The Problem of Selection of a Set of Partially Distinguishable Guards

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Abstract

In this paper we consider the problem of selection of a set of partially distinguishable guards. We describe an approach to solve the problem. This approach is based on an explicit reduction from the problem to the satisfiability problem.

Keywords: partially distinguishable guards, satisfiability, NP-complete

Visual landmarks problems has been extensively studied in robotics (see e.g. [1] – [3]). In particular, the following problem was proposed in [4].

Given a polygon $P$ and a finite set of candidate guard locations $N \subset P$, can one efficiently choose the guard set $S \subseteq N$ that minimizes the number of colors required?

A point $p \in P$ is visible from point $q \in P$ if the closed segment $[p, q]$ is a subset of $P$. Let $p \leftrightarrow q$ if and only if $p$ is visible from $q$. The visibility polygon $Vis(p)$ of a point $p \in P$ is defined as

$$Vis(p) = \{q \in P \mid p \leftrightarrow q\}.$$  

Since $N$ is a finite set, we can find $Vis(p)$, for any $p \in N$. Also, in view of finiteness of $N$, we can consider $P$ as a finite part of two-dimensional integer
grid. Therefore, we can consider the following decision version of the problem from [4].

**THE PROBLEM OF SELECTION OF A SET OF PARTIALLY DISTINGUISHABLE GUARDS (SG):**

**INSTANCE:** A grid graph \( P = (V, E) \), a finite subset \( N \) of the set of vertices of \( P \), \( Vis(p) \), for any \( p \in N \), and positive integer \( k \).

**QUESTION:** Are there a set \( S \subseteq N \) and function \( C : S \rightarrow \{1, \ldots, k\} \)

such that

\[ V = \cup_{p \in S} Vis(p) \]

and for any \( p, q \in S \), if \( C(p) = C(q) \), then \( q \notin Vis(p) \)?

Note that SG is \( \text{NP} \)-complete [4]. Encoding hard problems as instances of different variants of the satisfiability problem and solving them with very efficient satisfiability algorithms has caused considerable interest (see e.g. [5] – [18]). We consider an explicit reduction from SG to the satisfiability problem.

Let \( P = \{a_1, \ldots, a_{|P|}\} \), \( N = \{a_{t1}, \ldots, a_{t|N|}\} \),

\[
\varphi[1] = \land_{1 \leq i \leq |N|} \lor_{1 \leq j \leq k} x[i, j],
\]

\[
\varphi[2] = \land_{1 \leq i \leq |N|} \land_{1 \leq j[1] < j[2] \leq k} (\neg x[i, j[1]] \lor \neg x[i, j[2]]) ,
\]

\[
\varphi[3] = \land_{1 \leq i[1] < i[2] \leq |N|} (\neg w[i[1]] \lor \neg w[i[2]] \lor \neg x[i[1], j[1]] \lor \neg x[i[2], j[2]]) ,
\]

\[
\varphi[4] = \land_{1 \leq i \leq |V|} \lor_{1 \leq j \leq |N|} y[i, j],
\]

\[
\varphi[5] = \land_{1 \leq i \leq |V|} \land_{1 \leq j[1] < j[2] \leq |N|} (\neg y[i, j[1]] \lor \neg y[i, j[2]]),
\]

\[
\varphi[6] = \land_{1 \leq i \leq |V|}, (\neg y[i, j] \lor w[j]),
\]

\[
\xi = \land_{i=1}^{6} \varphi[i].
\]

It is easy to check that there are a set \( S \subseteq N \) and function \( C : S \rightarrow \{1, \ldots, k\} \) such that \( V = \cup_{p \in S} Vis(p) \) and for any \( p, q \in S \), if \( C(p) = C(q) \), then \( q \notin Vis(p) \) if and only if \( \xi \) is satisfiable. Clearly, \( \xi \) is a CNF. So, \( \xi \) gives us an explicit reduction from SG to SAT. Using standard transformations (see e.g. [19]) we can obtain an explicit transformation \( \xi \) into \( \zeta \) such that \( \xi \equiv \zeta \) and \( \zeta \) is a 3-CNF. It is easy to see that \( \zeta \) gives us an explicit reduction from SG to 3SAT.

We have designed a generator of natural instances for the problem SG. We consider our genetic algorithms OA[1] (see [20]), OA[2] (see [21]), OA[3] (see
Problem of selection

Table 1: Experimental results for SG.

<table>
<thead>
<tr>
<th></th>
<th>time</th>
<th>average</th>
<th>max</th>
<th>best</th>
</tr>
</thead>
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<tr>
<td>OA[1]</td>
<td>4.2 h</td>
<td>13.85 h</td>
<td>17.3 min</td>
<td></td>
</tr>
<tr>
<td>OA[2]</td>
<td>2.93 h</td>
<td>11.37 h</td>
<td>19.23 min</td>
<td></td>
</tr>
<tr>
<td>OA[3]</td>
<td>3.67 h</td>
<td>18.41 h</td>
<td>21.41 min</td>
<td></td>
</tr>
<tr>
<td>OA[4]</td>
<td>3.84 h</td>
<td>17.2 h</td>
<td>18.6 min</td>
<td></td>
</tr>
</tbody>
</table>

[22]), and OA[4] (see [23]) for SAT. We used heterogeneous cluster. Each test was runned on a cluster of at least 100 nodes. Selected experimental results are given in Table 1.

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References


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