{13}} = 0.8$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$S_{21}$</td>
<td>$\tilde{W}_{21} = $Low</td>
<td>$\tilde{R}<em>{21} = $Very High $w</em>{R_{21}} = 0.85$</td>
</tr>
<tr>
<td></td>
<td>$S_{22}$</td>
<td>$\tilde{W}_{22} = $Fairly High</td>
<td>$\tilde{R}<em>{22} = $Fairly High $w</em>{R_{22}} = 0.9$</td>
</tr>
<tr>
<td></td>
<td>$S_{23}$</td>
<td>$\tilde{W}_{23} = $Very Low</td>
<td>$\tilde{R}<em>{23} = $Medium $w</em>{R_{23}} = 0.9$</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$S_{31}$</td>
<td>$\tilde{W}_{31} = $Low</td>
<td>$\tilde{R}<em>{31} = $Fairly Low $w</em>{R_{31}} = 0.95$</td>
</tr>
<tr>
<td></td>
<td>$S_{32}$</td>
<td>$\tilde{W}_{32} = $Fairly High</td>
<td>$\tilde{R}<em>{32} = $High $w</em>{R_{32}} = 0.8$</td>
</tr>
<tr>
<td></td>
<td>$S_{33}$</td>
<td>$\tilde{W}_{33} = $Very Low</td>
<td>$\tilde{R}<em>{33} = $Fairly High $w</em>{R_{33}} = 1.0$</td>
</tr>
</tbody>
</table>
In the following, the proposed method is applied in dealing with the FRA problems.

[Step 1] Based on Equation (1), the calculation for probability of failure $\tilde{R}_i$ of component $S_i$ produced by manufacturer $A_i$ is shown as follows:

$$
\tilde{R}_i = [\tilde{R}_{i1} \otimes \tilde{W}_{i1} \oplus \tilde{R}_{i2} \otimes \tilde{W}_{i2} \oplus \tilde{R}_{i3} \otimes \tilde{W}_{i3}] \ominus [\tilde{W}_{i1} \otimes \tilde{W}_{i2} \oplus \tilde{W}_{i3}]
$$

$$
= (0.1659, 0.2803, 0.7463, 1.1545; 0.7).
$$

In similar manner, the probabilities of failures $\tilde{R}_2$ and $\tilde{R}_3$ of each component $S_2$ and $S_3$, produced by manufacturers $A_2$ and $A_3$, respectively, are obtained as follows:

$\tilde{R}_2 = (0.3221, 0.4949, 1.1392, 1.6373; 0.85)$, and

$\tilde{R}_3 = (0.3659, 0.5134, 1.1189, 1.5984; 0.8)$.

[Step 2] Based on fuzzy numbers $\tilde{R}_1$, $\tilde{R}_2$ and $\tilde{R}_3$, and the proposed ranking method, the values for $d_{ij}$ and $\beta_{ij}$ are obtained as follows:

$$
(d_{12}, \beta_{12}) = (0.066, 6.157) \quad \text{and} \quad (d_{23}, \beta_{23}) = (-0.058, 0.486)
$$

and

$$
(d_{13}, \beta_{13}) = (0.094, 4.252).
$$

[Step 3] Based on Observation 1, the pairwise ranking is given as follows:

$$
\tilde{R}_1 \prec \tilde{R}_2, \beta \in [0,1], \quad \begin{cases}
\tilde{R}_2 \succ \tilde{R}_3, \beta \in [0,0.486] \\
\tilde{R}_2 = \tilde{R}_3, \beta = 0.486 \\
\tilde{R}_2 \prec \tilde{R}_3, \beta \in (0.486,1]
\end{cases}
$$

Thus, the ranking order of the probabilities of failure $\tilde{R}_1$, $\tilde{R}_2$ and $\tilde{R}_3$ is

$$
\tilde{R}_1 \prec \tilde{R}_2 \prec \tilde{R}_3, \beta \in [0,0.486]
$$

$\tilde{R}_1 \prec \tilde{R}_3 = \tilde{R}_2, \beta = 0.486$. Further, the ranking order of the risk of the manufacturers $A_1$, $A_2$ and $A_3$ is

$$
\begin{cases}
A_1 \prec A_3 \prec A_2, \beta \in [0,0.486] \\
A_1 \prec A_3 = A_2, \beta = 0.486 \\
A_1 \prec A_2 \prec A_3, \beta \in (0.486,1]
\end{cases}
$$

The ranking result is affected by decision makers’ perspective which shows that the proposed method has a strong discrimination ability. The pessimistic decision makers produce three different ranking results with $A_1 \prec A_3 \prec A_2$ for $\beta \in [0,0.486)$, $A_1 \prec A_3 = A_2$ for $\beta = 0.486$ and $A_1 \prec A_2 \prec A_3$ for $\beta \in (0.486,1]$. So, the ranking result is affected by decision makers’ perspective which shows that the proposed method has a strong discrimination ability.
\[ \beta \in (0.486, 0.5) \]. On the other hand, neutral and optimistic decision makers produce \( A_1 \prec A_2 \prec A_3 \). The ranking result by the pessimistic decision makers with \( A_3 \prec A_1 \prec A_2 \) is consistent with [4], [5], [14], [7], [6], [8] and [1].

6 Conclusion

This paper presents a new method for FRA based on the proposed new ranking method for GTrFNs using Ochiai index and Hurwicz criterion. The proposed method takes into consideration all types of decision makers’ perspective which is crucial in solving decision-making problems. Besides, it can discriminate the ranking result of GTrFNs having the same mode and symmetric spread which fails to be ranked by some of the existing methods. The proposed ranking method is also applied in a new FRA algorithm for handling the FRA problems. The proposed method provides a useful way with strong discrimination ability in handling the FRA problems.

References


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